CHAPTER 133

NON-CONSERVATIVE WAVE INTERACTION WITH FIXED SEMI-IMMERSED RECTANGULAR STRUCTURES

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Introduction and Scope

Previous attempts to analytically describe wave reflection and transmission at surface penetrating structures have neglected losses due to flow expansion, contraction, and skin drag along the structure boundaries (Black and Mei, 1970; Ijima, et al., 1972). The model described in this study includes these effects and allows for the inclusion of a dissipative medium such as rubble or closely spaced piles in the region beneath the structure.

The problem of a fixed, two-dimensional structure in a train of monochromatic incident waves is modeled, as shown in Figure 1. The solution allows for 1) variable structure length and draft, 2) different depths in the regions fore, aft, and beneath the structure, 3) variable wave amplitude and period, and 4) turbulent and inertial damping in the region beneath the structure. An equivalent work technique is applied to linearize the damping beneath the structure, yielding a potential flow problem in all three regions. Amplitudes for the resulting series of eigenfunctions in each region are determined by matching pressure and horizontal mass flux at the region interfaces, orthogonalizing these expressions over the depth, and simultaneously solving the resulting equations to yield complex reflection and transmission coefficients. Complex horizontal and vertical force coefficients for the structure are also determined from the integrated Bernoulli equation.

The solution technique is computationally efficient. In general, five modes in the eigen series provide satisfactory convergence for the various hydrodynamic parameters. Approximately six-tenths of a computer system second are required to solve for a single wave-structure condition. The results compare favorably with variational methods used by others.

The effect of skin friction, expansion, and contraction losses tend to reduce both reflection and transmission coefficients by only a few percentage points over a wide range of wave frequencies. The addition

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Figure 1. Definition Sketch

of a dissipative medium beneath the structure, however, causes considerable reduction in wave transmission at all wave frequencies while increasing reflection at low frequencies and reducing reflection at high frequencies. Increased inertial damping and decreased porosity in the medium below the structure uniformly decrease transmission and increase reflection.

Comparison with experimental data and other theories is made. New experimental data are presented. Design curves for various structure and wave parameters are presented and discussed.

Theory

Equations of Motion

Newton's Law states that the vector summation of forces acting on a fluid parcel is equal to the resultant vector acceleration of that parcel. The significant forces affecting free surface phenomena may be summarized as

In equation form this statement becomes

| <u>d</u> | $\frac{q}{t} = -\frac{1}{\rho}$ | ⊽(p + | γz) - $\beta_1 q q - \beta_2 q - \beta_3 \frac{\partial q}{\partial t}$ | (1) |
|----------|---------------------------------|-------|--|-----|
| where | q | = | vector velocity | |
| | ρ | = | mass density of fluid | |
| | ∇ | = | gradient operator | |
| | р | = | pressure | |
| | γ | = | specific weight of fluid = ρg | |
| | g | = | acceleration due to gravity | |
| | β1, β2, β2 | | resistance coefficients | |

Convective accelerations may be ignored for small amplitude wave motions, thereby reducing the substantial acceleration to the local acceleration or

$$\frac{\partial \mathbf{q}}{\partial \mathbf{t}} = \frac{\mathbf{d}\mathbf{q}}{\partial \mathbf{t}}$$

Combining the local acceleration with the inertial damping term in Eq. (1) yields

$$(1+\beta_3) \frac{\partial q}{\partial t} = -\frac{1}{\rho} \nabla (p + \gamma z) - \beta_1 q |q| - \beta_2 q \qquad (2)$$

Now let

 $(1+\beta_3) = S \tag{3}$

where S is an inertial coefficient which includes the effects of local accelerations and additional accelerations caused by local obstructions such as rubble, piles, abrupt corners on the structure, etc.

