CHAPTER 129

WAVE TRANSMISSION THROUGH TRAPEZOIDAL BREAKWATERS

by

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INTRODUCTION

In a previous paper Madsen and White (1977) developed an approximate method for the determination of reflection and transmission characteristics of multi-layered, porous rubble-mound breakwaters of trapezoidal cross-section. This approximate method was based on the assumption that the energy dissipation associated with the wave-structure interaction could be considered as two separate mechanisms: (1) an external, frictional dissipation on the seaward slope; (2) an internal dissipation within the porous structure. The external dissipation on the seaward slope was evaluated from the semi-theoretical analysis of energy dissipation on rough, impermeable slopes developed by Madsen and White (1975). The remaining wave energy was represented by an equivalent wave incident on a hydraulically equivalent porous breakwater of rectangular cross-section. The partitioning of the remaining wave energy among reflected, transmitted and internally dissipated energy was evaluated as described by Madsen (1974), leading to a determination of the reflection and transmission coefficients of the structure.

The advantage of this previous approximate method was its ease of use. Input data requirements were limited to quantities which would either be known (water depth, wave characteristics, breakwater geometry, and stone sizes) or could be estimated (porosity) by the design engineer. This feature was achieved by the employment of empirical relationships for the parameterization of the external and internal energy dissipation mechanisms. General solutions were presented in graphical form so that calculations could proceed using no more sophisticated equipment than a hand calculator (or a slide rule). This simple method gave estimates of transmission coefficients in excellent agreement with laboratory measurements whereas its ability to predict reflection coefficients left a lot to be desired.

Accepting the assumptions of the validity of linear long wave

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theory, nonbreaking incident waves and the validity of the empirical relationships for the dissipative mechanisms the major assumption made in the previous approximate analysis is the separate treatment of external and internal dissipation. The present paper presents a more rigorous analysis of the wave-structure interaction in that external and internal dissipation are treated together rather than separately. Results obtained from this more elaborate analysis are compared and with results obtained from the previous simple procedure and with laboratory measurements of reflection and transmission characteristics of homogeneous, porous breakwaters of trapezoidal cross-section.

ANALYSIS

Governing Equations

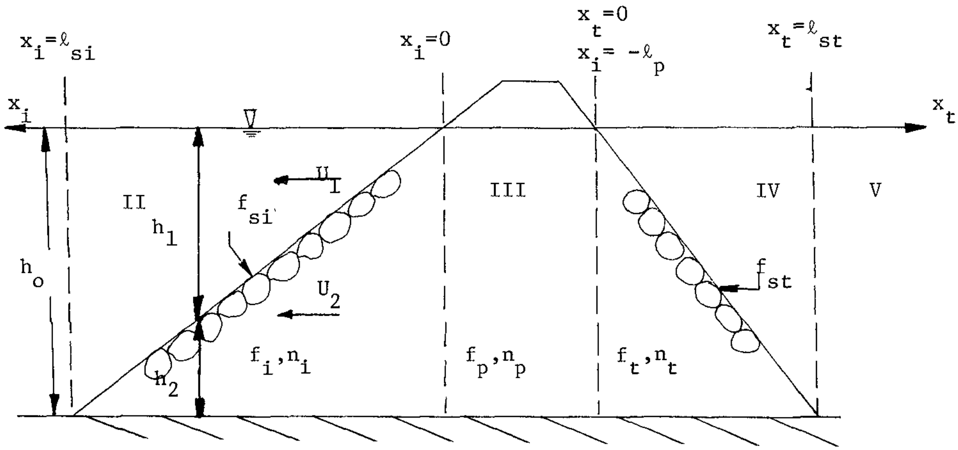

Figure 1: Definition Sketch

The analysis considers a homogeneous, trapezoidal breakwater subject to normally incident waves. The incident waves are assumed to be non-breaking and adequately described by linear long wave theory. Five different computational regions are identified in Figure 1. In regions I and V which are of constant depth, h_o, the general solutions for the wave motion is well known and consists of an incident and a reflected wave (Region I) and a transmitted wave (Region V). In Region III the general solution, after linearization of the flow resistance in the porous material, is given by Madsen (1974). Thus, we need only be concerned about the governing equations and the general solution for Regions II and IV, i.e., the slope regions.

