CHAPTER 97
SEDIMENT LOAD UNDER WAVES AND CURRENTS

by

D.H. Willis
Associate Research Officer
National Research Council of Canada

ABSTRACT

A technique is proposed for calculating sediment load, the mass concentration of sediment in motion, under combinations of waves and currents. The technique is based on the unidirectional flow method of Ackers and White, Ref. 1. The calibration of the technique against measurements of sediment load under waves has only just begun.

SEDIMENT LOAD

Sediment load is not sediment transport. It is an essential part of sediment transport, the concentration by mass of sediment in motion, but it requires the presence of a current to transport it from one place to another.

Although I will primarily deal with sediment load under waves, or waves and currents, the concept is best grasped for the case of currents alone. In this case there is, of course, always a current and therefore always a sediment transport. Sediment load is then the mass rate of sediment transport divided by the mass rate of current discharge.

The sediment load concept is however most useful when there are only very small currents, for example the mass transport associated with water waves. Here the sediment load due to the oscillatory wave motion may be quite high, but because of the very small currents involved, sediment transport calculations frequently fail. It is still handy to know how much sediment is available to fill in your navigation channel, even if you remain unsure of the rate at which infilling takes place.

We, at the Hydraulics Laboratory of the National Research Council of Canada, have therefore undertaken a program to develop a technique for calculating sediment loads under combinations of waves and currents, covering the range from currents only to waves only. We are initially aiming for a technique applicable seaward of the breaker line, but will not be at all disappointed if it turns out to be useful in the surf zone as well.

Like Swart in Ref. 13, I chose as a basis the method of Ackers and White, Ref. 1, for calculating sediment load in currents only. Their method has proved to be very successful in predicting sediment loads in rivers, see for example Fleming and Hunt, Ref. 3. The theoretical modification of the Ackers and White method for the presence of waves is virtually complete. But in making that modification, I added a new empirical coefficient, the calibration of which has only just begun. This then is a report on work in progress.

ACKERS AND WHITE METHOD

The technique of Ackers and White, see Ref. 1, was developed for calculating the sediment load in unidirectional flow over an alluvial bed. Ackers and White take a transporting power approach: the work done in moving sediment is the product of the power available to move the sediment and the efficiency of the system.

Despite its being a total load concept, their derivation does make a distinction between bed load and suspended load. But the distinction is made, not on the basis of position in the water column, but rather on dimensionless grain size,

$$D_{gr} = D\left[\frac{g(s-1)}{\nu^2}\right]^{\frac{1}{3}} \tag{1}$$

Coarse material, $D_{gr} \geq 60$, is considered to be moved as bed load. Grains are rolled along the surface of the bed by the component of bed shear parallel to the local bed surface,

$$\tau_{cg} = \rho \frac{V^2}{C_{h_{cg}}^2} \tag{2}$$

where $$C_{h_{cg}} = 5.75 \log \frac{11d}{D} \tag{3}$$

Fine material, $D_{gr} \leq 1$, is moved as suspended load. The turbulence which keeps the grains in suspension is a function of the total shear on the bed,

$$\tau_{fg} = \rho \frac{V^2}{C_{h_{fg}}^2} \tag{4}$$

where $$C_{h_{fg}} = 5.75 \log \frac{11d}{r} \tag{5}$$

The total shear includes components both parallel and normal to the local bed surface

The power per unit area available to move sediment then becomes

$$\begin{aligned} P_{cg} &= \tau_{cg} V \\ \text{and } P_{fg} &= \tau_{fg} V \end{aligned} \tag{6}$$