The laminar and turbulent friction terms are replaced by a single linear friction term which dissipates the same amount of energy over one wave period as the actual friction terms. This simplification permits an analytical solution to the problem without perturbing the equations of motion yet retains a non-linear dependence on wave amplitude. Then

 $-\beta_1 q |q| - \beta_2 q \text{ is replaced by } -f \omega q \tag{4}$

where
$$f =$$
 dimensionless friction coefficient
 $\omega =$ wave angular frequency (renders f dimensionless)

An additional condition is required to evaluate f since Eq. (4) is not satisfied by a simple equality. This condition is referred to as Lorentz's Condition of Equivalent Work, and it requires that both friction laws dissipate the same amount of energy over the region of interest during one wave cycle. In equation form this reads

$$f \omega \int_{\text{volume}} dV \int_{\text{period}} dt (q \cdot q) = \beta_1 \int_{\text{volume}} dV \int_{\text{period}} dt (q \cdot q |q|) + \beta_2 \int_{\text{volume}} dV \int_{\text{period}} dt (q \cdot q)$$
(5)

Then, on the average, the two friction laws are equivalent. Combining Eqs. (2), (3) and (4) yields

$$S \frac{\partial q}{\partial t} = -\frac{1}{\rho} \nabla(p + \gamma z) - f \omega q$$
(6)

A periodic fluid motion is sought for monochromatic waves, hence, the velocity time dependence becomes

$$q(x,z,t) = q(x,z) \exp(-i\omega t)$$

and

 $\frac{\partial q}{\partial t} = -i\omega q \tag{7}$

Substituting Eq. (7) into Eq. (6) and combining terms

$$\omega(f-iS)q = -\frac{1}{\rho} \nabla(p + \gamma z)$$
(8)

The curl of the right hand side of Eq. (8) is identically equal to zero, therefore

 $\nabla \mathbf{x} \mathbf{q} = \mathbf{0}$

and since

 $\nabla \mathbf{x} (\nabla \text{ anything}) = 0$

it is permissible to replace the vector velocity \boldsymbol{q} with the scalar velocity potential, $\boldsymbol{\phi},$ according to

$$\mathbf{q} = -\nabla \phi \tag{9}$$

Introducing Eq. (9) into Eq. (8) and combining terms

$$\nabla \left[-\omega (f-iS)\phi + \frac{1}{\rho} (p + \gamma z) \right] = 0$$
 (10)

The gradient of the bracketed term is equal to zero, therefore the term cannot be a function of spatial location. Requiring that the water surface displacement integrate to zero over one wave length further constrains the bracketed term, establishing that it must equal zero. Therefore, the gradient operator may be removed from Eq. (10), yielding the Bernoulli Equation, which may be solved for the pressure field.

$$\frac{\mathbf{p}}{\mathbf{o}} = -\mathbf{g}\mathbf{z} + \omega(\mathbf{f} - \mathbf{i}\mathbf{S})\phi \tag{11}$$

Note that Eq. (11) reduces to the linear wave theory Bernoulli Equation if no damping occurs (f = 0, S = 1.0).

Water is essentially incompressible in free surface flows. Consequently, conservation of mass reduces to the continuity equation which may be written as

$$\nabla^* \mathbf{q} = \mathbf{0} \tag{12}$$

Combining Eqs. (9) and (12) yields Laplace's Equation for irrotational, incompressible flow.

$$\nabla^2 \phi = 0 \tag{13}$$

Equations (11) and (13) are the appropriate equations of motion for the general problem of non-conservative, irrotational, incompressible flow. The velocity field is specified at all times, in all space by Eq. (13). Substitution of the solution to Eq. (13) into Eq. (11) prescribes the pressure field. In order to solve Eq. (11), however, boundary conditions are required to specify the integration constants.

Boundary Conditions

Laplace's Equation is a second order homogenous differential equation requiring two boundary conditions to specify the general solution. A third boundary condition is required to reference the pressure in Bernoulli's Equation. The three boundary conditions are determined by the physical restrictions imposed at the flow field boundaries.

