

## CHAPTER 115

### DIFFRACTION CALCULATION OF SHORELINE PLANFORMS

by

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#### ABSTRACT

A method is presented, and illustrated with examples, for calculating planforms for a class of "pocket" beaches. This type of pocket beach is formed of mobile sediment due to waves diffracting through an opening in erosion resistant material, for example behind a break in a revetment or behind closely spaced offshore breakwaters. The opening is considered as a series of sources with circular wavelets radiating landward from each element. The height and phasing of each wavelet is varied depending on whether the depth at the element limits the wave height and on the wave direction relative to the opening, respectively. Based on reasonable, but not complete, considerations of sediment transport, the equilibrium bathymetry is considered to exist when the wave front is everywhere tangent to the local bottom contours. A differential equation is developed for the local orientation of the contours; this equation is solved numerically starting from a reference transect and extending the contour until it intersects the line defining the opening.

Examples are presented illustrating the method for the following cases: (1) normally incident waves diffracting through a single relatively narrow opening, (2) obliquely incident waves diffracting through a single relatively narrow opening and (3) spiral bays which are contained by two relatively widely spaced headlands. In addition, a procedure is suggested for applying the results to the case of normally and obliquely incident waves diffracting through the openings formed by a series of offshore breakwaters.

#### INTRODUCTION

Wave diffraction is known to play an important role in the determination of the equilibrium planforms of pocket beaches particularly when the waves propagate through a restricted opening. An ability to predict the planforms for this type of beach would allow improved design for recreation and beach erosion protection and would also assist in the interpretation of the "effective" wave climates which form existing pocket beaches.

#### REVIEW OF PREVIOUS WORK

A number of studies have been concerned with the prediction of beach planforms under conditions where diffraction effects are significant. Yasso (1965) studied four beaches for which there is an updrift

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headland and very little sand supply. These features were termed "headland bay beaches" by Yasso and it was demonstrated that the planforms could be fit very well by a logarithmic spiral equation of the form

$$r = e^{\theta} \cot \alpha \quad (1)$$

in which  $r$  is the radius from the log spiral center,  $\theta$  is the horizontal angle from the origin, and  $\alpha$  is a characteristic of the particular logarithmic spiral and represents the angle between a radius vector and tangent to the curve at that point.

Silvester and his co-workers (1970), (1972) have conducted studies of spiral bays lending further credence to the empirical fit provided by the logarithmic spiral. Based on an examination of planforms in nature and those evolved through laboratory studies, the conclusion has been reached that the planforms consist of three segments: (1) a circular arc segment in the lee of the updrift headland, (2) a log-spiral segment, and (3) a straight segment extending to the downdrift headland or other littoral control. It was also concluded that both refraction and diffraction effects significantly influence the beach planform. Silvester and Ho (1972) have utilized the spiral bay concept in which artificial headlands were constructed of gabions and rubble in an attempt to stabilize a landfill.

LeBlond (1972) has developed a numerical model to simulate the evolution of a straight shoreline to a spiral bay. Numerical stability problems occurred and the model was not successful in predicting the equilibrium beach planform characteristics. O'Rourke and LeBlond (1972) utilized the equations of motion including the radiation stress terms to investigate the water circulation patterns in a semi-circular bay. No attempt was made in the latter study to infer the associated beach planform.

Rea and Komar (1975) have developed a numerical model to simulate the formation of a crenulate bay. Diffraction effects were parameterized with the wave crests in the "shadow zone" described as circular arcs with the relative wave height decreasing from unity outside the shadow zone to progressively reduced values further in the lee of the headland. The unique feature of this model is that shoreline elements of two different orientations were used. In proximity to the updrift headland, the displacements of the elements were parallel to the axis of the headland and approximately perpendicular to the local wave crest. Outside of the shadow zone, the elements were perpendicular to those just described so that again the shoreline displacement was generally perpendicular to the local wave front. Good qualitative agreement was found with the general form of logarithmic spiral beaches.