It is not my purpose to repeat the derivation by Ackers and White here. You are instead referred to Ref. 1. It will I hope suffice to say that they develop two sets of relationships, one for coarse grains and one for fine grains. Transition sizes, $1 < D_{gr} < 60$, are handled by mixing the relationships using an exponent, n. This is illustrated by the Ackers and White mobility number,

$$F_{gr} = \frac{u_{*fg}^{n} \, u_{*cg}^{1-n}}{\sqrt{gD(s-1)}} \tag{7}$$

where the shear velocity is as usual,

$$u_* = \sqrt{\frac{\tau}{\rho}} \tag{8}$$

An important feature of the method, and one which makes it almost unique, is the inclusion of a criterion for the beginning of sediment motion, the threshold of movement. This is expressed as a critical value of the mobility number, F_{gr_c}, below which no sediment motion takes place. They then derive the following expression for sediment load:

$$X = C \left(\frac{F_{gr}}{F_{gr_c}} - 1\right)^m \frac{sD}{d} \left(\frac{\rho^{\frac{1}{2}} \, P_{fg}}{\tau_{fg}^{\frac{3}{2}}}\right)^n \left(\frac{P_{cg}}{\tau_{cg} V}\right)^{1-n} \tag{9}$$

In the currents only case of Ackers and White, this reduces to

$$X = C\left(\frac{F_{gr}}{F_{gr_c}} - 1\right)^m \frac{sD}{d} \, C_{h_{fg}}^{n} \tag{9a}$$

At this point, the values of C, F_{gr_c}, m and n are still undefined. Ackers and White considered them to be empirical coefficients and calibrated them against over 1000 field and laboratory measurements of sediment load, obtaining

$$\log C = 2.86 \log D_{gr} - (\log D_{gr})^2 - 3.53, \quad 2.95 \times 10^{-4} \leq C \leq 0.025 \tag{10}$$

$$F_{gr_c} = \frac{0.23}{\sqrt{D_{gr}}} + 0.14, \quad 0.17 \leq F_{gr_c} \leq 0.37 \tag{11}$$

$$m = \frac{9.66}{D_{gr}} + 1.34, \quad 1.5 \leq m \leq 11.0 \tag{12}$$

$$n = 1 - 0.56 \log D_{gr}, \quad 0 \leq n \leq 1 \tag{13}$$

MODIFICATION FOR WAVES

My principal criterion in modifying the Ackers and White method for the presence of both waves and currents, was that the basic method as set out in Ref. 1 should remain intact when no waves were present. Ackers and White have, after all, calibrated their method against more than 1000 measurements, and such a wealth of data can not be lightly tossed aside.

Vector addition of wave and current velocities produces a shear relationship which meets that criterion. Shear is proportional to the instantaneous velocity squared:

$$V_{TOT}^2 = (V + u_o \sin \frac{2\pi t}{T} \cos \alpha)^2 + (u_o \sin \frac{2\pi t}{T} \sin \alpha)^2 \qquad (14)$$

Averaging V_{TOT}^2 over a wave period,

$$\overline{V_{TOT}^2} = V^2 + \frac{u_o{}^2}{2} \qquad (15)$$

which is independent of direction.

If we assume that the unidirectional Chézy friction factor can be applied to the unidirectional term only, the Jonsson wave friction factor, $f_w/2$ see Ref. 6, to the oscillatory term only, and that f_w is independent of wave phase, we obtain for the combined bed shear

$$\tau = \rho(\frac{V^2}{C_h{}^2} + \frac{f_w}{4} u_o{}^2) \qquad (16)$$

Power is a scalar, and therefore we only need obtain an expression for the wave power available to move sediment and add that to the unidirectional term, Equation (6). The wave power per unit area, available to move sediment is

$$P_w = E \frac{dC_g}{dx} + C_g \frac{dE}{dx} \qquad (17)$$

Since a locally horizontal bed is assumed in the Ackers and White method, C_g does not vary with distance and

$$P_w = C_g \frac{dE}{dx} \qquad (17a)$$

There are a number of factors affecting the wave energy attenuation, dE/dx: bed friction, bottom percolation, surface contamination, to name a few. But we are only interested in that part of dE/dx available to move sediment, the part due to bottom friction or bed shear. Therefore

$$P_w = C_g \rho \frac{f_w}{4} u_o{}^2 \qquad (17b)$$

The total power per unit bed area under waves and currents then becomes

$$P_w = \rho(\frac{V^2}{C_h{}^2} V + C_g \frac{f_w}{4} u_o{}^2) \qquad (18)$$

Subscripts have been dropped from Equations (16) and (18) but there are fine grain and coarse grain versions, analogous to Equations (2), (4) and (6). In both Equations (16) and (18), the first term is that for currents only, as in the Ackers and White method, with the second term adding the wave effect.