Referring to Figure 1, the bottom boundary condition requires that the vertical velocity component vanish at an impermeable horizontal boundary. Thus

$$w = -\frac{\partial \phi}{\partial z} = 0$$
 at $z = -h_1, -h_2$ and $-h_3$ (14)

Similarly, under the bottom of the structure

$$w = -\frac{\partial \phi}{\partial z} = 0 \text{ at } z = -d \tag{15}$$

At the free surface, $z = \eta$, the pressure must be equal to zero. Hence,

$$p = 0 \text{ at } z = \eta \tag{16}$$

Also, the surface must rise and fall at a rate equal to the vertical velocity to maintain continuity at the free surface. Hence

$$\frac{\mathrm{d}n}{\mathrm{d}t} = w = -\frac{\partial\phi}{\partial z} \text{ at } z = n \tag{17}$$

Small amplitude wave motions produce negligible convective changes in η and permit an evaluation of w at the still water level to avoid transcendental functions of η . These simplifications reduce Eq. (17) to

$$\frac{\partial n}{\partial t} = -\frac{\partial \phi}{\partial z} \text{ at } z = 0$$
(18)

Combining Eqs. (11), (16) and (18) yields the combined kinematic and dynamic free surface boundary condition.

$$\frac{\partial \phi}{\partial z} = -\frac{w}{g} (f - iS)\phi$$

In regions 1 and 3 of Figure 1, f = 0 and S = 1.0 so that

$$\frac{\partial \phi}{\partial z} = \frac{i\phi}{g} \text{ at } z = 0 \tag{19}$$

Boundary Value Problem

The boundary value problem for each of three regions fore, aft and beneath the structure is summarized in Figure 2. Each boundary value problem is prescribed by Laplace's Equation and the surface and bottom boundary conditions. Flow field boundaries are parallel to the coordinate axes, consequently, variable separation techniques may be used to solve Laplace's Equation. The boundary conditions are applied to evaluate the integration constants. The resulting solutions are presented in detail by Steimer (1977) and are summarized below. The incident wave is described by a single progressive wave in Eq. (20). The reflected and transmitted waves include a single progressive mode each and an infinite series of evanescent modes, as identified in Eqs. (21) and (22). Beneath the breakwater, the solution yields a single wave component corresponding to the progressive mode, Eq. (23), and two infinite series of evanescent modes, one decaying left to right, Eq. (24), and the other decaying right to left, Eq. (25).

$$\phi_{i} = \frac{ig}{\omega} A_{i} \exp \left[k_{11}(x+b) - \omega t \right] - \frac{\cosh k_{11}(z+h_{1})}{\cosh k_{11}h_{1}}$$
(20)



Figure 2. Boundary Value Problem

$$\phi_{\mathbf{r}} = \sum_{N=1}^{\infty} \frac{\mathbf{i}g}{\omega} A_{\mathbf{r}N} \exp\left(-\mathbf{i}[K_{1N}(\mathbf{x}+\mathbf{b})+\omega\mathbf{t}]\right) \frac{\cosh K_{1N}(\mathbf{z}+\mathbf{h}_{1})}{\cosh K_{1N}\mathbf{h}_{1}}$$
(21)

$$\phi_{t} = \sum_{N=1}^{\infty} \frac{ig}{\omega} A_{tN} \exp\left(i[K_{3N}(x-b)-\omega t]\right) - \frac{\cosh K_{3N}(z+h_{3})}{\cosh K_{3N}h_{3}}$$
(22)

$$\phi_{0} = \frac{ig}{\omega} \left(A_{a1} \frac{x}{b} + A_{b1} \right) \exp(-i\omega t)$$
(23)

$$\phi_{a} = \sum_{N=2}^{\infty} \frac{ig}{\omega(-1)^{N-1}} A_{aN} \exp[-K_{2N}(x+b) - i\omega t] \cos K_{2N}(z+h_{2})$$
(24)

$$\phi_{b} \approx \sum_{N=2}^{\infty} \frac{ig}{\omega(-1)^{N-1}} A_{bN} \exp[K_{2N}(x-b) - i\omega t] \cos K_{2N}(z+h_{2})$$
(25)

where: A_{1j} represent wave amplitudes in regions 1 and 3 and pressure head amplitudes in region 2; and K_{1j} represent separation constants or eigenvalue wave numbers in each region. The first subscript identifies the region, while the second subscript identifies the modal number.

