CHAPTER 65

HYDRODYNAMIC MODELLING OF THE SOUTHERN NORTH SEA

by

David Prandle*

ABSTRACT

Numerical modelling of rivers, estuaries and shallow seas has attracted increasing interest over the last two decades. The models have developed from one dimensional (1D) applications to tidal propagation and flood routing through two and, finally, three dimensional applications to motions ranging from "pseudo-turbulence" to annual-mean residual flows. The present account describes the development, over the last five years, of the modelling studies carried out by the author concerning the hydrodynamics of the southern North Sea and River Thames. The objective is to identify those major points which have emerged that may have a wider significance.

1. INTRODUCTION

The modelling work considered is restricted to one and two (horizontal) dimensional applications; explicit finite-difference solutions are used throughout. The motions involved are assumed to be represented by the following shallow water wave equations :

motion (-x) $$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} + g\frac{\partial Z}{\partial x} + \frac{KU(U^2+V^2)^{1/2}}{(D+Z)} - \Omega V = 0 \qquad (1)$$

motion (-y) $$\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} + g\frac{\partial Z}{\partial y} + \frac{KV(U^2+V^2)^{1/2}}{(D+Z)} + \Omega U = 0 \qquad (2)$$

continuity $$\frac{\partial Z}{\partial t} + \frac{\partial}{\partial x}\left[U(D+Z)\right] + \frac{\partial}{\partial y}\left[V(D+Z)\right] = 0 \qquad (3)$$

where U and V are depth-averaged velocities along the respective directions of the orthogonal axes x and y;
Z the elevation of the water surface above a fixed horizontal datum;

*Principal Scientific Officer, Institute of Oceanographic Sciences, Bidston Observatory, Merseyside, England.

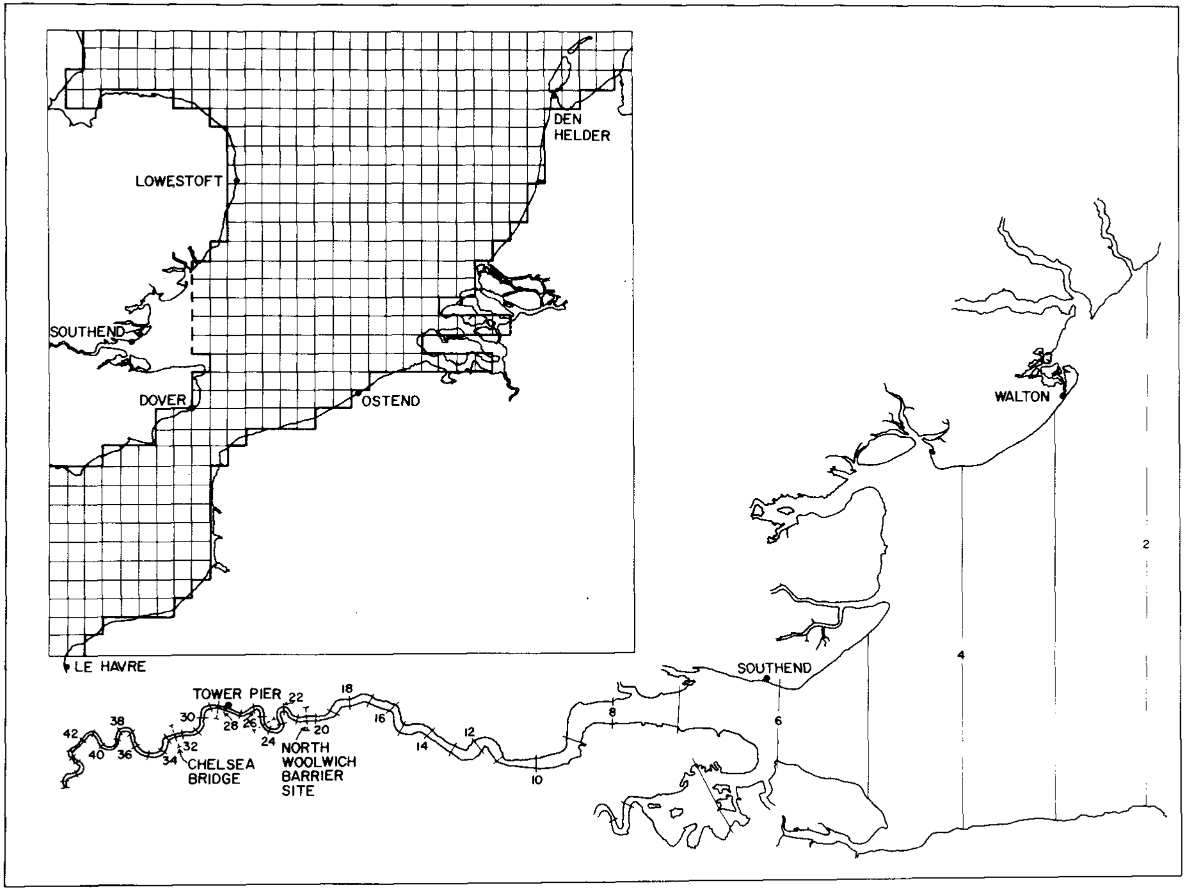


Figure 1. Schematic representation of the southern North Sea and River Thames.