For these regions the continuity equation reads

$$\frac{\partial \eta}{\partial t} + \frac{\partial (h_1 U_1)}{\partial x} + \frac{\partial (h_2 U_2)}{\partial x} = 0 \qquad (1)$$

where h_1, h_2 and U_1, U_2 refer to the depths and discharge velocities in the water and porous wedges, respectively; and η is the free surface elevation.

The linearized momentum equation for the fluid wedge reads

$$\frac{\partial U_1}{\partial t} + g\frac{\partial \eta}{\partial x} + \frac{\tau_b}{\rho h_1} = 0 \tag{2}$$

in which g is gravity, ρ is fluid density, and τ_b, the bottom shear stress ,is expressed in its nonlinear and linearized form as

$$\tau_b = \frac{1}{2}\rho f_w |U_1| U_1 = \rho f_s h_1 \omega U_1 \tag{3}$$

where f_w may be taken from the empirical expression developed by Madsen and White (1975), f_s is a linearized friction factor, and ω is the radian frequency of the incident waves. Substitution of (3) into (2) yields the linear momentum equation for the fluid wedge

$$\frac{\partial U_1}{\partial t} + g\frac{\partial \eta}{\partial x} + f_s \omega U_1 = 0 \tag{4}$$

The linearized momentum equation for the porous wedge is given by

$$\frac{1}{n}\frac{\partial U_2}{\partial t} + g\frac{\partial \eta}{\partial x} + f\frac{\omega}{n}U_2 = 0 \tag{5}$$

in which n is porosity and the flow resistance has been linearized by taking

$$\delta F = \rho(\alpha + \beta|U_2|)U_2 = \rho f \frac{\omega}{n} U_2 \tag{6}$$

Since the governing equations, (1), (4) and (5) are linear it is expedient to use complex variables. Thus, for a periodic motion of radian frequency ω we express the variables as the real part of the complex expressions given by

$$(\eta, U_1, U_2) = (\zeta(x), u_1(x), u_2(x))e^{i\omega t} \tag{7}$$

with $i = \sqrt{-1}$.

Substituting (7) into (4) yields

$$u_1 = \frac{ig}{\omega}\frac{1}{1-if_s}\frac{d\zeta}{dx} \tag{8}$$

whereas (5) reduces to

$$u_2 = \frac{ig}{\omega}\frac{n}{1-if}\frac{d\zeta}{dx} \tag{9}$$

which expresses the fluid velocities in terms of the slope of the free surface.

With (7), (8) and (9), equation (1) may be written

$$\frac{\omega^2}{gh_o}(1-if_s)\zeta + \frac{d}{dx}\left[\frac{h_1}{h_o}\frac{d\zeta}{dx}\right] + \frac{n(1-if_s)}{1-if}\frac{d}{dx}\left[\frac{h_2}{h_o}\frac{d\zeta}{dx}\right] = 0 \qquad (10)$$

Realizing that $h_1 + h_2 = h_o$ and therefore $h_1/h_o = x/\ell_s$ and $h_2/h_o = 1 - x/\ell_s$ for $0 \le x \le \ell_s$, and introducing the notation

$$\gamma = \frac{n(1-if_s)}{1-if} \qquad (11)$$

we may write (10) in the form

$$\frac{\omega^2}{gh_o}(1-if_s)\zeta + \frac{d}{dx}\left[\{\gamma + (1-\gamma)\frac{x}{\ell_s}\}\frac{d\zeta}{dx}\right] = 0 \qquad (12)$$

Here $\omega^2/(gh_o) = k_o^2$ is the long wave dispersion relationship for waves in constant depth, h_o. Introducing this notation and changing variable to

$$Z = \gamma + (1-\gamma)\frac{x}{\ell_s} \qquad (13)$$

we finally obtain the simple governing equation for the surface wave amplitude

$$\frac{d}{dZ}\left[Z\frac{d\zeta}{dZ}\right] + \frac{(k_o\ell_s)^2(1-if_s)}{(1-\gamma)^2}\zeta = 0 \qquad (14)$$

This equation reduces, as it should, to that of Madsen and White (1975) for the porosity $n \to 0$ or for the internal friction factor $f \to \infty$, both of which are limits resulting in an effectively impermeable slope.