The wave numbers are solved from dispersion equations in each region. The dispersion equations result from Eq. (19) in regions 1 and 3 and a combination of Eqs. (14) and (15) in region 2. The appropriate dispersion equation in regions 1 and 3 is

$$\omega^{2} = g K_{ij} \tanh (K_{ij}h_{i})$$
(26)

The K_{i1} wave numbers are the real roots of Eq. (26) and the K_{iN} wave numbers with N \geq 2 are the imaginary roots to Eq. (26). The appropriate dispersion equation in region 2 is

$$K_{2N} = \frac{(N-1)\pi}{(h_2 - d)} \text{ for } N \ge 2$$
(27)

Summarizing, the velocity potential in region 1 is

$$\phi_1 = \phi_i + \phi_r \tag{28}$$

The velocity potential in region 2 is

m

2

$$\phi_2 = \phi_0 + \phi_a + \phi_b \tag{29}$$

The velocity potential in region 3 is

$$\phi_3 = \phi_t \tag{30}$$

Orthogonalized Interfacial Boundary Conditions

Only the incident wave amplitude in Eq. (20) is known. The remaining amplitudes are unknowns and must satisfy pressure and mass flux

continuity at the interfaces between regions. Pressure continuity requires that the pressure field solutions in each region provide identical results at common boundaries between regions. Thus

$$p_1 = p_2 \quad \text{at } x = -b$$
 (31)

$$\mathbf{p}_2 = \mathbf{p}_3 \quad \text{at } \mathbf{x} = +\mathbf{b} \tag{32}$$

Referring to Eq. (11) and recognizing that f = 0, S = 1.0 in regions 1 and 3, Eqs. (31) and (32) become

$$\phi_1 = (S + if) \phi_2 \quad \text{at } x = -b \tag{33}$$

$$\phi_3 = (S + if) \phi_2 \quad \text{at } x = +b \tag{34}$$

Mass flux continuity requires that mass be conserved as flow proceeds from one region to another. Recognizing that region 2 may be occupied by a porous medium, the velocities within the pore spaces must increase inversely proportional to the porosity to maintain mass flux continuity at the interface. Thus, if ε is the porosity in region 2, the velocities normal to the interfaces will be related according to

$$\frac{\partial \varphi_1}{\partial x} = \varepsilon \frac{\partial \varphi_2}{\partial x} \quad \text{at } x = -b$$
 (35)

and

34

34

$$\frac{\partial \psi_3}{\partial x} = \varepsilon \frac{\partial \psi_2}{\partial x}$$
 at $x = +b$ (36)

Note that mass flux continuity reduces to velocity continuity if no porous medium exists.

Each interfacial boundary condition includes an infinite series of terms. The terms within the series may be separated to generate 4N equations to solve 4N unknown amplitudes by utilizing the orthogonal behavior of the z dependent separable functions. The boundary value problems in Figure 2 are all linear, homogenous, second order differential equations with linear, homogenous boundary conditions. Accordingly, they are properly posed Sturm-Liouville problems with orthogonal solutions having the useful property that products of two modal solutions, integrated between boundaries having homogenous boundary conditions, vanish unless the modes are identical (Hildebrand, 1965). Thus

$$\int_{0}^{M_{1}} \phi_{iM} \phi_{iN} = 0 \text{ unless } M \approx N$$
(37)

Equation (37) applies equally well to derivatives of ϕ .