Font, Sanabria and Silva (1976) have included the effect of the longitudinal gradients of breaking wave heights and the associated currents. The total longshore current consists of that due to the obliquity of the waves and that due to the gradient of the wave setup. The condition adopted for the equilibrium beach planform is zero total current. This is in contrast to the observations of Silvester (1970) that the waves break with crests parallel to the shoreline for

the equilibrium planform.

Walton (1977) has analyzed beach planforms of the spiral bay type and the associated waves and has concluded that the planform is everywhere perpendicular to the local wave energy vector. A family of possible beach planforms is determined and the planform selected which provides a best fit to the beach of interest. The method does not allow for diffraction and has the disadvantage of predicting that in the "gap" between two headlands, a wave system characterized by a single height and direction would result in a straight beach planform perpendicular to the incoming waves.

Dean and Maurmeyer (1977) have presented an approach to calculating the beach planforms resulting from "narrow" and "wide" openings between littoral controls. The method was compared with two measured planforms and demonstrated reasonably good agreement. One limitation of the method was that for the "narrow" gaps, it was necessary that the waves approach perpendicular to a line connecting the two headlands. The present paper presents an extension of that just described, including: (1) waves can approach the opening from any direction, and (2) an improved calculation procedure is utilized.

## METHODOLOGY

### General

The approximate method to be presented considers waves incident on an opening between two control points or headlands with a connecting sill of arbitrary depth, see Figure 1. The effects of diffraction are accounted for by using a simple representation of a series of wave sources distributed along the sill. The equilibrium beach and contour planforms are defined as coinciding with lines of constant wave phase; the effects of wave refraction are accounted for directly by the method in an approximate manner. The effect of currents due to gradients of breaking wave heights are not included. Although other authors, for example Font, Sanabria and Silva (1974) and LeBlond (1972) have emphasized the importance of these currents, other investigators, for example Silvester (1970), have noted that the equilibrium planform occurs when the wave breaks at normal incidence to the beach. My observations of beach planforms in apparent equilibrium tend to verify the latter assessment. Future studies will attempt to address the relative importance of these currents.

### Headlands and Connecting Sill

The two headlands are considered to be separated by a distance,  $b$ , and to be connected by a non-erodible sill. For computational purposes, the sill is subdivided into  $N$  segments, each of which will be considered as a wavelet source radiating outward with a circular crest.

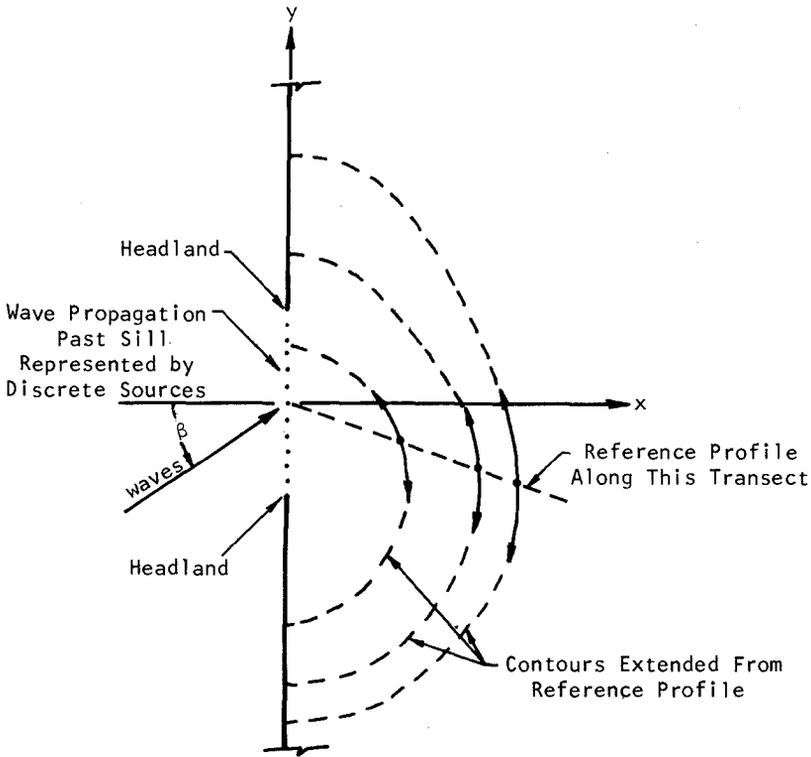


Figure 1. Definition Sketch Showing Method of Extending Contours From Reference Profile.