D the depth of the bed below that same datum;
t time;
g gravitational constant;
K friction coefficient;
and Ω the Coriolis parameter.

The schematic representation of the southern North Sea and River Thames is shown in figure 1.

In using these equations as the basis of the modelling work, the following assumptions have been made : (a) vertical accelerations are negligible (b) variations in the vertical velocity profiles may be neglected (c) density effects may be ignored and (d) energy dissipation may be described by the bed friction formula adopted. Amidst the growing sophistication of numerical models and the competing claims of various numerical methods it is sobering to recognise that the flows produced in a model are limited to those motions which may be described by the original equations. At best, the numerical technique approaches the true analytical solution to the equations subject to the imposed boundary conditions; it cannot in any way improve upon this solution. The discussion of the merits of the various techniques revolve around the question as to which methods minimise the inaccuracies and approximations of numerical solutions. In many three dimensional models it is necessary to specify some additional physical expression concerning the velocity profile or nature of the eddy viscosity. In such cases the dependence of the computed flow values upon this additional restriction should be clearly recognised.

2. DATA

(i) General

In a region as intensively studied as the southern North Sea much of the data is essentially historical providing an important generalised description of the area against which specific modelling applications may be formulated. Flow in this region is dominated by the effects of the M_2 tidal constituent, the amplitude of this constituent exceeding the sum of the amplitudes of all other tidal constituents. The amplitude of this constituent exceeds 3m, since the depth of water in this region is of the order of 10-50m, the region is characterised by strong tidal mixing with significant non-linearities producing tide-tide interactions. In the winter period surges of up to 3m in height occur fairly frequently with resulting surge-tide interactions. Frictional forces are important in this region, particularly in respect to current magnitudes for which the following general relationship applies :

$$U_{M_2} > U_{SURGE} > U_{RESIDUAL} \tag{4}$$

(ii) Specific

Specific data implies quantitative field measurements used in either calibrating or evaluating the model. The coastline of this region includes over 25 permanent tide gauge stations providing data of excellent quality extending over periods in excess of 50 years. In the off-shore region there have been numerous current measurements and a limited number of tide-gauge deployments, over the last decade the quality and duration of this off-shore data has improved markedly.

In the Dover Strait measurements of the voltage induced across the channel by the flow of sea water across the vertical component of the earth's magnetic field provide an estimate of the net flow through the channel. These measurements have been made using cross-channel telephone cables[6] and recordings are available over a period of 25 years.

Indications of residual flow patterns over a range of time scales are available from the following sources : (a) movement of drogues and bottom drifters (b) synoptic distributions of salinity and temperature (c) spread of radioactive tracers from the nuclear plant at Cherbourg and (d) observed sand wave movements.

In the following description of modelling studies many comparisons are made between computed and observed measurements, in assessing the validity of these comparisons due regard should be given to the prior filtering involved. In the modelling only a limited part of the spectrum of water movements is considered, the frequency band considered being limited by the validity of equations (1) to (3) and by the specification of the appropriate boundary conditions, some parametric representation of other scales of motion is implicitly included in the friction coefficients. In the case of observed data, filtering may be deliberately incorporated into the instrument design, for example the use of a tide gauge stilling well, or may be carried out at the data processing stage. Although such filtering techniques may be highly efficient it should be recognised that some mutual contamination of the energy spectra occurs in nature that may be impossible to include in the model. Obvious examples include the interaction between the highly energetic tidal and wind wave bands where tidal currents affect wave heights and, conversely, wind wave propagation is accompanied by changes in mean sea level. Interactions of this type mean that certain errors must be accepted between model results and field measurements, thus manipulation of model parameters to force a more and more exact fit may, in reality, reduce the accuracy to which the model simulates the specific phenomenon.

