CHAPTER 124

BEACH GROUND-WATER OSCILLATIONS

Andrzej LEWANDOWSKI *
Ryszard B.ZEIDLER **

ABSTRACT

Oscillations of ground-water table were measured in perforated piezometers distributed across the Baltic beach at the coastal research station of Lubiatowo. From spectral and correlation analysis it appears that the most pronounced wind-wave oscillations having a period of about 6 seconds have not been detected in the beach, while longwave oscillations, dominant in the beach spectrum, correlate well with nearshore long waves. Among the longwave oscillations, it is only those having periods about or slightly longer than 100 seconds that obey the Boussinesq law. However, some longer waves may also be generated by the interaction of shorter waves. Oscillations with periods from 50 to 100 s are very coherent throughout the beach. In the absence of stochastic noise and variety of inputs, low values of coherence for other periods indicate presence of nonlinear effects. More light should be shed on the origin of this nonlinearity and theoretical tools should be tried to complement the Boussinesq model.

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* Polish Academy of Sciences'Institute of Hydro-Engineering, Gdansk, Poland, Sen.Asst.

** ibidem, Asst.Prof.

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INTRODUCTION

During the past decade, extensive measurements of hydro- and lithodynamical factors were continued in the Polish Academy of Sciences' coastal research station at Lubiatowo, stretching along a rectilinear coastline section some 100 km northwesterly of Gdańsk.
A set of nine measuring posts (Fig. 1), reaching a depth of 6 m at a distance of 700 m from shore line , permits investigations of waves, currents, turbulent diffusion, sediment transport etc.

Fig.1 - Coastal Research Station at Lubiatowo

The data collected in two international expeditions of 1974 and 1976, together with relevant scientific results, can be found in References 1 and 2.

About five years ago the motion of ground water was added to the phenomena studied at Lubiatowo. Oscillations of ground-water table (GWT) were measured in the network of piezometers shown in Fig.2 .

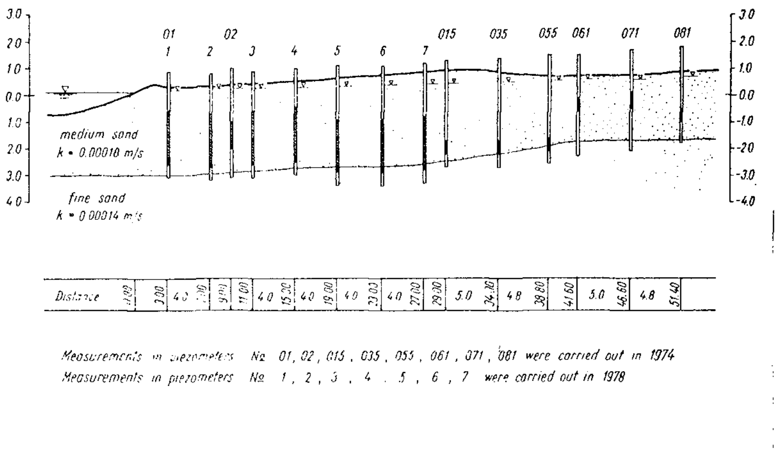

Fig.2 - Layout of Piezometers at Lubiatowo

All piezometers were made of 4-inch steel pipes, with filters as perforated sections lined with a filter gauze.
The diameter of the gravel layer around the filter sections was 8 inches. The instrumentation for measurements of oscillations consisted of three resistance wave probes, electronic system HAG-1 (designed at the Institute) and recorder. Calibration of the piezometers in a wave flume has indicated that free surface oscillations having periods of 1 second and longer are undistorted.

Changes in the sea water level, both slow and fast, bring about oscillations of ground water in the beach, which can be felt far away from shore line, e.g. in case of strong storm surges. Recognition of long oscillations has economic bearings because of floods, agriculture of hindlands, water sources etc. Shortwave oscillations are also important, as they determine the dynamics and stability of coastline, penetration of water and beach consolidation etc. Studies of these oscillations can elucidate hydro- and lithodynamical processes about civil engineering structures, e.g. distribution of pressures, dynamics of dikes and so forth.