One further modification is necessary to compensate for the fact that the threshold of sediment motion is different under waves than under unidirectional flow. If the expression for F_{gr_c}, Equation (11), is modified, as Swart has done in Ref. 12, then it ceases to be a function of D_{gr} alone; it also becomes a function of the flow conditions. However, the flow conditions are already contained in the mobility number, F_{gr} Equation (7), by way of the modified shear stress of Equation (16). I therefore decided to leave the threshold of movement criterion alone, and further modify the shear stress to compensate for differences in wave and current thresholds. A new empirical coefficient, W_c, was added to the wave terms of Equation (16) and (18), which become respectively:

$$\tau = \rho\left(\frac{V^2}{C_h{}^2} + W_c{}^2 \frac{f_w}{4} u_o{}^2\right) \tag{19}$$

$$P = \rho\left(\frac{V^2}{C_h{}^2} V + W_c{}^2 C_g \frac{f_w}{4} u_o{}^2\right) \tag{20}$$

THRESHOLD OF MOVEMENT

Equations (19) and (20) not only reduce to their unidirectional flow forms when no waves are present, but when no currents are present they also reduce to their wave terms. It should therefore be possible to calibrate W_c against measurements of sediment load under waves alone.

Twenty-seven suitable measurements of sediment load under waves were found in the literature, 4 field measurements of which one included a current velocity, and 23 laboratory measurements. "Suitable" simply means that enough data were presented to allow an Ackers and White calculation of sediment load to be made and a value of W_c to be determined. Even for such "suitable" measurements, it was usually necessary to make one or more of the following assumptions:

1. Mass density of sea water, 1020 kg/m^3, or of fresh water, 1000 kg/m^3.
2. Mass density of sand, 2650 kg/m^3.
3. Kinematic viscosity of laboratory water, 1 x 10^{-6} m^2/s (20°C), or of sea water 1.3 x 10^{-6} m^2/s (10°C).
4. Ripple dimensions given by the design curves of Mogridge, Ref. 10.

The 27 measurements are summarized in Table I. In all cases, sediment load was presented as a plot of concentration vs. depth. I integrated these plots, either analytically or numerically, from water surface to bed to obtain the total load, X, shown on Table I. The column headed "F_{gr_c} Required" is the value of the threshold of movement necessary to make the computed sediment load agree with the measured, and "Equivalent W_c" is the adjustment required to the wave shear.

The computed thresholds are plotted on Fig. 1 as Shields parameter, θ, against dimensionless grain size, D_{gr}. The Shields parameter is simply the square of the Ackers and White mobility numbers, F_{gr} or F_{gr_c}. The following are also plotted on Fig. 1.

1. The Shields curve, Ref. 11, for threshold of movement under unidirectional flow over a plane bed. Threshold was determined by observation and is somewhat subjective. It may be useful to note that, plotted in this way with D_{gr} as the abscissa rather than shear Reynolds Number, the Shields curve can be used without iteration to determine a critical shear stress for a given sediment.
2. The Ackers and White threshold of movement criterion, Equation (11), for unidirectional flow over a rippled bed. This criterion has been inferred from measured rates of sediment transport, and may therefore be more objective than the Shields curve.
3. Thirty-five points presented by Komar and Miller, Ref. 8, for threshold of movement under oscillatory flow. These points come from a variety of sources, but in general are the result of observation of the initiation of motion on a plane bed.

Most of the oscillatory flow points lie above the two curves for unidirectional flow. However, the data of Bhattacharya, Ref. 2, which forms the majority of my 27 calibration points, seems to require a significantly lower threshold of movement than either the unidirectional criteria or the rest of the oscillatory flow data. Thirteen of the 21 points from Ref. 2 fall below both the Shields and the Ackers and White curves, while only one of the remaining 41 points does so. This may be a bed slope effect, Bhattacharya did his measurements on a laboratory beach, but whatever the reason, it was with some regret that I decided to ignore his data for the rest of the analysis.