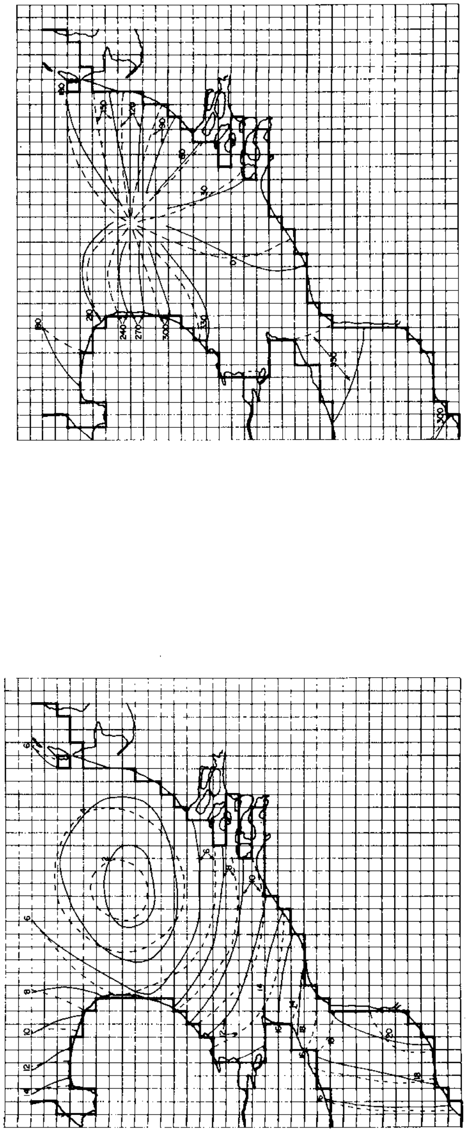
3. TIDES

Because of the predominance of the M_2 tidal constituent it can be shown that it is valid to simulate this constituent in isolation. Likewise in simulating any other tidal constituent it is sufficient to consider only the interaction with M_2.

(i) Boundary Conditions

Tidal elevations along the two open boundaries of the model were specified using data from the Admiralty Charts and from the German Navy tidal atlas. These charts were originally constructed using the method outlined by Proudman and Doodson[12] whereby interpolation between coastal data points is achieved by using off-shore current measurements. Thus the accuracy of the tidal distributions along the open boundaries is questionable and it seems valid to make any minor adjustments that might improve the agreement between model and recorded results in the interior of the region. In the present application, adjustments to the chart data were found to be ineffective.

At the other extreme, in certain applications the paucity of available data may make it necessary to concoct the tidal distribution along the open boundary by a trial and error procedure



(a) (b)

Figure 2. Comparison of (a) co-range (ft) and (b) co-tidal charts; ------- Admiralty, ——— computed.

based on improving the observed fit within the model. Some recent investigations have shown that, mathematically, this procedure presents an ill-posed problem and that numerical solutions may well not converge to the true solution.

(ii) Major tidal constituents

The reproduction of the observed M_2 co-tidal and co-range distributions are shown in figures 2(a) and 2(b) respectively.[4] The agreement shown was achieved using a constant value for the friction factor. Having thus determined a suitable value for this friction factor it is possible to extend the simulation to other major tidal constituents. Tide-tide interactions must then be accounted for by incorporating the influence of the M_2 tide, thus an accurate simulation of, say, S_2 requires the model to be operated for a full 15 days to simulate the complete cycle of $M_2 - S_2$ interactions. As an alternative, since the interaction via the quadratic friction is predominant it is possible to simulate S_2 alone by using a linearised friction term in which the friction coefficient at each position is determined by reference to the M_2 tidal velocities.

(iii) Higher harmonics

The main concern with higher harmonics is the reproduction of M_4 and M_6, in the present model it can be shown that these constituents are generated largely by the non-linear shallow water and friction terms, in other regions the convective terms may be of greater significance.[3] Although the overall spatial distribution of these constituents in the model is in reasonable agreement with observations, this agreement tends to deteriorate in the very shallow regions where the constituents are largest e.g. in the River Thames as shown in figure 3. In such regions it may be shown that the usual assumption that the shallow water term produces M_4 and the friction term M_6 is invalid, figure 4 shows that M_4 and M_6 may be produced by both terms.

The inaccuracy in the reproduction of these constituents is attributed to the inadequacy of the bed friction formula. In some applications it may be justifiable to include additional terms in the friction formula to force an agreement between model and observed