This behavior is utilized by multiplying Eqs. (33) and (34) by ϕ_2 and integrating from $z = -h_2$ to z = -d. Similarly, Eq. (35) is multiplied by $\frac{\partial \phi_1}{\partial x}$ and integrated from $z = -h_1$ to z = 0; and Eq. (36) is multiplied by $\frac{\partial \phi_3}{\partial x}$ and integrated from $z = -h_3$ to z = 0.

Note that
$$\frac{\partial \phi_1}{\partial x}$$
 and $\frac{\partial \phi_3}{\partial x}$ equal zero above $z = -d$ and below $z = -h_2$.
Evaluating these integrals provides the six equations listed below.

$$\sum_{N=1}^{\infty} C_{rN} \left[\frac{K_{11} \cosh K_{11}h_{1}}{K_{1N} \cosh K_{1N}h_{1}} \right] \left[\frac{\sinh K_{1N}(h_{1}-d)-\sinh K_{1N}(h_{1}-h_{2})}{\sinh K_{11}(h_{1}-d)-\sinh K_{11}(h_{1}-h_{2})} \right] + (C_{a1}-C_{b1}) \left[\frac{(S+if)(h_{2}-d) K_{11} \cosh K_{11}h_{1}}{\sinh K_{11}(h_{1}-d)-\sinh K_{11}(h_{1}-h_{2})} \right] = -1 \quad (38)$$

$$\sum_{N=1}^{\infty} C_{rN} \left[\frac{\cosh K_{11}h_{1}}{\cosh K_{1N}h_{1}} \right] \frac{K_{1N}}{K_{11}} \left[\frac{(-1)^{(M-1)}\sinh K_{1N}(h_{1}-d)-\sinh K_{1N}(h_{1}-h_{2})}{(-1)^{(M-1)}\sinh K_{11}(h_{1}-d)-\sinh K_{11}(h_{1}-h_{2})} \right] \left[\frac{K_{11}^{2}+K_{2M}^{2}}{K_{1N}^{2}+K_{2M}^{2}} \right] - [C_{aM}+C_{bM}\exp(-2K_{2M}b)] \left[\frac{(h_{2}-d)(S+if)(\cosh K_{11}h_{1})(K_{11}^{2}+K_{2M}^{2})}{2K_{11}[\sinh K_{11}(h_{1}-d)+(-1)^{M}\sinh K_{11}(h_{1}-h_{2})]} \right] = -1 \quad (39)$$

$$\sum_{N=1}^{\infty} C_{tN} \left[\frac{K_{11} \cosh K_{11}h_1}{K_{3N} \cosh K_{3N}h_3} \right] \left[\frac{\sinh K_{3N}(h_3-d) - \sinh K_{3N}(h_3-h_2)}{\sinh K_{11}(h_1-d) - \sinh K_{11}(h_1-h_2)} \right] - (C_{a1}+C_{b1}) \\ \left[\frac{(S+if)(h_2-d)(K_{11} \cosh K_{11}h_1)}{\sinh K_{11}(h_1-d) - \sinh K_{11}(h_1-h_2)} \right] = 0$$
(40)
$$\left[\cosh K_{11}h_1 \right] K_{2N} \left[(-1)^{(M-1)} \sinh K_{2N}(h_2-d) - \sinh K_{2N}(h_2-h_2) \right] \left[K_{11}^2 + K_{2N}^2 \right] \right]$$