A general description of the dynamics of beach ground water, using hydrochemical and hydrophysical indicators, has been given in the doctor's thesis of the junior author. This paper presents only the results concerning the ground-water oscillations. The measurements carried out under the thesis program have been revised and supplemented by this year's findings. The available experimental material is limited to longshore homogeneity, stationary conditions at the land boundary far off shore, and lack of atmospheric precipitation.

BASIC CHARACTERISTICS

Even a cursory glance at the records of ground-water oscillations exposes the effect of sea level oscillations on GWT. Under conditions of heavy storms (wave height H about 2 m some 350 m from shore line) this effect is pronounced as far as 50 m landwards (piezometers 051 - 055) . Haphazard measurements in piezometer 061 have not detected shortwave GWT oscillations. The effect of lower sea waves (H = 1 m) can be distinguished by 30 m from shore line.

Since GWT oscillations are random, the theory of random processes has been harnessed in their analysis. Nevertheless, the stochastic findings have been combined with theoretical considerations for non-random ground-water filtration. - Fig.3 shows spectra of waves measured at stations D2 and D5 (150 and 350 m off shore) and GWT oscillations in piezometer P.01, located about 2 m from shore line. Note different ordinate scales. It can be seen that the frequency band of maximum energy (sea versus beach) do not overlap. In general, the spectral peak of wind waves and swell at Lubiatowo occurs at the linear frequency f about $0.16 s^{-1}$, which corresponds to

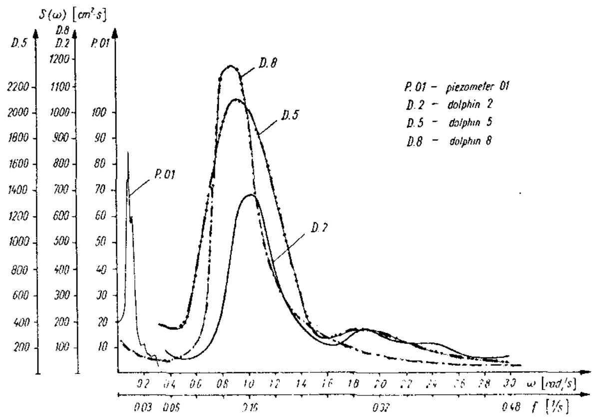

Fig.3 - Spectra of Sea Surface and GWT Oscillations

a period T of 6 s. These oscillations do not penetrate too far into land: the spectrum of P.01 does not contain significant 6-second components. On the other hand, because of technical shortcomings, the initial measurements of 1974 did not point to the presence of long sea waves, corresponding to those detected in the beach. It was only in this year's measurements, including recordings of sea surface displacements over hours, that it was possible not only to expose the longwave oscillations of water level in the nearshore zone but also to look at their correlation with the GWT oscillations.

The correlation and spectral analysis of GWT oscillations provides basic characteristics of the behavior of ground water in the beach. Fig.4 presents spectral densities measured in piezometers 01 , 02 and 015. One can note characteristic confinement and displacement of the spectral band with distance, increasing from shore. The further landwards the given spectrum, the narrower its band and the more it is shifted towards lower frequencies. As they travel across the beach, shortwave oscillations become gradually filtered out, by the mechanism of damping. - Similar conclusions can be drawn from

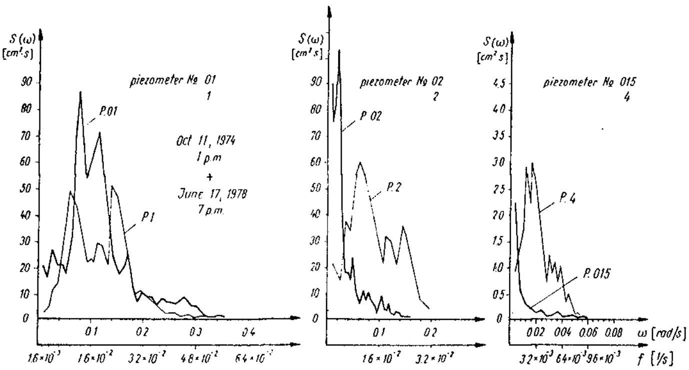

Fig.4 - GWT Oscillation Spectra

review of autocorrelation functions. The typical form of the latter (sinusoid(s) plus exponential attenuation) suggests the existence of a narrow-band random process with one (or two) sinusoidal signals. Principal parameters of these signals, determined from the spectral density peaks and clear-cut sinusoids in the autocorrelation functions, are given in Table 1.