General Solution

Equation (14) is of the Bessel type (Hildebrand, 1965) and its general solution may be expressed as

$$\zeta = A\,J_o(\text{Arg}) + B\,Y_o(\text{Arg}) \qquad (15)$$

where Arg in the original variables is given by

$$\mathrm{Arg} = \frac{2k_o\ell_s}{1-\gamma}\sqrt{1-if_s}\sqrt{\gamma + (1-\gamma)x/\ell_s} \tag{16}$$

From the property of Bessel Functions we further obtain the derivative

$$\frac{d\zeta}{dx} = -\frac{k_o\sqrt{1-if_s}}{\sqrt{\gamma + (1-\gamma)x/\ell_s}}\left[A\ J_1(\mathrm{Arg}) + B\ Y_1(\mathrm{Arg})\right] \tag{17}$$

where Arg again is given by (16).

The fluid velocities are obtained from (8) and (9) as

$$u_1 = -i\sqrt{\frac{g}{h_o}}\ \frac{1}{\sqrt{1-if_s}}\ \frac{1}{\sqrt{\gamma + (1-\gamma)x/\ell_s}}\left[A\ J_1(\mathrm{Arg}) + B\ Y_1(\mathrm{Arg})\right] \tag{18}$$

for the water wedge, and

$$u_2 = -i\sqrt{\frac{g}{h_o}}\ \frac{n\sqrt{1-if_s}}{1-if}\ \frac{1}{\sqrt{\gamma + (1-\gamma)x/\ell_s}}\left[A\ J_1(\mathrm{Arg}) + B\ Y_1(\mathrm{Arg})\right] \tag{19}$$

for the porous wedge.

From the general solutions presented above the solutions for the seaward and landward slopes are obtained by taking $x=x_i$ and $x=x_t$, respectively, and the appropriate ℓ_s, f_s, and n values (see Figure 1).

For Region III which is entirely porous the governing equations consist of the momentum equation which in its linearized form is identical to (5) and the continuity equation which reads

$$n\frac{\partial \eta}{\partial t} + h_o\frac{\partial U}{\partial x} = 0 \tag{20}$$

As previously mentioned the general solution to equations (5) and (20) was given by Madsen (1974) and in complex form it may be expressed as

$$\zeta_{III} = A_{pi}e^{ikx_i} + B_{pr}e^{-ik(x_i + \ell_p)} \tag{21}$$

where

$$k = \frac{\omega}{\sqrt{gh_o}}\sqrt{1-if_p} = k_o\sqrt{1-if_p} \tag{22}$$

and

$$u_{III} = -\sqrt{\frac{g}{h_o}}\ \frac{n_p}{\sqrt{1-if_p}}\ [A_{pi}e^{ikx_i} - a_{pr}e^{-ik(x_i + \ell_p)}] \tag{23}$$

Outside the structure the general solution is in Region I

$$\zeta_I = a_i e^{ik_o x_i} + a_r e^{-ik_o x_i} \tag{24}$$

$$u_I = -\sqrt{\frac{g}{h_o}}\ [a_i e^{ik_o x_i} - a_r e^{-ik_o x_i}] \tag{25}$$

where a_i is the incident wave amplitude and a_r is the complex amplitude of the reflected wave.

For Region V the motion consists of only the transmitted wave of complex amplitude a_t, i.e.,

$$\zeta_V = a_t e^{-ik_o x_t} \tag{26}$$

$$u_V = \sqrt{\frac{g}{h_o}}\ a_t e^{-ik_o x_t} \tag{27}$$

Matching Conditions

Assume for the moment that all linearized friction factors, porosities and incident wave characteristics are specified. The general solutions developed in the preceeding section then contain 8 unknown constants of which the magnitudes of a_r, the reflected wave amplitude, and a_t, the transmitted wave amplitude, of course, are of primary interest in the present context. To determine the constants the general solutions for the surface elevation, ζ, and the horizontal discharge velocity, u, are matched at their common boundaries, i.e., at $x_i = \ell_{si}$, 0, $-\ell_p$ $(x_t = 0)$; and at $x_t = 0$ $(x_i = -\ell_p)$ and ℓ_{st}. This yields 8 equations from which the 8 unknowns may be determined.