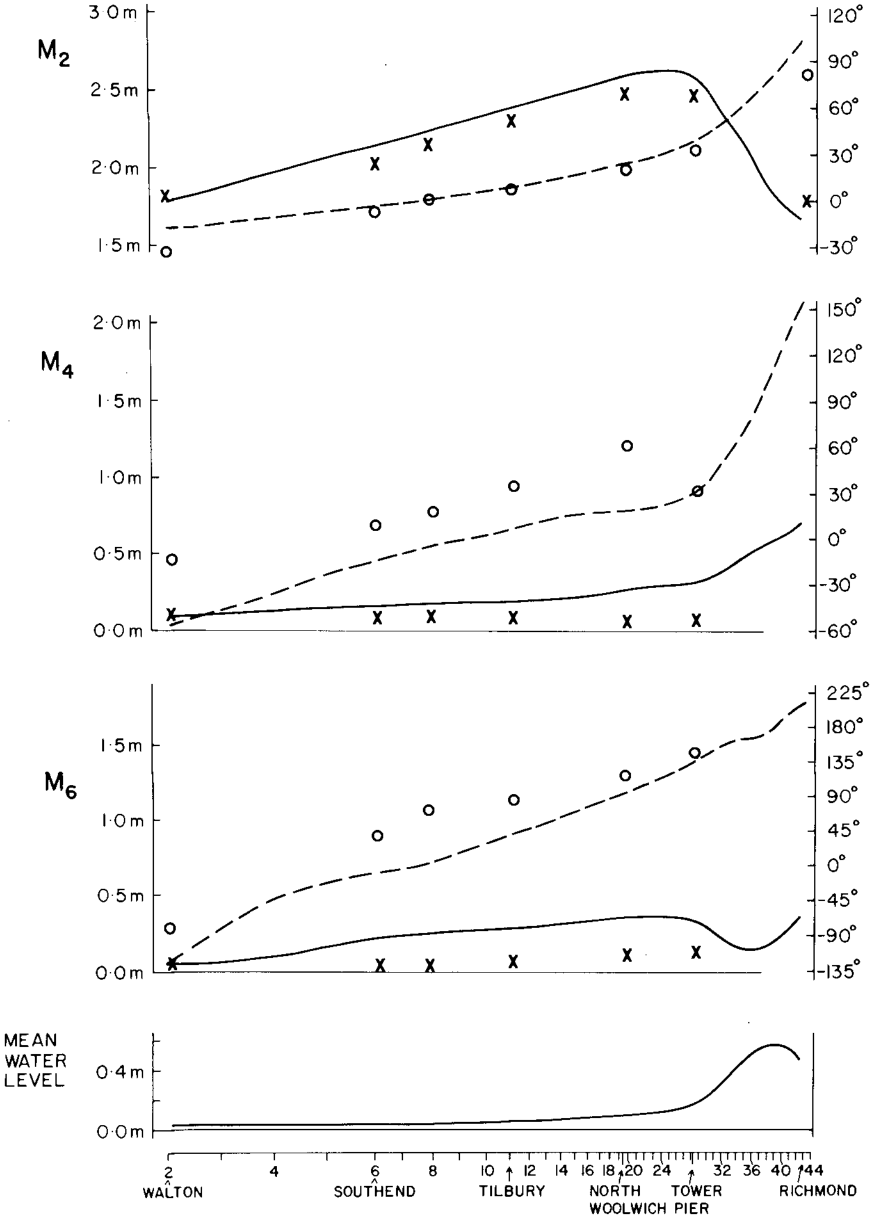


Figure 3. Computed and recorded values of M_2, M_4 and M_6 along the Thames. Tidal amplitude : ———————— computed, x observed; tidal phase : -------------- computed, o observed.