The spectral density functions shown in Figures 5 and 6, normalized with regard to variances, expose some common features of the measured random functions. It appears that $S(\omega) \sim S(f)$ are decreasing power functions with an exponent about 3, which also falls with distance from shore line(3.1 at P.02 versus 2.7 at P.01) . Attention should be called to the maxima of S(f) (for f about 0.01 s^{-1}), especially in Fig. 5 for P.01. They might indicate the inflow of energy from outside (from the sea), conveyed by oscillations having periods $T = \frac{1}{f}$ about 100 seconds. The band of maxima in Fig.6, for P.02, is more smeared, which seems to point to the generation of longer GWT oscillations in the beach, which in turn could receive energy from shorter waves. This observation is

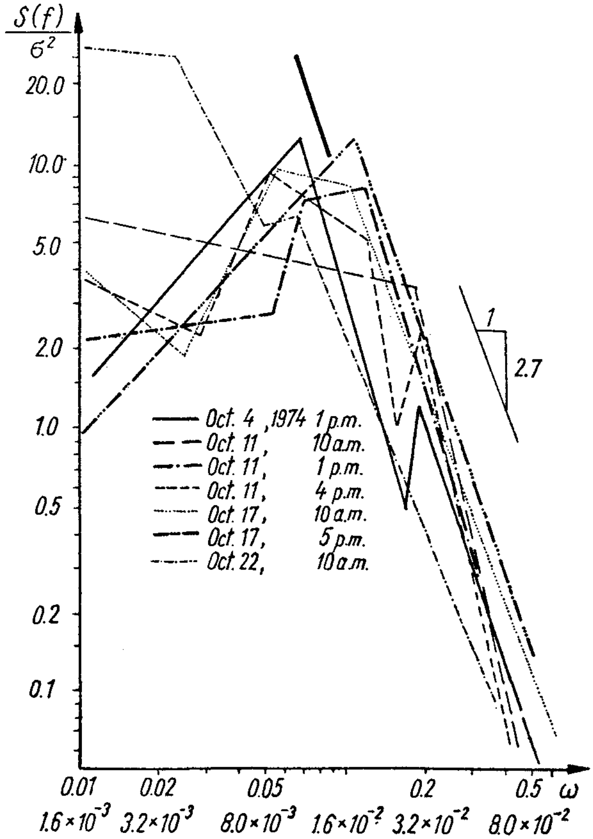

Fig.5 - Normalized GWT Spectra of P.01

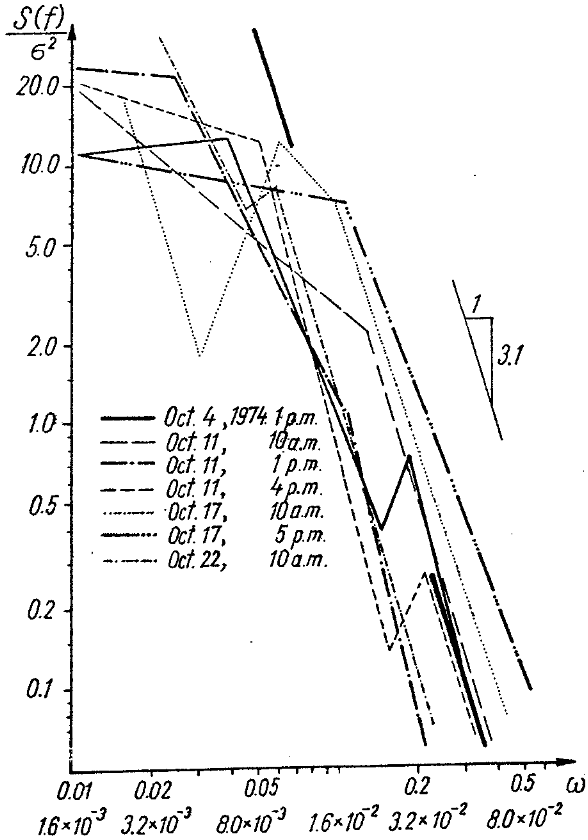

Fig.6 - Normalized GWT Spectra of P.02

Table 1 - Characteristics of Most Pronounced Situations with Wind Waves and GWT Oscillations

