CHAPTER 140

LOAD ANALYSIS FROM WAVE GROUPS

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At the present time sea wave is described by means of two theoretical models, the first is based on regular waves, components of which do not change in time and space, and the second model is based on irregular waves, components of which are randomly changed. The latter coincides to the greater extent with the rolling sea, but even this model does not characterize it to the full. Taking sea wave for a random process, the model of irregular waves does not take into account the sequence of their alternation. From the point of view of probability, on which the model of irregular waves is based, the maximum wave may be followed by the minimum wave, and the greater period may be followed by the smallest one. The real sea wave, especially in shore zone, where the main engineering constructions are placed, is characterized by clearly expressed group structure, which includes alternation of a number of great and small waves and the maximum wave is always followed by the wave having almost the same parameters. Moving through the pointed measuring section, and also depending on the wave generating condi-

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tions, the number of waves in the group is changed.

There is a tale about "the 9-th bar" - the greatest wave in the group consisting of nine waves, which coincides to a certain extent to the observations in situ. Accoroding to the linear potential theory, simple wave group may be obtained by means of addition of two sinusoidal waves with different but almost the same wave numbers and frequencies. In nature it is much more complex, but the nature of wave group structure is the same and is based on the existance of the several systems of quasi - regular waves in the field of the wind irregular waves which are characterised by their own frequencies, that the main energy of this system is concentrated on. These systems, interacting with each other, form groups of waves, and the spectrum of such wavering, when the peaks of the energy maxima of wave systems are widely distributed in their freguencies, is multymodal.

The main energy maximum of the spectrum has the least angular frequency (rad/sec), and the rest are placed in order of energy level decreasing according to increasing frequencies. The energy of spectrum components with the frequencies greater than the frequency of the third maximum, as a rule, is very small in general energy balance. The example of such energy spectrum of waves in the area Sheskharis (the Black sea) is shown in fig.1 (the angular freguency in rad/sec is presented on the X -axis and the spectrum density, normalized on the dispersion - sec, is on the Y-axis).[x]

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[x] The existance in the wind wave field several systems of waves, characterized by their own wave length (phase speed) is mentioned in the works by L.F.Titov, Newman, Phisical explanation of such wave field structure is presented in paper [1].

In this area the wave properties and also pressures on the breakwater of vertical type were measured. Waves near the vertical wall were characterezed by narrow spectrum of directions and thus, data on preassure measured in situ should be reffered to the wave interaction when they frontally approached the wall. The waves parameters were measured outside of the interference zone and also just near the wall. The pressure was registered on seven subsurface levels. The number of waves in one oscillogram was about one hundred. The storms with maximum wave height from 1,0 to 2,5 m were registered. The loads acting on the wall were defined according to the pressure values. The object was to compare the values measured in situ with the values calculated by theoretical formula. With this aim the maximum wave from one hundred registed waves (1% provision in the wave system) was chosen in the oscillogram. The calculations were made by the following formula:

$$\frac{p}{\gamma} = Z - \eta(t) - hn \sin \sigma t [ch\, \varphi - ch\, a\, (H-Z)] + \\ + a h^2 m\, [ch\, 2\varphi - ch\, 2a\, (H-Z)] \qquad (1)^{x}$$

where $\frac{p}{\gamma}$ - pressure in the point defined by the coordinate Z ;

h - the incident wave heigth;

$\eta(t)$ - position of the free water surface at the wall in respect to the calm water level;

$\sigma = \frac{2\pi}{\tau}$ - angular freguency;

$a = \frac{2\pi}{\lambda}$ - wave number;

τ - wave period;

λ - wave length;

H - water depth near the wall.

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x The results of calculation according to the formula (1) coincide with the calculations made with the help of charts, given in the Construction Codes of the USSR (SNiP 11-57-75).

$\alpha = ah$

$\beta = aH$

t - time

$\varphi = a(H - \eta)$

$$n = \frac{1 + 2\alpha \sin \delta t}{ch\,\beta \cdot \ell^{(\varphi - \beta)}} \quad ; \quad m = \frac{\cos^2 \delta t}{2\,sh\,2\beta \cdot \ell^{2(\varphi - \beta)}}$$

The intercepting point of the coordinates is situated on the calm water level and the axis 0-Z is directed downward.

The formula (1) was preliminarily checked in the laboratory conditions on the regular wave and provided the coincidence with the experimental data within 10%.

Comparison of calculations made with the help of formula (1) with the data obtained in situ showed that the calculated load values were 30% greater than measured values (in some cases - 50%). In connection with that the assumption was made that there is some influence of group wave structure on pressure values and the mathematical model of wavering process near the vertical wall was as:

$$\eta(t) = \sum_{1}^{K} h_i \sin \delta_{io} \left[1 + f(t)\right] t \qquad (2)$$

where K - the number of wave systems (within 3) which is defined according to the number of characteristic energy maxima of frequency spectrum;

$\delta_{i,o}$ - the main frequency of system;

h_i - equivalent wave height of the system.