#### Incident Wave Representation

The incident wave,  $\eta(x,t)$ , is represented as the sum of  $M$  incident waves of height,  $H_{I_m}$ , angular frequency,  $\sigma_m (= 2\pi/\text{wave period})$ , wave number,  $k_m (= 2\pi/\text{wavelength})$ , and phase,  $\epsilon_m$ , propagating in a direction,  $\beta_m$ , with respect to a line joining the two headlands forming the pocket beach, see Figure 1.

#### Reference Profile

The computations are initiated and extended from specified depths along a reference transect as shown in Figure 1. The depths,  $h(x)$ , along this reference transect could be based on measurements or could be based on an idealized form

$$h(x) = A(x_{\max} - x)^{2/3} \quad (2)$$

as found by Dean (1977) in an analysis of 502 beach profiles from the Atlantic Coast and the Gulf of Mexico. In Eq. (2), the parameter, A, depends on sediment properties, primarily the fall velocity.

Wave Propagation Into Bay

The wave height,  $H_s$ , radiating out from the sill elements is considered to be equal to the incident wave height if no breaking at the sill occurs or to be limited by breaking in accordance with the usual assumptions for spilling breakers.

$$\left. \begin{aligned} H_{S_m} &= \kappa h_s, H_{I_m} > \kappa h_s \\ H_{S_m} &= H_{I_m}, H_{I_m} \leq \kappa h_s \end{aligned} \right\} \quad (3)$$

As noted, the wave height and phase at any location in the bay are determined from the linear sum of all contributions from the sill elements. The resulting water surface displacement in the bay can be expressed as

$$\eta(x,y,t) = [\eta(x,y)]_{\max} \cos[\sigma t - \epsilon(x,y)] \quad (4)$$

in which  $\epsilon(x,y)$  is the phase angle associated with the maximum water surface displacements as defined by

$$\tan \epsilon = \frac{S_2}{S_1} \quad (5)$$

where

$$S_1 = \sum_{m=1}^M \sum_{n=1}^N \frac{H_{n,m}}{2} \cos \mu_{n,m} \quad (6)$$

$$S_2 = \sum_{m=1}^M \sum_{n=1}^N \frac{H_{n,m}}{2} \sin \mu_{n,m} \quad (7)$$

and

$$\mu_{n,m} = k_{s_{n,m}} x_{n,m} \cos \beta_m + k_{s_{n,m}} y_{n,m} \sin \beta_m + \sigma_m r_n / \bar{C}_m \quad (8)$$

In these equations, the outer and inner summations are carried out over the number of incident wave components, M, and sill elements, N, respectively. The subscript s refers to conditions at the sill,  $\bar{C}_m$  is the average propagational speed of the m<sup>th</sup> wave component from source element to contour point of interest, a distance  $r_n$ , where

$$r_n \equiv \sqrt{(x - x_{s_n})^2 + (y - y_{s_n})^2} \quad (9)$$

The average propagational speed for a shallow water wave directed normally along a profile as given by Eq. (2) can be shown to be

$$\bar{c} = \frac{(\Delta x)_* \sqrt{gA^3}}{1.5[h_* - h']} \quad (10)$$

where  $h_*$  and  $h'$  are the depths just bayward of the sill and at the water depth of interest, respectively, and  $\Delta x_*$  is the normal distance along that contour separating the two end depths,  $h_*$  and  $h'$ . It is assumed that the propagational speed from the sill element to the location of interest is the same as if the wave were propagating normally along a profile given by Eq. (2).