	Station D.2,8*			Piezometer No.P.01			Piezometer No. P.02				Piezometer No.015,4*		
	h cm	T s	f Hz	h cm	T s	f Hz	h cm	T s	f_1 Hz	f_2 Hz	h cm	T s	f Hz
4.10.1974, 16.00	75	4.1	0.24	5.9	72	0.014	3.8	137	0.007				
6.10.1974, 7.00	52	4.5	0.22				3.7	227	0.004				
9.10.1974, 19.00	53	4.3	0.23	10.6	52	0.019	1.5	110	0.009				
11.10.1974, 10.00	67	3.7	0.27	11.0	45	0.022	3.8	63	0.016	0.0011			
11.10.1974, 13.00	50	2.7	0.37	7.2	70	0.014	4.3	162	0.006	0.0015	0.4	58	0.017
11.10.1974, 16.00	72	3.8	0.26	7.6	59	0.017	5.1	131	0.0076				
11.10.1974, 19.00				5.5	62	0.016	4.0	128	0.0078				
17.10.1974, 10.00	70	3.4	0.29	6.3	52	0.019	2.5	116	0.0086				
17.10.1974, 17.00	64	4.1	0.24	3.8	67	0.015	1.5	104	0.010				
22.10.1974, 10.00				3.6	360	0.028	2.0	360	0.0028				
22.10.1974, 19.00				6.0	100	0.010	3.0	243	0.0041				
23.10.1974, 10.00				5.2	55	0.018	3.7	90	0.011	0.0010			
11.06.1978, 10.00	70*	5.00*	0.20*	6.8	72	0.014					0.6*	480*	0.0021*
13.06.1978, 19.00	58*	4.03*	0.25*	7.8	42	0.024					0.8*	430*	0.0023*
17.06.1978, 19.00	71*	4.42*	0.22*	9.8	44	0.023	5.6	79	0.013		1.4*	480*	0.0021*

supported by the results for some autocorrelation functions, with the periods of sinusoidal signals growing longer, or even doubling, over a short distance between piezometers. Some secondary long waves, superimposed on primary osicllations close to the shore line and becoming uncreasingly distinct at greater distances, are shown in Table 1 for piezometer P.02.

Thus, tentative analysis of GWT oscillations (their simple stochastic characteristics) indicates that the waves having periods about 100 seconds are somehow critical: they travel across the beach without strong attenuation, so characteristic for shorter periods. On the other hand, longer waves can come from secondary sources (can be generated by interaction mechanisms). Deeper insight into the physical structure of GWT oscillations can be provided by a bit more sophisticated considerations presented below.

UNSTEADY FILTRATION AND SUPPLEMENTARY STOCHASTIC CHARACTERISTICS

Two-dimensional unsteady filtration of free-surface ground water in soil having inhomogeneous hydraulic properties is described by the Boussinesq equation

$$\frac{\partial \zeta}{\partial t} = \frac{\partial}{\partial x}\left[\frac{k}{\partial \mu}\left(\zeta - h_s\right)\frac{\partial \zeta}{\partial x}\right] + \frac{\partial}{\partial y}\left[\frac{k}{\mu}\left(\zeta - h_s\right)\frac{\partial \zeta}{\partial y}\right] + \frac{w}{\mu} \qquad \ldots 1$$