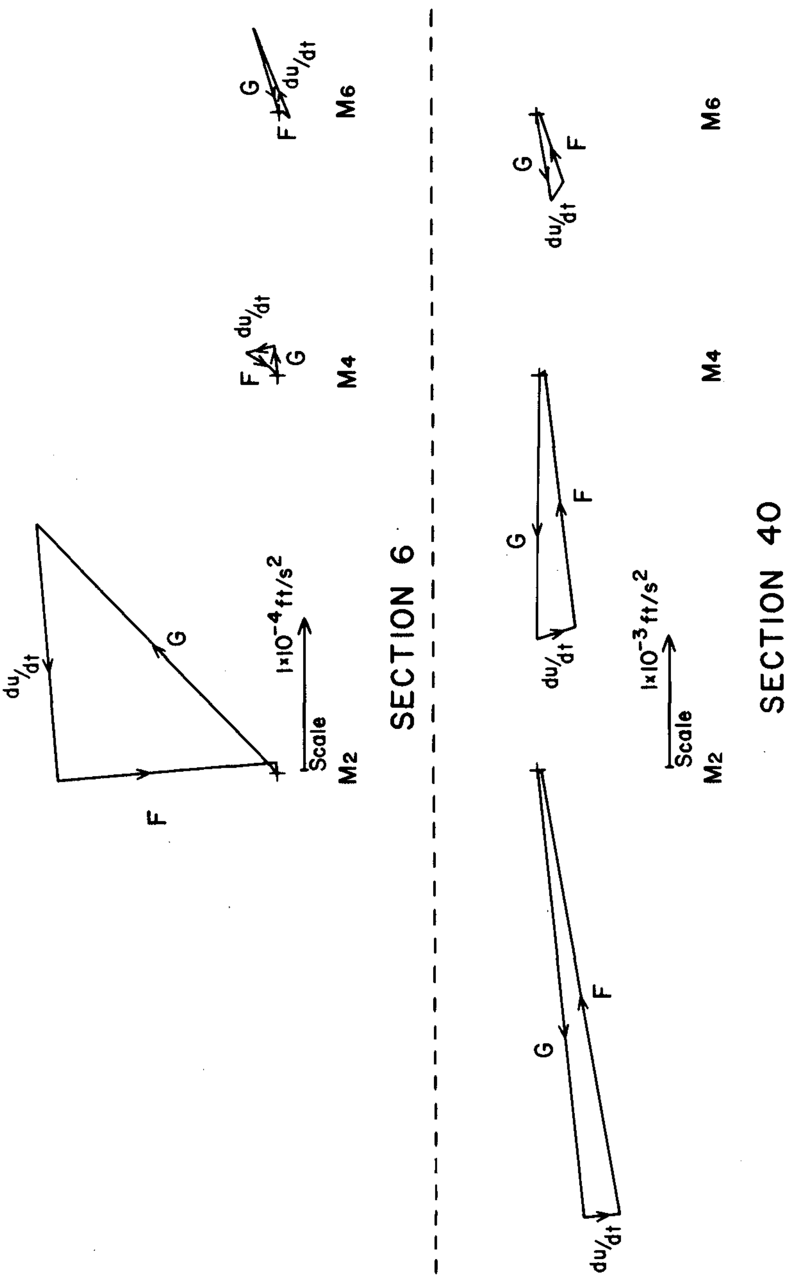


Figure 4. Harmonic constituents for each term in the equation of motion at two positions in the Thames. F - friction term, G - surface gradient, convective term is not labelled.

results. However where there is no clear physical basis for such additional terms the model must be suspect when used for alternative applications i.e. in an extrapolative rather than interpolative mode. An obvious example of this danger is in the simulation of the impact of tidal barriers where, for the River Thames, it has been shown that changes to high and low waters due to barrier construction are more influenced by changes in the M_4 and M_6 constituents than by changes in the predominant M_2 tide.

4. SURGES

(i) 1953 Surge

A detailed examination has been made of the disastrous storm surge of January/February 1953.[5] The simulation of the surge event involved the addition of four components to the tidal model previously described : (a) the observed surge heights along the northern boundary (b) the observed surge heights along the western boundary (c) the wind stress over the surface of the region and (d) the observed horizontal variation in atmospheric pressure. A comparison of observed and computed surge heights showed that the model could accurately simulate surge propagation. Simulating successively (a) the surge alone, (b) tide plus surge and (c) tide alone, then comparing the results from (a) with the values given by (b) - (c) it was possible to identify the importance of surge-tide interaction.

(ii) Sensitivity analyses

An analysis of the influence of each of the four components, (a) to (d) above, was carried out by operating the model of tide plus surge a further four times and on each occasion omitting a single component. In this way it was shown that the surge from the North is of most significance in the Thames followed by the effect of the local wind field. The rapidity of the water level response, in the Thames, to this local wind field demonstrated the difficulty in using statistical methods for storm surge forecasting.

The influence of the surge component from the West is more significant along the Dutch coast than along the English coasts, this explains why the surge forecasting for the Dutch coast takes account of this component while the English system does not.

Sensitivity analyses of this type are subject to misuse, strictly it is only appropriate for linearly independent systems. However it is useful in weakly non-linear processes where each of the separate terms omitted is, by itself, a relatively small part of the total system. A frequent application of this approach is to operate models with and without the Coriolis term, the difference in the results is then assumed to represent the influence of the earth's rotation. However the Coriolis term also influences the boundary conditions and hence the physical interpretation of such sensitivity analyses must be questionable.

(iii) Energy budget

The mechanics of energy transfers during the course of the storm were examined, one facet of this examination is shown in figure 5 where the spatial distribution of energy dissipation by bed friction is plotted. This distribution is a representation of $\int u^3 dt$, in a similar fashion functions such as the stratification parameter u^3/D may be readily displayed.

5. RESIDUALS

Interest in residual flows has grown in recent years for such applications as siting of power plants, sewage outfalls, fisheries studies and so on. Field measurements of residual currents suggested considerable spatial and temporal variability in these flows combined with an obvious dependence on wind forcing. However, prior to the modelling study[7], the mechanics of the residual circulation and its sensitivity to effects such as wind forcing were poorly understood.

(i) Tidal residuals

The non-linear terms present in the equations of motion which describe the propagation of the M_2 tide produce a time invariant (or residual) flow component that accompanies the oscillating component. An examination of these non-linear terms shows that their magnitude is proportional to the square of the tidal range. Once again, because of the predominance of M_2 it may be shown that