Although not necessary for computation of the equilibrium planform contours, it is noted that the resultant wave height,  $H(x,y)$ , at any location is

$$H(x,y) = 2 \sqrt{S_1^2 + S_2^2} \quad (11)$$

Since the consideration for the locus of an equilibrium contour is that the contour be parallel to the wave crest, this is equivalent to establishing the locus of isolines of constant  $\epsilon(x,y)$  (or  $\tan \epsilon(x,y)$ ). Expressed analytically, using Eqs. (5), (6), and (7),

$$d(\tan \epsilon) = \frac{\partial(\tan \epsilon)}{\partial x} dx + \frac{\partial(\tan \epsilon)}{\partial y} dy = 0 \quad (12)$$

which can be simplified to the differential equation for the locus of a line of constant  $\epsilon(x,y)$

$$\frac{dy}{dx} = - \frac{S_1 \frac{\partial S_2}{\partial x} - S_2 \frac{\partial S_1}{\partial x}}{S_1 \frac{\partial S_2}{\partial y} - S_2 \frac{\partial S_1}{\partial y}} \quad (13)$$

#### Contour Determination

The phase  $\epsilon(x,y)$  to be held fixed along a given contour is first determined at the contour of interest on the reference profile through direct application of Eq. (5). The contour is next extended from the reference profile by a selected increment  $\Delta s$  in accordance with the direction established by Eq. (13), see Figure 2. The incremental  $\Delta x$  and  $\Delta y$  are

$$\left. \begin{aligned} \Delta x &= \Delta s \cos \delta \\ \Delta y &= \Delta s \sin \delta \end{aligned} \right\} \quad (14)$$

where

$$\delta = \tan^{-1} \left( \frac{dy}{dx} \right) \quad (15)$$

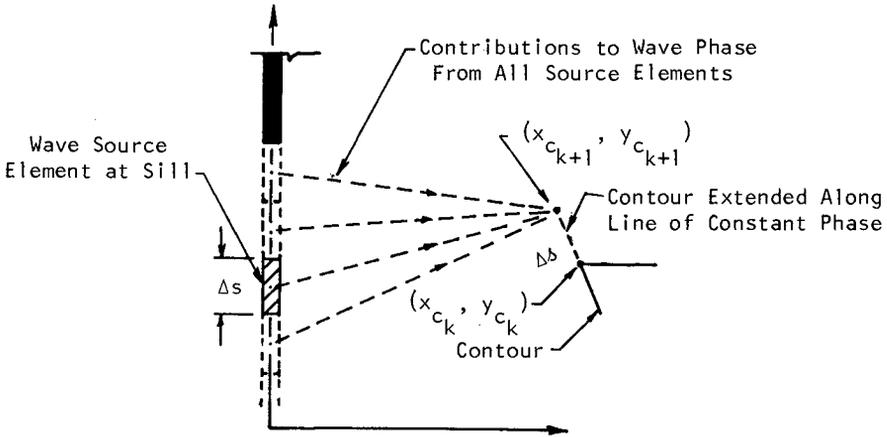


Figure 2. Step-Wise Extension of Contour Along Line of Constant Wave Phase.

The values of the (k+1) coordinates are now

$$\begin{aligned} x_{k+1} &= x_k + \Delta x \\ y_{k+1} &= y_k + \Delta y \end{aligned} \tag{16}$$