in which

$\zeta = \zeta(x,y,t)$ = elevation of ground-water table

$h_s = h_s(x,y,t)$ = ordinate of floor of acquiferous stratum

$k = k(x,y)$ = coefficient of permeability

$\mu = \mu(x,y)$ = coefficient of yield

$w = w(x,y,t)$ = source function

Equation (1) can be substituted by an explicit finite difference scheme with time as a parameter. The equivalent equation was solved in ALGOL on an ODRA 1204 computer. For 160 by 220 m section of the Lubiatowo beach the boundary conditions were taken as constant water level far away from the sea, a sinusoidal variation of sea level at shore line and impermeability along the two remaining sides of the rectangle. The source function was taken as inhomogenous atmospheric precipitation measured in situ. All the boundary conditions have resulted from two-year investigations, the data of which was used for verification of the computations. The verification (comparison of computations with yearlong field measurements) has shown that the Boussinesq equation describes well the behavior of long waves having periods of several hours, or even longer. Moreover, it turned out that the filtration processes in the Lubiatowo beach are almost unidirectional, i.e. do not vary considerably along the beach over distances of hundreds of meters. Thus, they can be described fairly by the Boussinesq one-dimensional equation with constant coefficients:

$$\frac{\partial \zeta}{\partial t} = \frac{k}{\mu} h_s \frac{\partial^2 \zeta}{\partial x^2} \qquad \ldots 2$$

For the boundary conditions

$$\begin{array}{lll} \zeta = \zeta_o \sin \omega t & \text{at} & x = 0 \\ \zeta = 0 & \text{at} & x \to \infty \\ \zeta = 0 & \text{at} & t = 0 \end{array}$$

a solution of Eq.2 for sufficiently long t assumes the form

$$\zeta = \zeta_o \exp(-bx) \sin(\omega t - bx) \qquad \ldots 3$$

in which

$$b = \left(\frac{\omega \cdot \mu}{2k \cdot h_s}\right)^{1/2}$$

The Fourier transform of Eq.2 in the time domain is

$$\frac{k \cdot h_s}{\mu} \cdot \frac{\partial^2 S(f,x)}{\partial x^2} - j f S(f,x) = 0 \qquad \ldots 4$$

From its solution with respective boundary conditions (or through the Fourier transformation of the response function included in Eq.3) one obtains the following spectral density of GWT oscillations

$$S(f,x) = S_o(f) \exp\left(-\sqrt{\frac{f}{k \cdot h_s}}\; x\right) \qquad \ldots 5$$

in which $S_o(f)$ determines the spectrum of sea surface oscillations at the input to the area of ground-water motion (x = 0).

From solution (5) it follows that the spectrum of GWT oscillations should curve down to the abscissae axis: in the bilogarithmic system of Figures 5 and 6, because of the exponential factor in Eq.5, the straight lines S(f) should become curvilinear with increasing f. Similarly, for higher x the curves S(f) must be steeper.

Although the second tendency is fairly distinct (compare slopes of the curves for piezometers P.01 and P.02), the former trend is difficult to expose. From this fact it can be inferred that the Boussinesq equation is inadequate for the description of shortwave oscillations in the beach. At least three additional factors point to inadequacy. Firstly, the shortwave oscillations ($f \sim 0.01\ s^{-1}$) measured at Lubiatowo are attenuated much more strongly than it is given by the attenuation factor b in formula (5).

Secondly, the experimental data shows that the frequency modulation predicted by the Boussinesq $\sin(\omega t - bx)$ is unrealistic. The factor b computed for the known beach characteristics is much greater than that encountered in the piezometers. Finally, the Boussinesq model does not shed light on the origin of very long oscillations (having however periods shorter than the diurnal oscillations mentioned with reference to Eq.1) ; these oscillations seem to be generated by interactions of shorter waves.

Further information about the gound-water motion is borne by such statistical characteristics as coherence and transfer functions. The coherence functions γ_{xy} computed for two different piezometers indicate that it is only GWT oscillations with periods about 100 seconds that come from the same source (read: from the sea, as sea surface oscillations) . This is illustrated by examples given in Fig.7.