$$\sum_{N=1}^{\infty} C_{tN} \left[\frac{\cosh K_{11}h_{1}}{\cosh K_{3N}h_{3}} \right] \frac{K_{3N}}{K_{11}} \left[\frac{(-1)^{(M-1)}\sinh K_{3N}(h_{3}-d)-\sinh K_{3N}(h_{3}-h_{2})}{(-1)^{(M-1)}\sinh K_{11}(h_{1}-d)-\sinh K_{11}(h_{1}-h_{2})} \right] \left[\frac{K_{11}^{2}+K_{2M}^{2}}{K_{3N}^{2}+K_{2M}^{2}} \right] - [C_{aM} \exp(-2K_{2M}b)+C_{bM}] \left[\frac{(h_{2}-d)(S+if)(\cosh K_{11}h_{1})(K_{11}^{2}+K_{2M}^{2})}{2K_{11}(\sinh K_{11}(h_{1}-d)+(-1)^{M}\sinh K_{11}(h_{1}-h_{2}))} \right] = 0$$
for $M \ge 2$

$$(41)$$

$$C_{\rm rM} \left[\frac{\cosh K_{11}h_1}{\cosh K_{1M}h_1} \right] \left[\frac{2K_{1M}h_1 + \sinh 2K_{1M}h_1}{2K_{11}h_1 + \sinh 2K_{11}h_1} \right] + C_{\rm al} \left[\frac{-4i\varepsilon \cosh K_{11}h_1[\sinh K_{1M}(h_1-d)-\sinh K_{1M}(h_1-h_2)]}{b K_{1M}(2K_{11}h_1 + \sinh 2K_{11}h_1)} \right] + \sum_{N=2}^{\infty} \left[-C_{\rm aN} + C_{\rm bN} \exp(-2K_{2N}b) \right] \left[\frac{K_{2N}K_{1M}}{K_{1M}^2 + K_{2N}^2} \right] \left[\frac{-4i\varepsilon \cosh K_{11}h_1[\sinh K_{1M}(h_1-d)+(-1)^N \sinh K_{1M}(h_1-h_2)]}{2K_{11}h_1 + \sinh 2K_{11}h_1} \right] = \delta_{\rm M1} \quad (42)$$

$$-C_{tM} \left[\frac{\cosh K_{11}h_1}{\cosh K_{3M}h_3} \right] \left[\frac{2K_{3M}h_3 + \sinh 2K_{3M}h_3}{2K_{11}h_1 + \sinh 2K_{11}h_1} \right] + C_{a1} \left[\frac{-4i\epsilon \cosh K_{11}h_1[\sinh K_{3M}(h_3^{-d}) - \sinh K_{3M}(h_3^{-h_2})]}{b K_{3M}(2K_{11}h_1 + \sinh 2K_{11}h_1)} \right] + \sum_{N=2}^{\infty} \left[-C_{aN} \exp(-2K_{2N}b) + C_{bN} \right] \left[\frac{K_{2N}K_{3M}}{K_{3M}^2 + K_{2N}^2} \right] \left[\frac{-4i\epsilon \cosh K_{11}h_1[\sinh K_{3M}(h_3^{-d}) + (-1)^N \sinh K_{3M}(h_3^{-h_2})]}{2K_{11}h_1 + \sinh 2K_{11}h_1} \right] = 0 \quad (43)$$

Six equations result, rather than four, because orthogonalizing pressure continuity with respect to the inner modes generates two additional unique equations, one at each interface, for the propagating mode beneath the structure. Each equation is summed on N but is repeated for each Mth eigenvalue. Equations (38) and (40) apply to the M=1 mode beneath the structure while Eqs. (39) and (41) complete the pressure continuity requirements for $M \ge 2$. Equations (42) and (43) are statements of mass flux continuity, orthogonalized with respect to the outer modes of regions 1 and 3, and apply to all $M \ge 1$. Thus 4M equations are produced to solve for 4M unknown amplitudes: M reflected waves, M transmitted, and M waves in both the +x and -x directions under the structure.

In Eqs. (38) through (43), the complex amplitudes A_{ij} have been rendered dimensionless by division with the incidental wave amplitude A_i and are denoted as C_{ij} . The Kronecker delta appears as δ_{M1} . The system of 4M equations becomes finite by establishing a finite upper limit for the N summation. The amplitude coefficients, C_{ij} , become smaller as j increases and experience with this set of equations, for most practical structural configurations, has demonstrated that summing on five modes is sufficient. Errors due to this finite summation are on the order of a few percent or less. Up to 20 modes have been utilized for unusual configurations such as thin plates.