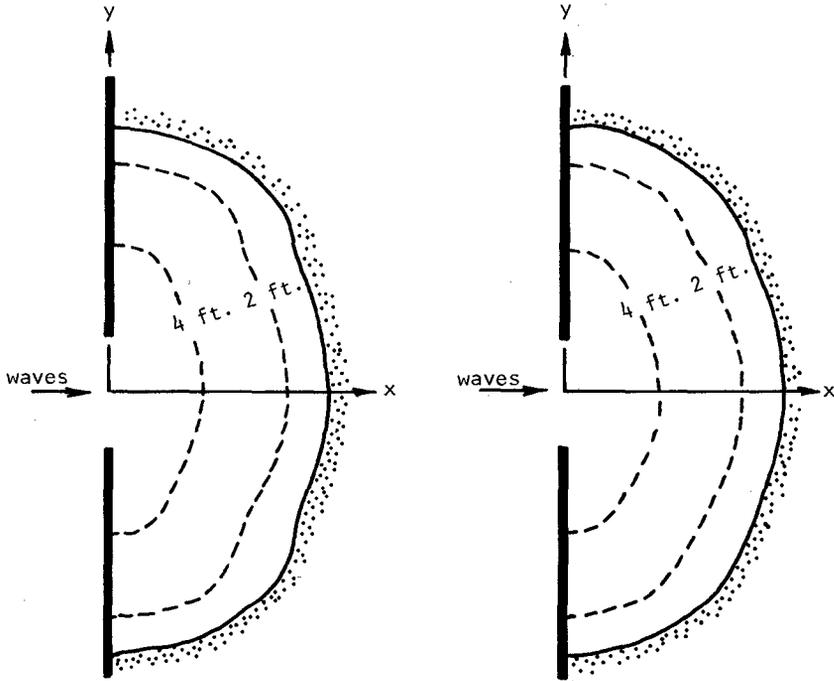
With estimates of the location of the extended contour now available, improved values of  $(x_{k+1}, y_{k+1})$  are determined by applying a Newton-Raphson procedure repeatedly to establish the "target" value of  $\epsilon(x,y)$ . This involves modifying  $(x_{k+1}, y_{k+1})$  along a direction which is perpendicular to the local value of  $(dy/dx)$  as determined from Eq. (13). In the results to be presented, this correction procedure was applied eight times for each point. It was usually possible to achieve the "target" value of  $\epsilon(x,y)$  within  $0.5^\circ$ . In this manner the contour is progressively extended until it intersects the line connecting the two headlands. Each successive contour of interest is then calculated in the manner described. This completes the calculation of the embayment planform.

APPLICATIONS

In this section the method will be applied to several idealized cases of interest.

Normal Wave Incidence

A number of cases of varying relative gap width have been computed for normally incident waves. For an incident wave of a single period, it was found that the contours were somewhat irregular; the irregularities were reduced when the incident wave was represented by two components of slightly different periods. Figure 3a presents the planform associated with an incident wave of a single period of 8 seconds.



a) Planform for a Single Incident Wave,  $T = 8$  sec.      b) Planform for An Incident Wave System Comprising Two Components. Reference Wave Period is 8.0 sec.

Figure 3. Calculated Pocket Beach Planforms. Distance Between Headlands is 120 ft.

The contours contain slight irregularities characteristic of diffracted wave fields. Figure 3b presents the planform for the same case except that the incident wave system comprises two individual waves of equal height but of different periods: 8.0 and 8.8 seconds. For all examples presented hereafter, the incident wave system will consist of two components of the same height and two periods equal to 1.0 and 1.1 times the "reference" period.

The effect of wave period, relative gap opening, and sill discretization were examined. It was found that for the longer

periods, the planforms were smoother and less elliptical (i.e. more circular) in shape. Figure 4 presents the calculated planform for the same case as Figure 3b, except the reference period is 24 seconds, and the differences noted above are evident.

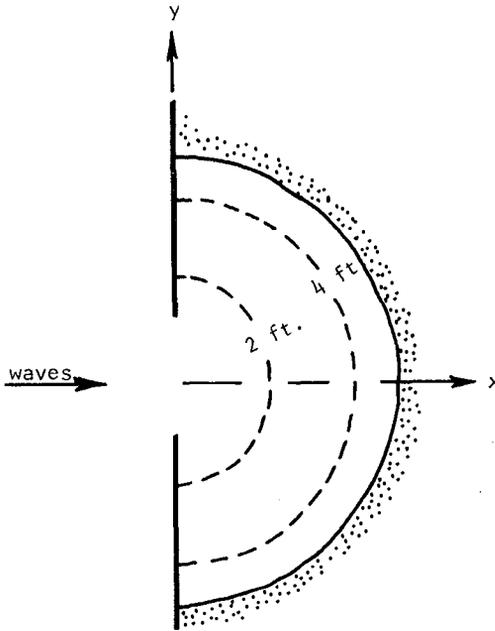


Figure 4. Calculated Planform for a Reference Wave Period of 24 Seconds. Distance Between Headlands is 120 ft.