It is of special interest to note that the solution for the velocity in the water wedge, i.e., u_1, is matched to the solution outside the structure at $x_i = \ell_{si}$ and $x_t = \ell_{st}$, whereas the velocity in the porous wedge, i.e., u_2, is matched to the solution inside the structure, u_p, at x_i and $x_t = 0$. It is also noted that the value of the argument of the Bessel Functions in the general slope solution remain finite at x = 0 and x = ℓ_s since

$$\text{Arg} \quad \begin{matrix} 1 & \text{for } x = \ell_s \\ \sqrt{\gamma} & \text{for } x = 0 \end{matrix} \tag{28}$$

Evaluation of Linearized Friction Factors

Once the solution for given values of the linearized friction factors has been obtained Lorentz principle is invoked in order to obtain a new estimate for the friction factors. For the external dissipation Lorentz' principle takes the form of the integral

$$\frac{1}{T}\int_0^T dt \int_0^{\ell_s} \tau_b U_1 dx \tag{29}$$

being independent of whether τ_b is expressed by its nonlinear or linear form given by (3). Since f_w in (3) is taken from Madsen and White (1975) this leads to an equation for f_s. The integration is carried out on a high speed computer.

For the internal dissipation Lorentz' principle is expressed by the invariance of the integral

$$\frac{1}{T}\int_0^T dt \int_{\forall} (\delta F) U_2 \delta\forall \tag{30}$$

with frictional law used to express δF (equation 6). The integral in (30) is evaluated numerically over the appropriate volume, $\forall$, by high speed computer. Since the nonlinear friction law in (6) contains two parameters, α and β, these must be specified before (3) yields a determination of the linearized friction factor f. The empirical expressions

$$\begin{aligned} \alpha &= \alpha_o \frac{(1-n)^3}{n^2} \frac{\nu}{d^2} \\ \beta &= \beta_o \frac{(1-n)}{n^3} \frac{1}{d} \end{aligned} \tag{31}$$

in which d is the stone diameter, ν is the kinematic viscosity of the fluid. The constants α_o and β_o vary with the characteristics of the stones with a range of $800 < \alpha < 1500$ and $1.8 < \beta_o < 3.6$. Since the stones used in the present experiments were highly angular (crushed quarry stones) values of $\alpha_o = 1500$ and $\beta_o = 3.6$ were used in (31).

With updated values for all friction factors the governing equations are again solved and renewed application of Lorentz' principle yield new values for the friction factors. From experience a final solution is obtained after 3 to 5 iterations.

COMPARISON WITH EXPERIMENTS

A set of 54 experiments on the reflection and transmission characteristics of homogeneous trapezoidal breakwaters were carried out in a 24 m long wave flume in the R.M. Parsons Lab. at MIT. The water depth was 30 cm and three wave periods (1.6, 1.8 and 2.0 sec.) were used. For each period 6 different values for the incident wave height were run on the structure having front slopes of 1 on 2.5, 2.0 and 1.5 with the rear slope remaining constant at 1 on 1.5. The stone material was crushed quarry stones with an equivalent diameter of d = 1.5 cm. Porosity was found to be 0.4.

The experiments were run continuously and incident reflected and transmitted wave amplitudes were determined by the accurate procedure developed and described by Madsen and White (1975). The structure was located approximately 12 m from the wave generator and reflection from the end of the wave flume was minimized by a 1 on 6 sloping absorber beach covered by horsehair.

Experimental results for reflection and transmission coefficients are shown in Figures 2a, b, c for a wave period of 2.0 sec. and front slopes of 1 on 1.5, 2 and 2.5, respectively. The experimental results are compared with predictions by the "new" method described in this paper (indicated by subscript N) as well as with predictions of the "old" method (indicated by subscript O) developed by Madsen and White (1977). From Figures 2a,b,c it is evident that the "new" as well as the "old" methods are quite successful in their prediction of transmission coefficients. It is, however, painfully obvious that both methods are quite weak in their ability to yield accurate predictions of the reflection coefficient. In fact, the "old" method yields high estimates of reflection coefficients whereas the "new" method yields low values. The discrepancy between the predicted and observed reflection coefficients is seen to increase with decreasing front slope.