Wave heights h_i were derived from the assumption that maximum wave height according to the given realization (h_{max}) is the result of addition of equivalent system wave heights and also according to the fact that the energy is proportional to the square wave height. For the three wave systems the equations of connection will be following:

$$h_{max} = h_1 + h_2 + h_3 \tag{3}$$

$$\frac{h_1}{h_2} = \sqrt{\frac{S(\sigma_{1,0})}{S(\sigma_{2,0})}}; \quad \frac{h_1}{h_3} = \sqrt{\frac{S(\sigma_{1,0})}{S(\sigma_{3,0})}}, \quad \frac{h_2}{h_3} = \sqrt{\frac{S(\sigma_{2,0})}{S(\sigma_{3,0})}}, \tag{4}$$

$S(\sigma_{1,0})$, $S(\sigma_{2,0})$, $S(\sigma_{3,0})$ – the values of spectrum density maxima at frequencies $\sigma_{1,0}$ $\sigma_{2,0}$ $\sigma_{3,0}$ (fig. 1).

With the help of the in situ occillograms it was shown, that it is possible to select a periodical function:

$$f_i(t) = A \cos Bt \tag{5}$$

with which on definite t (the interval of order 100 waves), the function $\eta(t)$, computer calculated according to (2), has a group structure, and its statistical and spectral characteristics are close to the natural irregular process. In the expression (5) B – the frequency of wave groups appeareance – rad/sec (under suggestion of A. I. Kuznetsov).

Euler's equation was used to determine the pressure (on axis OZ).

$$\frac{\partial V_z}{\partial t} + \frac{\partial V_z}{\partial z} V_z = g - \frac{1}{\rho} \cdot \frac{\partial P}{\partial z} \tag{6}$$

In the course of its integration the value of vertical speed component according to the linear wave theory was written down as follows:

$$V_z = -\sum_1^3 \frac{\sigma_i h_i \, sh\, a_i (H-z)}{sh\, \beta_i} \cos \sigma_i t \tag{7}$$

As a result of integration (6) the formula for pressure value in the point of ordinate Z (with 3 components of wave field) was obtained:

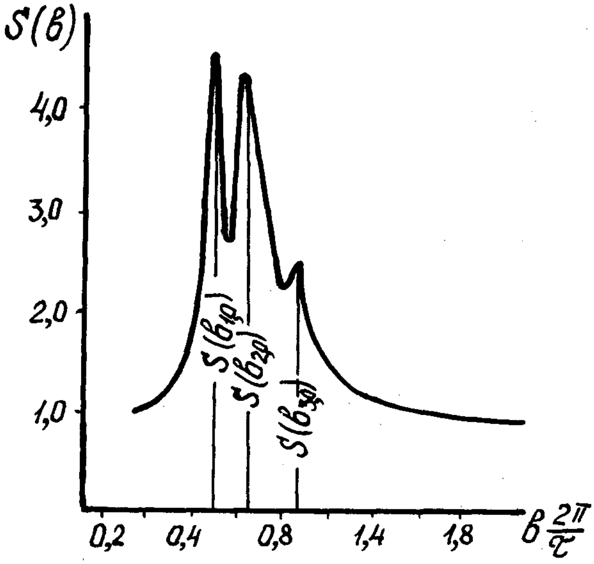

Fig.1. Example of wave oscillations spectrum in the area of Sheskharis

$$\frac{p}{\gamma} = z + \eta(t) - \sum_1^3 h_i \cdot n_i \cdot \sin \sigma_i t \left[ch\, \varphi_1 - ch\, a_i (H - z)\right] +$$

$$+ \sum_1^3 a_i h_i^2 m_i \left[ch\, 2\varphi_i - ch\, 2 a_i (H - z)\right] -$$

$$-2h_1 h_2 \sqrt{\frac{a_1 \cdot a_2}{sh\, 2\beta_1 \cdot sh\, 2\beta_2}} \left[sh\, \varphi_1 \cdot sh\, \varphi_2 - sh\, a_1 (H-z) \cdot sh\, a_2 (H-z)\right] \cdot \cos \sigma_1 t \cdot \cos \sigma_2 t$$

$$-2h_1 h_3 \sqrt{\frac{a_1 \cdot a_3}{sh\, 2\beta_1 \cdot sh\, 2\beta_3}} \left[sh\, \varphi_1 \cdot sh\, \varphi_3 - sh\, a_1 (H-z) \cdot sh\, a_3 (H-z)\right] \cos \sigma_1 t \cdot \cos \sigma_3 t$$

$$-2h_2 h_3 \sqrt{\frac{a_2 \cdot a_3}{sh\, 2\beta_2 \cdot sh\, 2\beta_3}} \left[sh\, \varphi_2\, sh\, \varphi_3 - sh\, a_2 (H-z) \cdot sh\, a_3 (H-z)\right] \cos \sigma_2 t \cdot \cos \sigma_3 t \qquad (8$$

Notations in (8) are similar to those of (1).