The ratio of the major-to-minor semi-axis,  $a_2/a_1$ , was evaluated as a function of ratio of half gap width to minor semi-axes,  $b/2a_1$ , for three reference wave periods. Based on simple geometric considerations, an approximate relationship for these variables is

$$\frac{a_2}{a_1} = 1 + \frac{b}{2a_1} \tag{17}$$

Figure 5 presents the calculated planform results where it is seen that Eq. (17) represents an upper limit. For any given relative gap width, the ratio of major to minor semi-axis ( $a_2/a_1$ ) is larger for the shorter periods. The irregularities in the results are believed to be associated with the previously described characteristics of diffraction patterns.

The effect of finer discretization of the sill width was found to reduce slightly the irregularities in the calculated planforms.

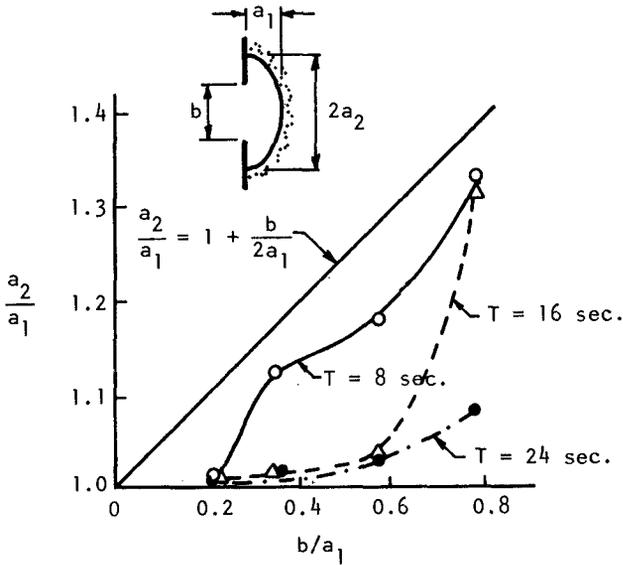


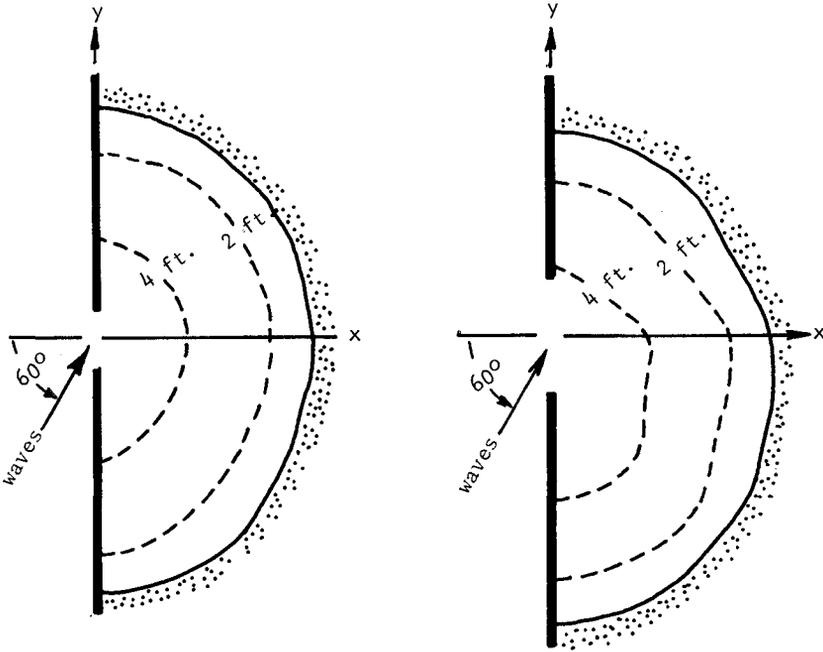
Figure 5. Variation in Calculated Ratios of Major to Minor Semi-Axes for Cases of Normal Wave Incidence and Various Wave Periods.