The remaining oscillatory flow points on Fig. 1 show no clear trend, other than that there appears to be a higher threshold of movement under oscillatory than under unidirectional flow. Madsen and Grant, Ref. 9, concluded that the Shields curve adequately described the threshold in oscillatory flow. Certainly for large grains, $D_{gr} \geq 100$, the Shields

TABLE I

SUMMARY OF CALIBRATION DATA

Ref.	Water Depth D	Wave Height H	Wave Period T	Current Velocity V	Sediment Load X	D_{gr}	F_{gr_c} Req'd	Equiv. W_c	Symbol on Fig. 1
--	m	m	s	m/s	--	--	--	--	--
5	11.3	2.05	8.0	0.25	1.07×10^{-4}	1.99	0.574	0.528	+
14	4.6	0.6	7.0	0.0	3.1×10^{-5}	1.54	0.433	0.751	⊙
14	6.4	0.6	7.0	0.0	3.9×10^{-5}	1.54	0.443	0.734	⊙
7	0.25	0.146	1.0	0.0	4.4×10^{-4}	3.54	0.302	0.868	△
4	0.3	0.165	1.3	0.0	7.2×10^{-4}	2.53	0.314	0.907	⊡
4	7.8	1.05	6.2	0.0	2.4×10^{-5}	5.22	0.243	0.992	⊡
2	0.137	0.119	1.5	0.0	7.8×10^{-4}	5.31	0.284	0.843	*
2	0.155	0.116	1.5	0.0	6.3×10^{-4}	5.31	0.295	0.813	*
2	0.184	0.110	1.5	0.0	5.8×10^{-4}	5.31	0.258	0.928	*
2	0.209	0.107	1.5	0.0	6.6×10^{-4}	5.31	0.220	1.090	*
2	0.223	0.104	1.5	0.0	4.3×10^{-4}	5.31	0.224	1.069	*
2	0.244	0.101	1.5	0.0	4.1×10^{-4}	5.31	0.204	1.177	*
2	0.279	0.101	1.5	0.0	2.5×10^{-4}	5.31	0.203	1.180	*
2	0.300	0.098	1.5	0.0	2.8×10^{-4}	5.31	0.181	1.325	*
2	0.140	0.107	1.04	0.0	9.0×10^{-4}	5.31	0.263	0.911	*
2	0.157	0.122	1.04	0.0	9.8×10^{-4}	5.31	0.262	0.914	*
2	0.189	0.098	1.04	0.0	6.2×10^{-4}	5.31	0.199	1.204	*
2	0.198	0.094	1.04	0.0	8.3×10^{-4}	5.31	0.171	1.401	*
2	0.221	0.091	1.04	0.0	6.3×10^{-4}	5.31	0.158	1.519	*
2	0.238	0.101	1.04	0.0	4.4×10^{-4}	5.31	0.175	1.373	*
2	0.250	0.091	1.04	0.0	3.2×10^{-4}	5.31	0.162	1.482	*
2	0.143	0.101	1.04	0.0	1.8×10^{-3}	5.31	0.208	1.155	*
2	0.171	0.129	1.04	0.0	4.8×10^{-4}	5.31	0.300	0.800	*
2	0.210	0.105	1.04	0.0	8.0×10^{-4}	5.31	0.180	1.332	*
2	0.218	0.110	1.04	0.0	6.8×10^{-4}	5.31	0.188	1.274	*
2	0.261	0.115	1.04	0.0	4.8×10^{-4}	5.31	0.174	1.377	*
2	0.304	0.105	1.04	0.0	2.8×10^{-4}	5.31	0.151	1.587	*