A possible explanation for this discrepancy could be associated with an artificial discontinuity introduced in the analysis. Referring back to equation (1) it is seen that the storage term in the continuity equation is taken to be $\partial\eta/\partial t$ for the slope regions, i.e., storage is in the form of pure water on the slopes even in the areas where the water wedge pinches out. At the same time immediately inside the structure the storage term in the continuity equation, equation (20), indicates that storage takes place within the porous medium. This discontinuity was removed artificially by evaluating the slope solution at a value of $x = H_i/\tan\delta$, where δ is the slope, rather than at $x = 0$. It was found that this change had a relatively minor effect on the predicted transmission coefficients whereas the effect on predicted reflection coefficients was substantial. There was, however, no clear trend in the results obtained in this manner. Sometimes the reflection coefficient prediction improved, sometimes it got worse.

It is worthwhile to note that the experimental results for the

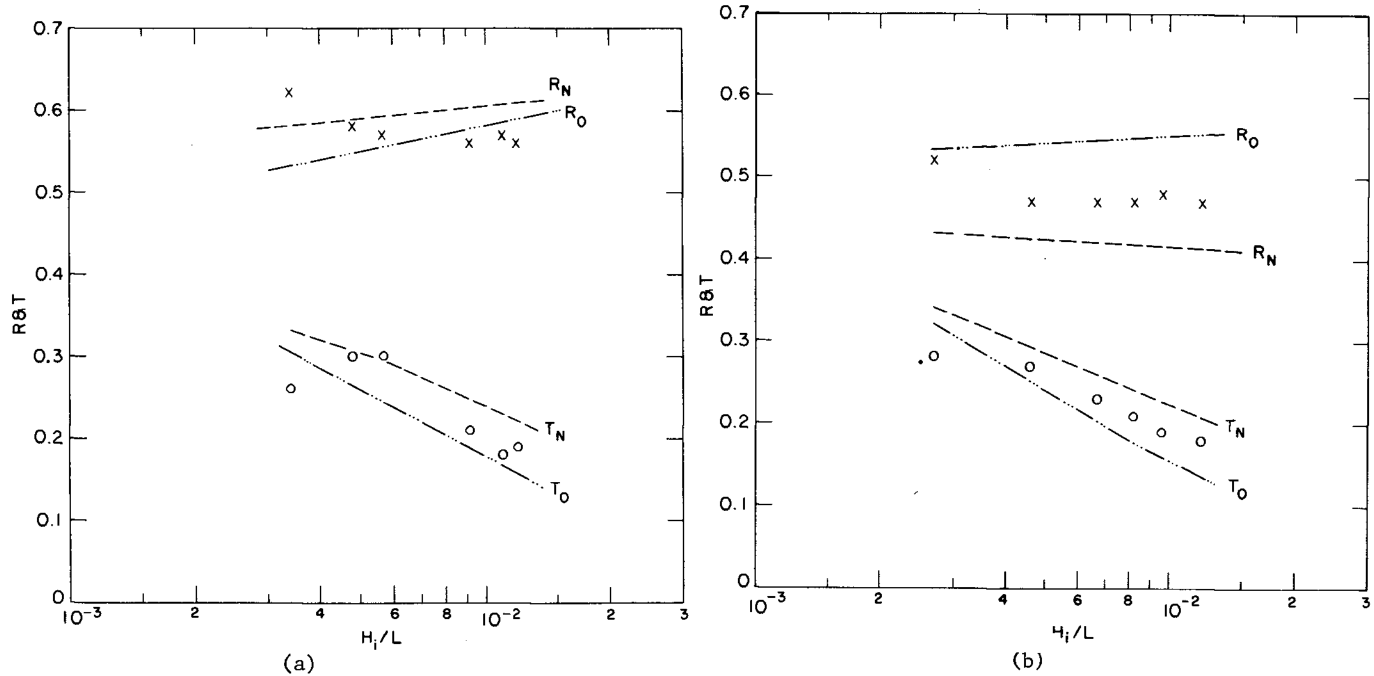

Figure 2: Comparison of Predicted and Observed Reflection (R) and Transmission Coefficients of Porous Trapezoidal Breakwaters.—·—— Predictions by present method.—···—— Predictions by previous method. Wave Period 2.0 sec., Water Depth 30.5 cm. Front slope 1 on 1.5(a), 1 on 2.0(b), 1 on 2.5(c).