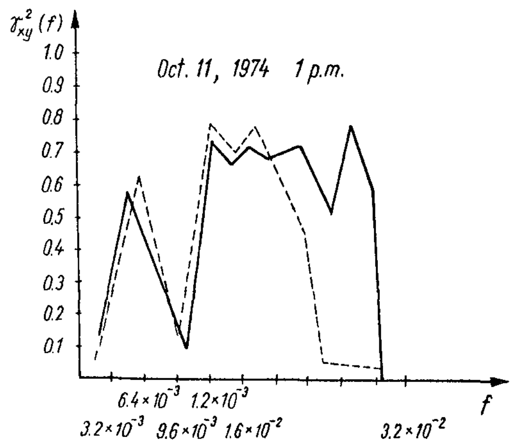

Fig.7 - Coherence Functions

The filtered data was obtained by the well-known filtering technique with a cosine kernel. - It can be seen that high values of coherence occur only in a narrow frequency band. Oscillations with other frequencies have coherences between 0 and 1.

For the ideal case of a linear constant-parameter dynamic system with single, strictly defined, input and output (a "deterministic" system), the coherence function is equal to one. If γ_{xy} lies between 0 and 1, at least one of the three following cases is possible:

a) results of measurements are subject to errors due to external noise
b) the system which couples the input and output signals, x(t) and y(t) is nonlinear
c) the output signal y(t) is a resultant of a few input signals, instead of a single x(t).

Analysis of the physical processes and experimental techniques inherent in this study of GWT oscillations makes it possible to exclude the cases (a) and (c), as irrelevant under the Lubiatowo conditions. Thus, the only acceptable explanation of the low coherence of most GWT oscillations is confined to nonlinear effects. The values of γ_{xy} measured in various situations indicate that the nonlinear effects are strong both in the band of "short" waves having periods T below 50s, in which GWT oscillations cannot be treated analytically by the Boussinesq equation, and in long waves, with T above 100 s, which might obey the Boussinesq approach but are subject to interaction effects.

The finding that the period T about 100s is somehow critical is also supported by the measured values of the amplification factor.

$$|H(f)| = H(f) \cdot \exp\left[i\,\theta(f)\right] \qquad \ldots 6$$

From the examples presented in Fig.8, with $|H(f)|$ computed for original GWT level series prior to and after filtering, it can be seen that attenuation of waves is smallest (high $H(f)$) for $f \simeq 0.01\mathrm{s}^{-1}$.

Analysis of the phase angle θ(f) shows that the waves passing across the beach (piezometers P.01 ... P.02 etc.) travel toward the dunes, and not along the shore (as should be in case of the edge wave type interaction waves). The phase velocities of these waves vary from 50 to 100 cm per s, in good agreement with the values of ω/b for waves with periods longer than 100 seconds, another indication of the critical character of these oscillations.

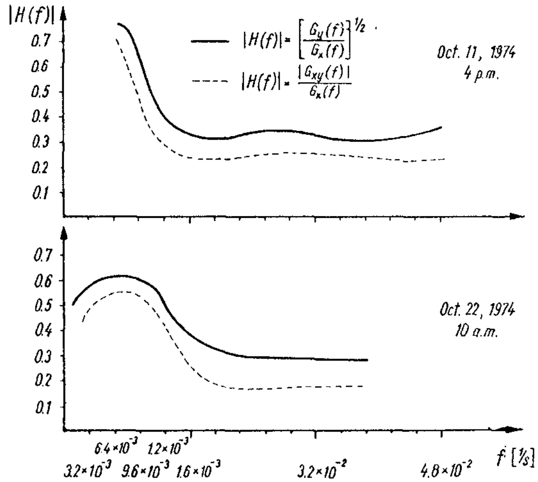

Fig.8 - Amplification Factor

ACKNOWLEDGEMENTS

This study was financially supported by the Polish Academy of Sciences under MR I.15.3.1 program. Dr. T.Piwecki of the Gdańsk Technical University is gratefully acknowledged for his computer routine, which permitted analysis of unsteady filtration.

REFERENCES

1 "Properties and Transformation of Hydrodynamical Processes in Coastal Zone of Nontidal Sea", Results of the international project LUBIATOWO 1974 in the coastal zone of the Baltic Sea, Raporty seria R,Nr 2a, Morski Instytut Rybacki; Gdynia 1976.

2 "Coastal Processes of Nontidal Sea", Results of the international expedition LUBIATOWO 1976, PRACE Inst. Bud. Wodnego PAN,No.5a,Gdańsk 1978.