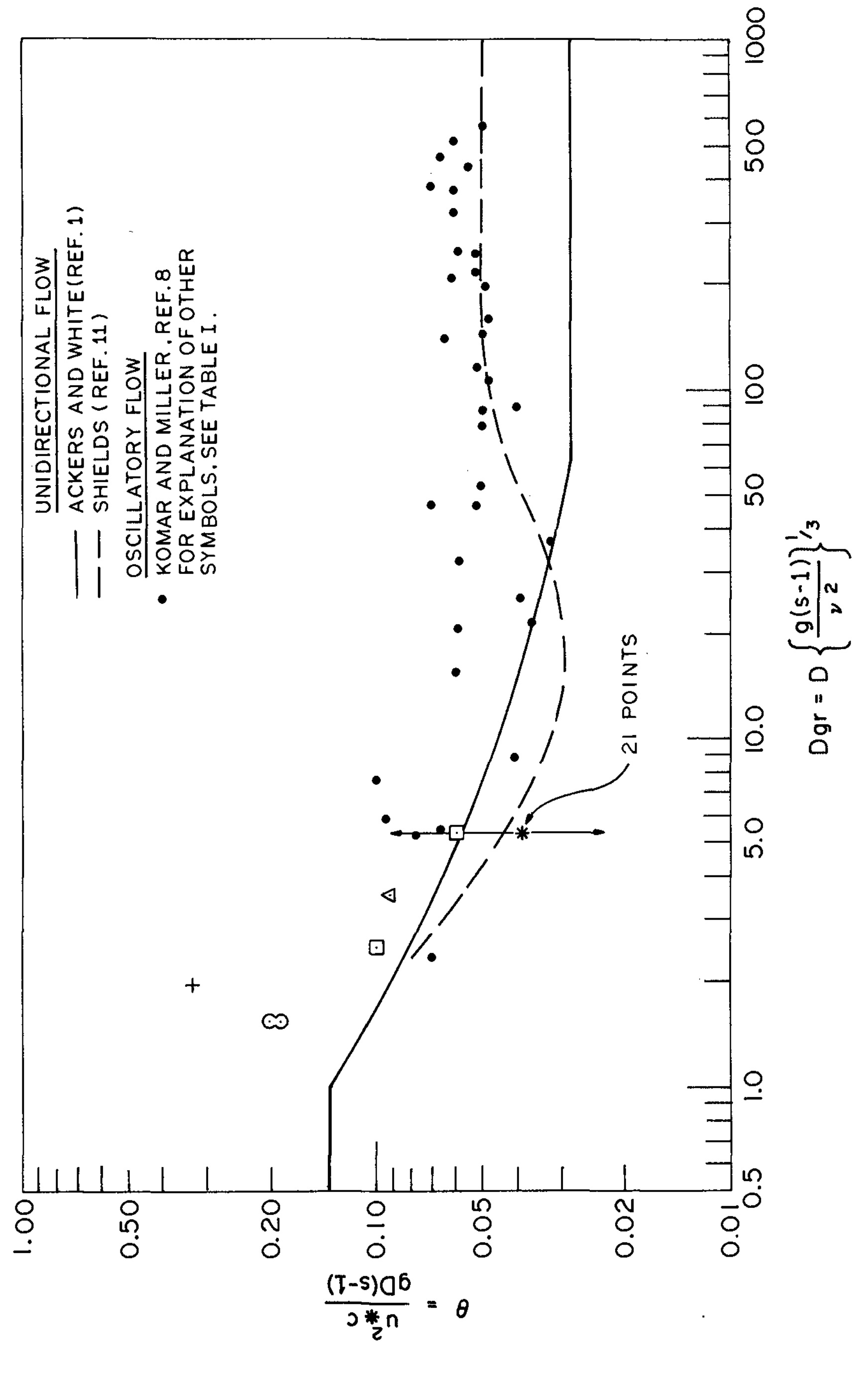

FIG. 1 THRESHOLD OF MOVEMENT

curve is a better lower envelope to the oscillatory flow threshold than the Ackers and White curve. The value of W_c needed to make the two curves coincide here is 0.76. However, at the lower end, $D_{gr} \leq 10$, the Ackers and White criterion is a better lower envelope, implying $W_c = 1$. Perhaps overall W_c varies inversely with D_{gr}, but as an interim measure, I have taken it as a constant, $W_c = \sqrt{0.6}$, in Equations (19) and (20), making them

$$\tau = \rho\left\{\frac{V^2}{C_h^2} + 0.6\ \frac{f_w}{4}\ u_o^2\right\} \tag{21}$$

$$P = \rho\left\{\frac{V^2}{C_h^2}\ V + 0.6\ C_g\ \frac{f_w}{4}\ u_o^2\right\} \tag{22}$$