The 4M system of linear algebraic, complex equations is solved via the IBM Scientific Subroutine SIMQ, modified to accept complex coefficients. Amplitudes and phases for each modal amplitude are determined by calculating the modulus and argument, respectively, of each C_{ij} . The reflection and transmission coefficients for the propagating modes are simply $|C_{r1}|$ and $|C_{t1}|$, respectively. Approximately six-tenths of a computer system second are required to solve for a single wavestructure condition described by a five mode series.

Theoretical Behavior

Predicted wave response, quantified as reflection and transmission coefficients, is presented for a hypothetical structure as a function of

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dimensionless wave number in Fig. 3-a, b and c. The results show that shorter waves experience more reflection and less transmission at the structure. In addition, as the resistance beneath the structure is increased via an increase in the friction coefficient, f, inertial coefficient, S, or decrease in porosity, ε , reflection is increased, attenuation is increased and transmission is decreased. Figure 3-d demonstrates that increased frictional resistance beneath the structure leads to increased energy dissipation. However, an upper limit for energy dissipation is reached at 1/2 the available incident wave energy. Further increases in frictional resistance simply cause 50 percent of the wave energy to be reflected rather than completely absorbed and dissipated beneath the structure.

The results of the present analysis compare favorably with those predicted by other investigators utilizing alternative solution techniques. Sample comparisons are presented in Figs. 4 and 5. Reflection and transmission coefficients results are compared in Fig. 4-a, b, c and d for various structural configurations. The variational technique of Black (1970) in Figs. 4-a, c and d displays essentially identical results to the present theory with negligible resistance (f=0, S=1.0, ε =1.0). John's shallow water dock theory (Ippen, 1966) also concurs. The matched asymptotic expansion theory of Tuck (1971) for a narrow slit in a thin, infinitely deep plate also agrees well with the present theory if finite values are used for the depth and plate thickness. Even the long wave behavior is reproduced well, contrasting the variational results in Fig. 4-b.

The horizontal and vertical forces induced by waves acting on the structure are evaluated by integrating the pressure distribution on vertical and horizontal structure surfaces, respectively. The vertical dynamic wave force component is

$$F_{VD} = \int_{-b}^{+b} p dx = \rho \omega (f-iS) \int_{-b}^{+b} \phi_2 dx$$

Substituting for ϕ_2 and evaluating the integral yields

$$F_{VD} = \gamma(S+if) \exp(-i\omega t) \left(2b A_{b1} + \sum_{N=2}^{\infty} \left[\frac{1-\exp(-2K_{2N}b)}{K_{2N}} \right] (A_{aN}+A_{bN}) \right)$$
(44)

The horizontal dynamic wave force component is the difference in forces on the two sides of the structure.

$$F_{H} = \int_{0}^{-d} p_{1}dz - \int_{0}^{-d} p_{3}dz = \int_{0}^{-d} \rho i\phi_{1}(\theta x = -b)dz + \int_{0}^{-d} \rho i\phi_{3}(\theta x = b)dz$$

Substituting for $\phi^{}_1$ and $\phi^{}_3$ and evaluating the integrals yields







(b)



Figure 5. Predicted Wave Forces Compared to Other Theories (a) Horizontal Component, (b) Vertical Component

$$F_{H} = \gamma \exp(-i\omega t) \left[\frac{A_{i} [\sinh K_{11}(h_{1}-d)-\sinh K_{11}h_{1}]}{K_{11} \cosh K_{11}h_{1}} + \sum_{N=1}^{\infty} \frac{A_{rN} [\sinh K_{1N}(h_{1}-d)-\sinh K_{1N}h_{1}]}{K_{1N} \cosh K_{1N}h_{1}} - \sum_{N=1}^{\infty} \frac{A_{tN} [\sinh K_{3N}(h_{3}-d)-\sinh K_{3N}h_{3}]}{K_{3N} \cosh K_{3N}h_{3}} \right]$$
(45)

Dimensionless force components from Eqs. (44) and (45) have been graphed in Fig. 5-a and b. Results taken from Black (1970) include comparisons with the work of Garrison and Haskind. The comparisons are quite favorable, with only slight differences for very thin plates.