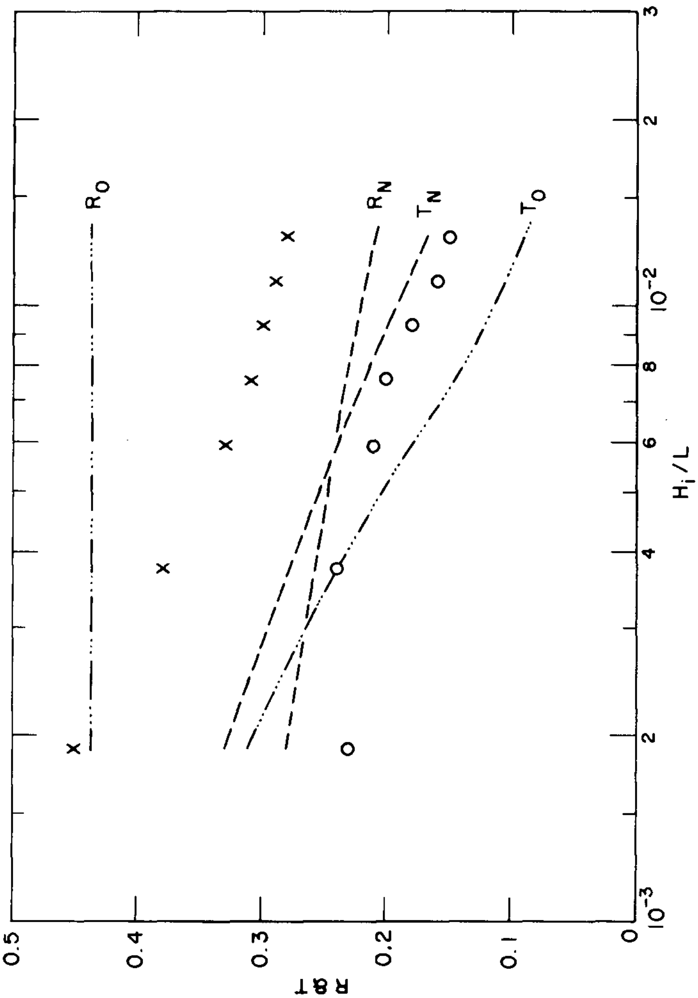

Figure 2(c): For legend see previous page.

reflection coefficients, which are believed to be quite accurately determined, on the gentler slope show a trend of decreasing reflection coefficient with increasing wave steepness (Figure 2). This feature, which cannot be reproduced by the "old" method appears to be represented by the results obtained from the present theory.

As a final test of the procedure developed in this paper predicted values of reflection and transmission coefficients are compared with the experimental results obtained by Sollitt and Cross (1972) for a multi-layered trapezoidal breakwater. To apply the present methodology the multi-layered breakwater is modeled as a homogeneous breakwater of the same geometry. The uniform stone size of the homogeneous breakwater is chosen so that the two breakwaters are hydraulically equivalent in the sense defined by Madsen and White (1977). The results are shown in Figure 3 (from Madsen and White, 1977) as dashed and dotted lines. Again the predicted transmission coefficients are in close agreement with measurements, whereas predicted reflection coefficients, although better than predictions afforded by the "old" method, still leave somewhat to be desired.

CONCLUSIONS

The present investigation was undertaken to improve on a previous simplified model for the determination of reflection and transmission characteristics of porous breakwaters of trapezoidal cross-section. The improvement consisted of a more rigorous analysis of the wave-structure interaction on the seaward and landward slopes of the breakwater thereby removing the somewhat artificial treatment in the previous model of external and internal dissipation as separate mechanisms.

The resulting theory retains the practicality of the previous theory in that only parameters which may be expected to be approximately known are needed to carry out the computations. However, the new theory is more elaborate and does require calculations to be carried out on a high speed computer.