By means of integrating (8) in the limits from $\eta(t)$ to H the load on the wall was determined.

All calculations were performed with the help of a computer.

As could be seen from the formula (8), besides members, which characterize each wave system separatly, the pressure value is influenced also by cross members, which reflect the interaction of wave field components (the last three members of formula (8). This expresses the influence of the group wave structure on the value of wave pressure. When $h_2 = h_3 = 0$ and $A = 0$ we have a system of regular waves and formula (8) is transformed into (1). The influence of the group wave structure on the pressure and loads values was checked according to the data obtained in situ in the course of several storms. Here are the results for the case of the greatest deviation of in situ data from calculated with the help of formula (1) at the maximum wave of 1% provision. Wave actions were characterized by the average wave height $h = 0,75$ m, and the wave heigth of 1% provision $h_{1\%} = 1,8$ m. The spectral density function of the wavering had three suffisiently spaced maxima (fig. 2-a), from which those situated at high frequencies, were smoothed at the load spectrum, and especially at the wave pressure spectrum near the bottom at point with coordinate 11,2 m (fig. 2-b, 2-c). The mathematical model of the process was described by means of three wave systems with equivalent heights $h_1 = 0,78$, $h_2 = 0,54$, $h_3 = 0,48$ m and with frequencies respectively $b_{1,0} = 0,65$, $b_{2,0} = 1,10$, $b_{3,0} = 1,47$, parameter $A = 0,03$ and group frequency $B = 0,1$. Fig. 3 shows one of the most characteristic areas of in situ oscillogram of wave and wave pressures. The model of this recording together with the chronogram of the loads calculated by means of a computer according to formula (8) are presented at fig. 3-b and 3-c.

It could be seen from the comparison that the recording model somewhat lacks high freguency oscillations, but the group structure character in model and in situ are similar. In the course of comparison it should be kept in mind that the scale factors for models and for experiments performed in situ are quite different. The functions of spectrum density obtained by means of calculations illustrate the coincidence of energy distribution in accordance with frequencies in situ and in model (see fig.2 - dashed line). Finaly, for the given example there is a comparison of wave pressure diagrams from the wave of 1% provision obtained in situ and also calculated considering (formula 8) and without consideration (formula 1) of the group wave structure, fig. 4. This comparison shows that pressure and load calculations without considering the group structure according to the theory of regular waves provide sufficient and sometimes excessive margin. Consideration of group structure of waves in the analitical description of the process makes it possible to draw closer in situ and computed data. But formula (8) could not be considered as a formula for

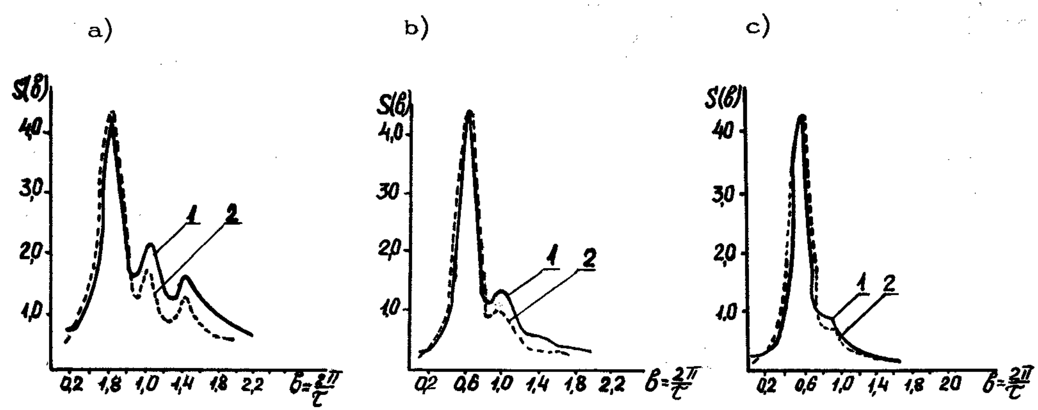

Fig. 2 In situ and calculated spectra:
a) wave oscillations;
b) total loads;
c) pressures (near the bottom at point 11,2 m)
1 - in situ spectrum
2 - calculated spectrum