Experimental Results

A small scale experimental program was conducted at Oregon State University to supplement the available data on wave reflection and transmission at rectangular structures. The experiment apparatus is described in Fig. 6-a and the geometric configurations tested are identified in Fig. 6-b. Each model was exposed to a variety of wave amplitudes and frequencies. Incident and reflected waves were resolved with a traversing wave gauge from measurements of the partial standing wave envelope. Transmitted wave measurements were acquired with a stationary wave gauge.

Transmission coefficient results are displayed in Fig. 7. Four combinations of draft, depth and step height are shown with results expressed relative to dimensionless wave numbers. Theoretical results are expressed for frictionless conditions. In general the agreement between experiment and theory is quite good. In Figs. 7-a, b and c, the theory tends to slightly overpredict the transmission coefficient. A non-zero value for the friction coefficient would tend to improve this correlation, indicating that real fluid effects may be modifying the experimental results. Figure 7-d shows the theory underpredicting transmission slightly for long waves. This model configuration has zero draft so that the long wave trough passes under the model without surface contact. The theory does not allow for the separation of the water and model surfaces, a condition which is unlikely to occur in the prototype.

Experimental and theoretical reflection coefficients are presented in Fig. 8-a and b. In general the theory tends to overpredict the measured reflection coefficients. Again, this correlation can be improved by utilizing a non-zero value for the friction coefficient, f, to account for real fluid effects. Small increases in f tend to decrease both the reflection and transmission coefficient as indicated in Figs. 3-a and 8-c. The frictional condition displayed in Fig. 8-c evaluates energy loss due to skin drag over the structure surface and expansion/contraction losses at the abrupt corners of the structure. The corrections indicated improve the correlation with the experimental data, however, larger losses need to be identified to further correct the predicted reflection



TOP VIEW

(a)



(b)

| RUN # | b(in.) | S(in.) | h(in.) | d(in.) |
|-------|--------|--------|--------|--------|
| 1 | 4 | 0 | 18 | 81/8 |
| 2 | 4 | 0 | 18 | 6 |
| 3 | 4 | 0 | 18 | 23/4 |
| 4 | 4 | 81/4 | 18 | 2 3/4 |
| 5 | 4 | 81/4 | 18 | 51/2 |
| 6 | 4 | 81/4 | 18 | 0 |
| 7 | 4 | 0 | . 18 | 0 |

Figure 6. Experimental Program (a) Apparatus, (b) Model Configurations

efficients (a) Run #1, 2, 3, (b) Run #4, 5, 6, (c) Predicted Wave Response With and Without Friction, (d) Predicted Design

Curves

coefficients. Evidently the steady flow relationships for skin friction and expansion/contraction losses are not adequate for addressing the same condition instantaneously in unsteady flows. A more complete discussion of friction coefficient evaluation is presented by Steimer (1977).

Design curves for various drafts and step heights have been prepared for the frictionless condition. A sample curve is presented in Fig. 8-d.

Conclusions

The theoretical analyses presented in this study correlates well with other rigorous analytical procedures and with available experimental data. The theory can accommodate a variety of rectangular structural configurations and it provides a rational method for incorporating real fluid losses. The solution technique is numerically efficient in providing reflection, transmission and force coefficients. Additional experimental work is required to validate force predictions and to suggest alternative friction laws to evaluate real fluid effects.

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