From a comparison with experimental data it appears that the new as well as the old method are quite successful in their determination of transmission coefficients. Neither the new nor the old method is very successful in their prediction of reflection coefficients. For relatively gentle seaward slopes (1 on 2.0 to 2.5) the new method underpredicts whereas the old method overpredicts the reflection coefficient. For relatively steep seaward slopes both methods give reflection coefficients in reasonable agreement with observations. Slight changes in the assumed geometry of the breakwater have an insignificant effect on predictions of transmission coefficients by both the new and the old method. Whereas the old method is quite insensitive to breakwater geometry also for prediction of reflection coefficients, the new method is in this aspect very sensitive which suggests that a major renewed effort is needed if one wishes to obtain better predictions of reflection coefficients.

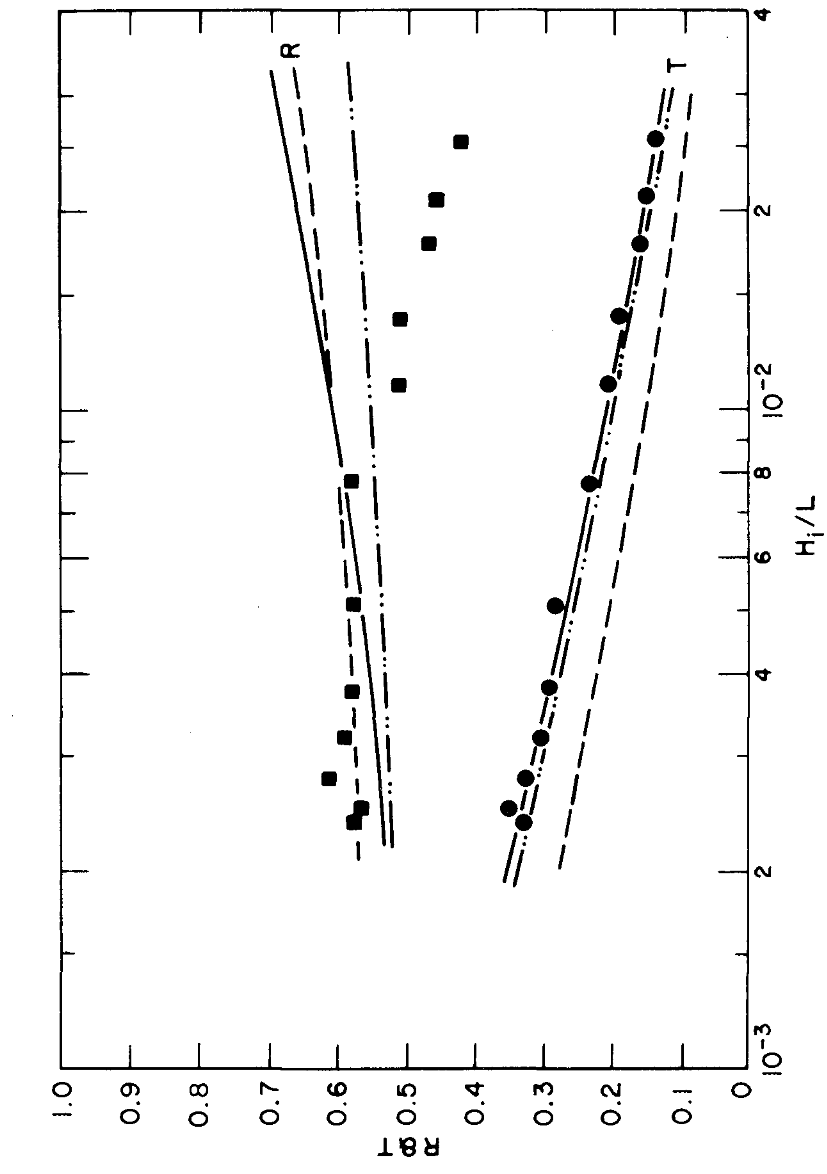

Figure 3: Comparison of predicted and observed reflection and transmission coefficients of trapezoidal, multi-layered breakwater tested by Sollitt and Cross (1972). — — — and ——— prediction by Madsen and White (1977); —··— prediction by present method.

ACKNOWLEDGMENTS

The research presented here was supported by the National Science Foundation, Water Resources, Urban and Environmental Engineering Program, under Grant No. ENG76-08982 A01. The junior author (Ms. Hanson) participated in the project as part of MIT's Undergraduate Research Opportunities Program (UROP) whose assistance is gratefully acknowledged.

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