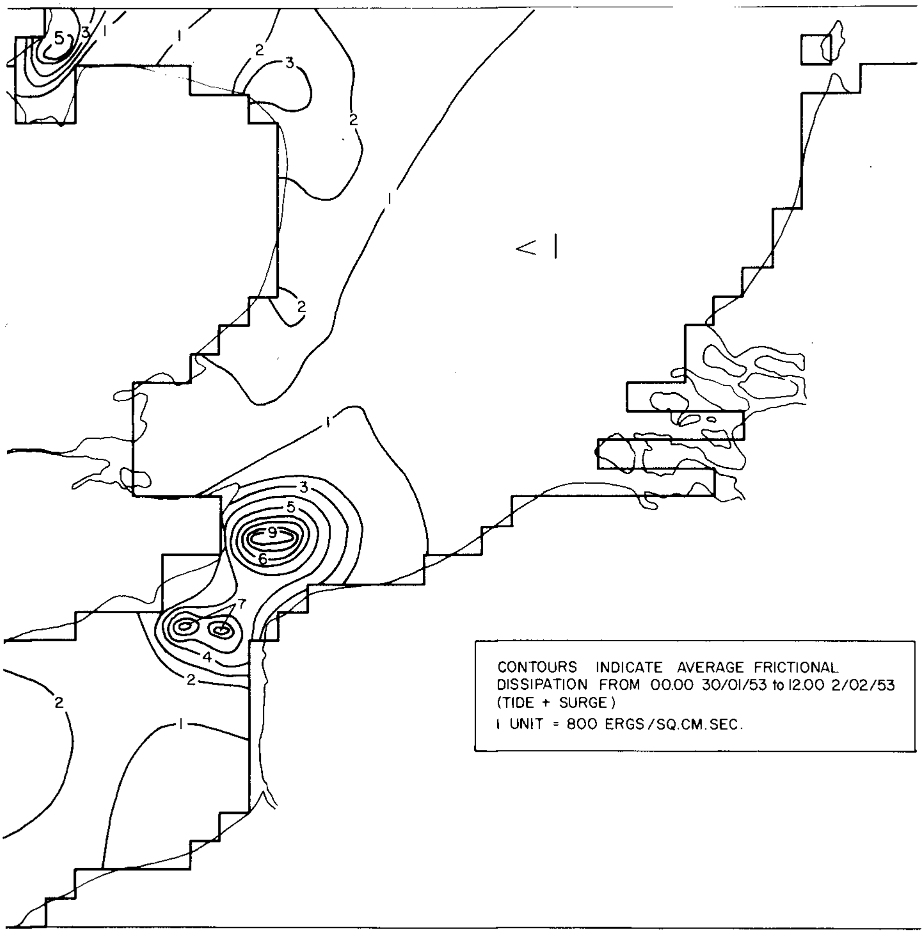


Figure 5. Spatial distribution of frictional dissipation. Mean values over the period 30 Jan to 2 Feb 1953, one unit represents 80 $\mu J\ cm^{-2}s^{-1}$.

the average value of the square of the tidal range is equal to the square of the M_2 tidal range and hence an estimation of the M_2 tidal residuals should approximate the net tidal residual.

In extracting residual flow from a time-stepping simulation of the M_2 tide an erroneous residual circulation develops in the vicinity of the open-boundaries. This circulation is due to the assumption of a constant mean sea level along these boundaries, i.e. the neglect of the tidal stress component.[2] A technique of incorporating a radiation boundary condition was developed to overcome this problem. The tidal residual flow was computed as $8 \times 10^4 m^3 s^{-1}$ into the North Sea.

(ii) <u>Wind driven residuals</u>

Since the residual flows are much smaller than the tidal flows it is possible to describe these flows using linearised equations in which the friction coefficient is determined by reference to the tidal velocities. It is then possible to use linear superpositioning to combine the separate components of residual flow.

In computing the annual-mean wind driven flow three points of general significance emerged.

(a) Wind data from off-shore (in this case Light Vessels) were found to be much more coherent and therefore representative of spatially averaged surface wind stress than corresponding data from adjacent shore-based stations.

(b) The value of monthly mean wind stress calculated from $\sum_{i=1}^{N} W_i^2/N$ for successive 3 hourly wind speed observations W_i was found to be a factor of around 4 times the corresponding value calculated from $\left\{\sum_{i=1}^{N} W_i\right\}^2/N$

(c) The model responds in an irregular fashion to winds from different directions. Thus in order to compute the annual-mean wind driven flow it was necessary to compute separately the response for those periods when the wind was blowing from a particular sector. The net effect was then obtained by adding vectorially the results from each sector. A figure of $4 \times 10^4 m^3 s^{-1}$ was estimated as the annual-mean wind driven flow. The variability of this component over a twenty five year period has been examined.[10]

(iii) Sea surface gradient

Residual flows due to horizontal density gradients were shown to be negligible and, hence, the only remaining residual component was from sea surface gradients. This remaining component was determined by comparing the observed variation in mean sea level along the Continental coast with the corresponding values obtained for the sum of the level variations computed for the tidal and wind driven residual flows. An additional component was then included to obtain the best fit between observed and computed mean sea levels, this component consisted of an imposed sea surface gradient between the two open-boundaries of the model. The flow produced by this additional component was approximately $4 \times 10^4 m^3 s^{-1}$.