FUTURE WORK

There is clearly a need for more data, in particular for a consistent set of sediment load measurements covering the full range of dimensionless grain sizes. We at the National Research Council of Canada are presently collecting that data in a wave flume, varying grain size and density, water depth, and height and period of the regular waves. Sediment load is determined by counting particles in the narrow column illuminated by a vertical laser beam.

I must emphasize again that the work reported here is still very much in progress. The quoted value of $W_c = \sqrt{0.6}$ must be regarded as a temporary measure until either confirmed or replaced by something better as a result of the on-going flume tests.

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APPENDIX I

GENERAL NOTATION

C coefficient in sediment load function (Equation 10)

C_g wave group velocity $\left(\frac{L}{T}\right)$

C_h dimensionless Chézy coefficient

D typical grain diameter [L]

D_{gr} dimensionless grain size

d water depth [L]

d_o maximum water particle excursion at the bed [L]

E wave energy $\left(\frac{M}{T^2}\right)$

F_{gr} sediment mobility number

f_w Jonsson wave friction factor, Ref. 6

g acceleration due to gravity $\left(\frac{L}{T^2}\right)$

H wave height, trough to crest [L]

m exponent in sediment load function (Equation 12)

n transition exponent (Equation 13)

P power per unit bed area $\left(\frac{M}{T^3}\right)$

r ripple roughness, a function of ripple geometry, see for example Ref. 13 [L]

s ratio of mass density of sediment to that of the fluid

u_o maximum wave orbital velocity at the bed $\left(\frac{L}{T}\right)$

u_* shear velocity (Equation 8) $\left(\frac{L}{T}\right)$

V mean unidirectional flow velocity $\left(\frac{L}{T}\right)$

W_c empirical wave shear coefficient

X sediment load

x horizontal co-ordinate [L]

α horizontal angle between wave orbital velocity and mean current velocity

θ Shields parameter, see Fig. 1

ν kinematic viscosity of the fluid $\left(\frac{L^2}{T}\right)$

ρ mass density of the fluid $\left(\frac{M}{L^3}\right)$

τ shear stress at the bed $\left(\frac{M}{LT^2}\right)$

Subscripts

c critical for initiation of sediment motion

cg coarse grain

fg fine grain

TOT total

w due to waves

APPENDIX II

SUMMARY OF METHOD

1. Calculate D_{gr} from Equation (1)

2. Calculate empirical coefficients, C, F_{gr_c} m and n from Equations (10), (11), (12) and (13)

3. Calculate the dimensionless Chézy coefficients $C_{h_{cg}}$ and $C_{h_{fg}}$ using Equations (3) and (5).

4. Calculate the Jonsson wave friction factors,

$$\left.\begin{aligned} f_{w_{cg}} &= \exp\left(-5.98 + 5.21\left(\frac{d_o}{2D}\right)^{-0.19}\right) \quad (23) \\ f_{w_{fg}} &= \exp\left(-5.98 + 5.21\left(\frac{d_o}{2r}\right)^{-0.19}\right) \quad (24) \end{aligned}\right\} f_w \leq 0.30$$

These approximate formulae are from Swart, Ref. 13.

5. Calculate bed shears, τ_{cg} and τ_{fg}, using Equation (21) and convert to shear velocities, $u_{*_{cg}}$ and $u_{*_{fg}}$, with Equation (8).

6. Calculate powers, P_{cg} and P_{fg}, using Equation (22).

7. Calculate the mobility number, F_{gr}, using Equation (7).

8. Calculate sediment load by Equation (9).