#### Oblique Wave Incidence

Figure 6 presents the calculated planforms for waves propagating at a  $60^\circ$  angle with respect to the x-axis and for two gap openings. For the relatively narrow gap opening ( $b/2a_1 = 0.11$ ), it is seen that there is little asymmetry in the two major semi-axes, the ratio of the down-wave to up-wave semi-axes being 0.88. For the wider gap opening ( $b/2a_1 = 0.29$ ), the ratio of the major down-wave to up-wave semi-axes is reduced to 0.71. It is also of interest that in the vicinity of the opening, the 4 ft. contour is oriented approximately normal to the incident wave direction for the wider gap.

#### Spiral Bays

The method was applied to the calculation of two spiral bay planforms for incident wave angles of  $30^\circ$  and  $60^\circ$  respectively, see Figures 7 and 8. The planforms are in qualitative agreement with the characteristics of spiral bays as produced in laboratory experiments and as found in nature.



- a) Planform for Relatively Narrow Opening,  $b/2a_1 = 0.11$ .      b) Planform for Relatively Wide Opening,  $b/2a_1 = 0.29$ .

Figure 6. Calculated Beach Planforms for Oblique Wave Incidence. Reference Wave Period is 8.0 sec. The Minor Semi-Axis = 235 ft. in Both Cases.

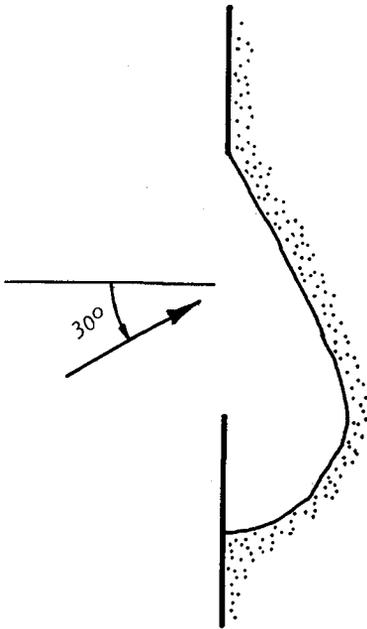


Figure 7. Calculated Spiral Bay Type Planform for  $30^\circ$  Wave Obliquity. Reference Period = 16 sec. Distance Between Headlands = 840 ft.

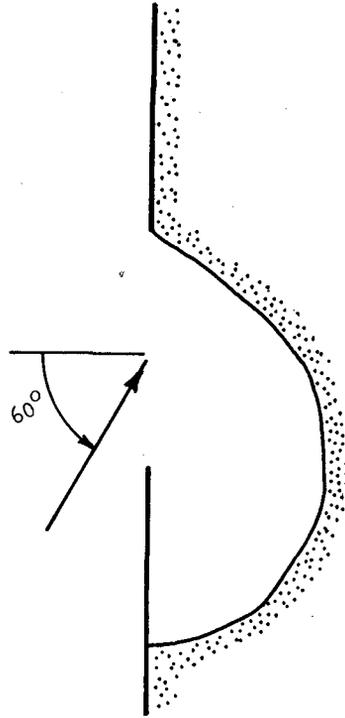
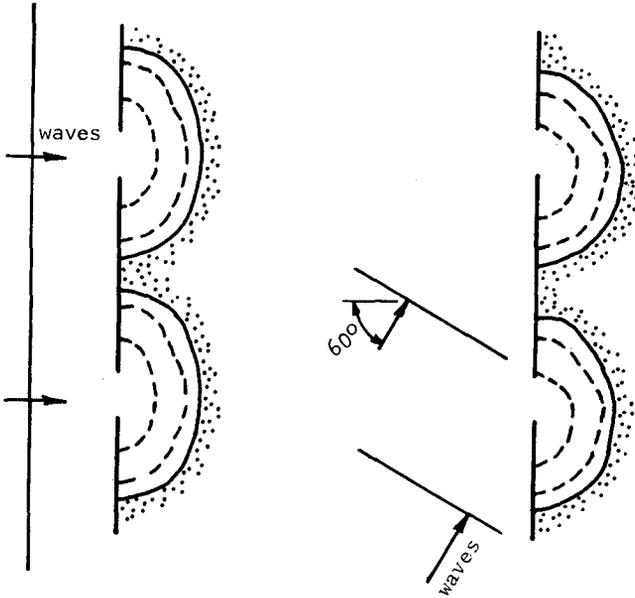


Figure 8. Calculated Spiral Bay Type Planform for  $60^\circ$  Wave Obliquity. Reference Wave Period = 16 sec. Distance Between Headlands = 380 ft.