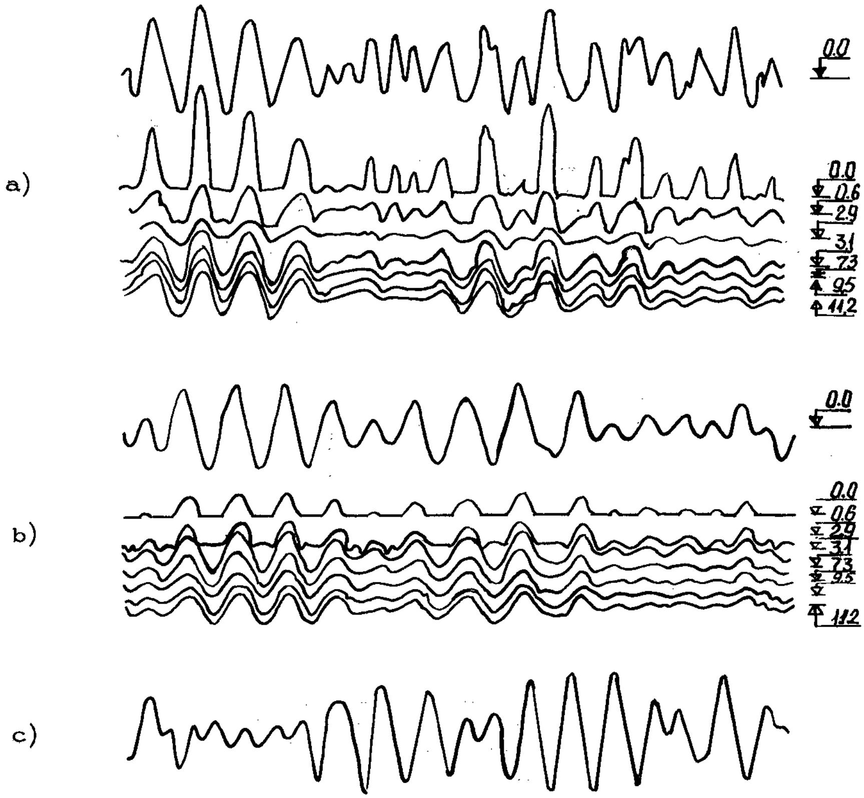

Fig. 3. Comparison between in situ and calculated realizations:
a) oscillographic recording in situ;
b) realization of wave oscillations and pressures obtained by means of calculation according to formula (8);
c) calculated load chronograph trace.

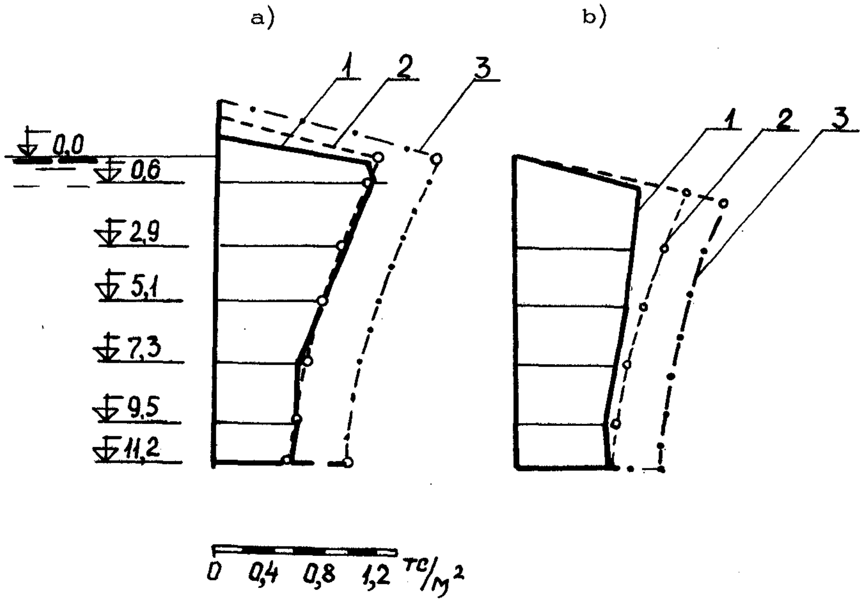

Fig. 4. Wave pressures epures:
a) in the phase of wave crest;
b) in the phase of wave through.
1 - measurings done in situ;
2 - calculation according to formula (8);
3 - calculation according to formula (1).

calculations, but only as a proof of the influence of wave group structure on the pressure and load values.

At present time according to the results of modelling wave actions on the constructions in wave tank with irregular waves, there is some information that hydrodynamic pressure and reactions in the elements of constructions (for pile structures) under the action of waves with group structure according to the intensity of action, depending on the number of waves in the group, take the intermediate position between regular and purely random waves.

The regular waves produce maximum pressure and reaction values and the irregular-minimum [2] . Above mentioned consideration of group structure draws the calculations quite close to the data obtained in situ that is why modelling of wave action on the constructions with regard to actions of only irregular waves without considering their group structure, could lead to the too low results.

Consideration of group structure in the course of wave loads calculation and wave modelling needs further investigation.

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