(iv) Total residual flow

Combining the three components gives the total residual flow ($16 \times 10^4 m^3 s^{-1}$ into the North Sea) and the spatial variation in mean sea level (figure 6). It was then possible to make four quantitative checks on these results.

(a) A comparison of observed and computed variation in mean sea level along the English Coast.

(b) An estimate of the level difference between the British levelling datum (O.D.N.) and the French levelling datum (I.G.N.) was calculated and compared with an earlier independent estimate.

(c) The total residual flow was compared with a corresponding value obtained from 8 years of continuous cable recordings.

(d) The residual flow in the absence of wind forcing was compared with current meter measurements made in the Dover Strait under tranquil conditions.

6. INTERACTION

An intriguing feature of surge propagation in the Thames is that large surge peaks almost always occur 3 to 4 hours before high tide. This is precisely the time when flood contingency plans have to be decided upon. The existing prediction techniques implicitly assume that the observed surge peak will decrease as high tide approaches.

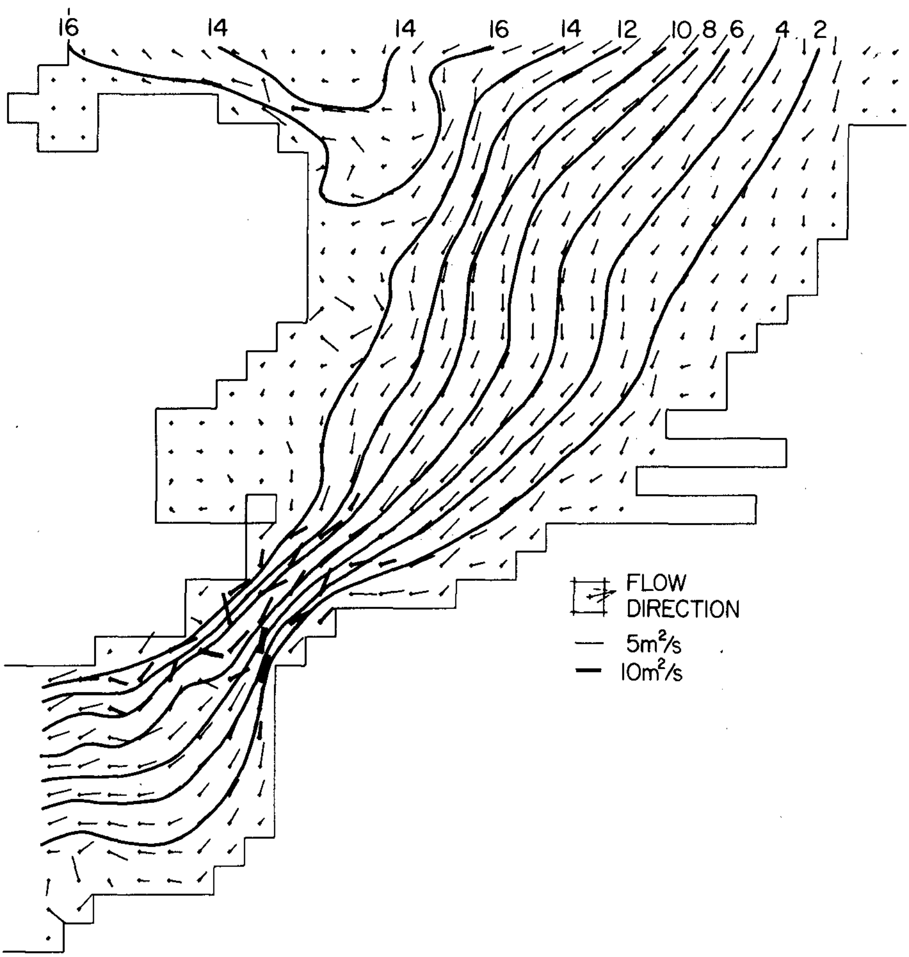


Figure 6. Total residual flow. Streamlines represent $\Psi/(10^4 m^3 s^{-1})$ = constant.

However no clear explanation for this phenonomen had been presented and no guidance was available regarding those circumstances under which the surge peak might be sustained.

(i) Surge statistics

A careful examination[8,9] was made of surge statistics for a number of ports along the east coast of the British Isles and also along the Thames estuary. This showed that surges tend to develop a peak on the rising tide in the Thames irrespective of the phase relationship between the tide and the initial surge development in the northern North Sea. It also showed that although interaction tended to develop progressively as the surges propagated southwards there was an important exception at Lowestoft where interaction appeared to be reduced.