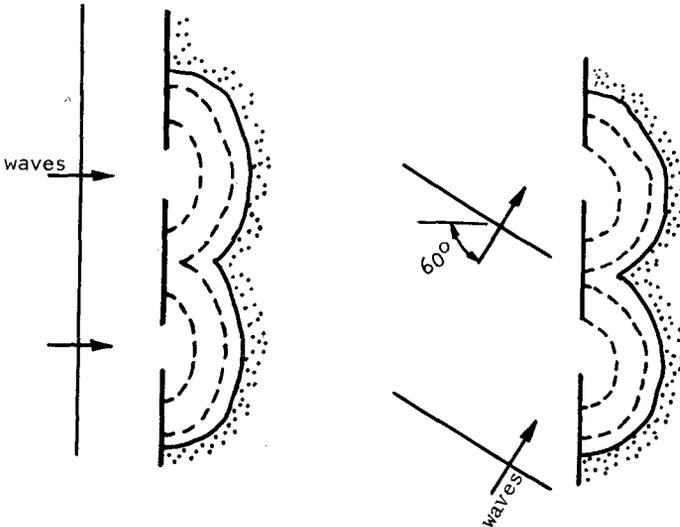
#### Application to Planforms Associated With Offshore Breakwaters

The planforms computed as described herein can be extended in a very approximate graphical manner to the case of offshore breakwaters. The procedure is first applied to planform computation for a single opening, the wave conditions of interest and with the reference transect representing the proper offshore distance of the breakwater. These planform results are then transferred to an overlay with the offshore breakwaters shown. If the spacing of the breakwater gap centerlines is greater than the major axis of the planform, then a tombolo is indicated, see Figure 9. If the spacing of the breakwater gap centerlines is less than the major axis of the planform, then the breakwater is not connected to the shoreline, see Figure 10. In order for this approach to be approximately valid, the reference transect must be correct and there must be adequate sand in the system to form an equilibrium planform in the proper geometric relationship to the profile along the reference transect.



a) Normal Wave Incidence.                      b) Oblique Wave Incidence.

Figure 9. Shore-Connected Planforms Associated With Offshore Breakwaters. Gap Opening is 120 ft. Reference Wave Period is 8 sec.



a) Normal Wave Incidence.                      b) Oblique Wave Incidence.

Figure 10. Planforms Associated With Offshore Breakwaters With a Relatively Large Ratio of Gap Opening to Breakwater Length. Gap Opening is 120 ft. Reference Wave Period is 8 sec.

SUMMARY AND CONCLUSIONSSummary

A method has been presented and illustrated with examples for calculating the equilibrium planform characteristics of pocket beaches contained by erosion resistant features, herein called headlands. In accordance with the observations of some investigators, the method considers the planform contours to coincide with lines of equal wave phase. The opening between the headlands is represented as a series of sources from which circular wavelets radiate landward. The water surface displacement at any location in the bay is considered as the linear sum of the contributions from all sill elements. Starting from the known contours on a reference transect, the contour is extended in accordance with a differential equation defining the locus of that contour. Examples presented include planforms for relatively narrow openings with normally and obliquely incident waves. In addition planforms are calculated for spiral bays which are associated with reasonably wide openings.

Conclusions

There are irregularities associated with the calculated planforms of lines of equal wave phase. These are found to be reduced somewhat for an incident wave system comprising two waves of differing periods. It appears that planforms in nature may be smoothed by a range of effective wave periods and directions. The general characteristics of the calculated planforms are reasonably representative of those found in nature. However, a full evaluation of the method will require additional laboratory and field data.

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