(ii) Parallel models

A new conceptual approach was used to examine the physics of surge-tide interaction, this involved two separate models, one of surge propagation and the other of tidal propagation. The two models are operated simultaneously and interaction, via the non-linear terms, is introduced by cross-linking between the models. To illustrate this consider the quadratic friction term :

$$\underset{\text{(combined model)}}{K\,(U_S+U_T)\,|U_S+U_T|} = \underset{\text{(surge model)}}{K\,U_S\,|U_S+\underline{\underline{U_T}}|} + \underset{\text{(tidal model)}}{K\,U_T\,|\underline{\underline{U_S}}+U_T|} \qquad (5)$$

subscripts S and T refer to surge and tide respectively.

Thus in the model of surge propagation the friction term, is modified by the additional term underlined in (5). Similarly the parameter D in the surge model is modified to $D' = D + Z_T$ to represent the shallow water influence of tidal heights on surge propagation. The validity of this approach rests upon the results from these separate models combining to approximate the results from a combined simulation (subscript c), this requires that

$$Z_c = Z_S + Z_T \qquad (6)$$

$$U_c = U_S + U_T \qquad (7)$$

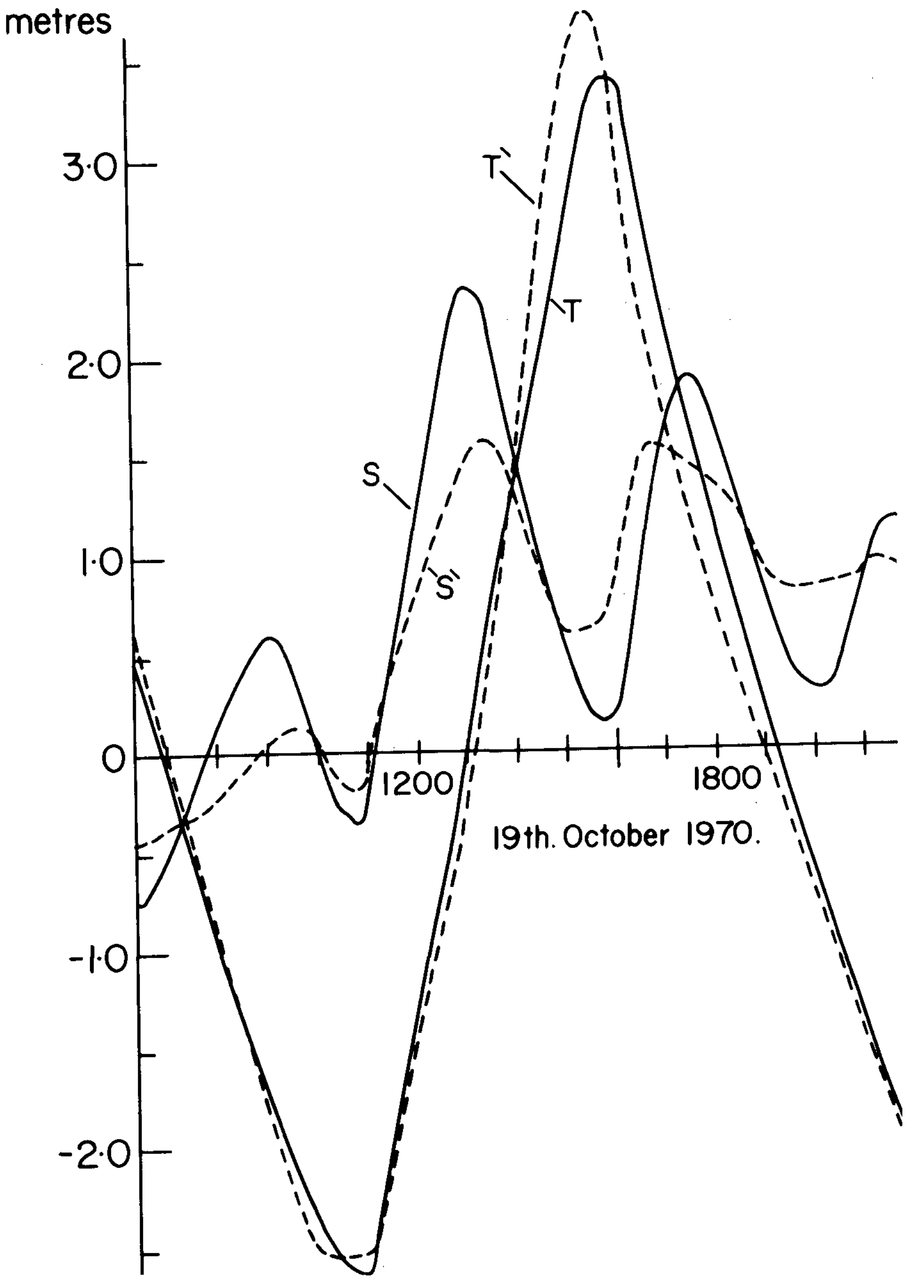


Figure 7. Elevations at Tower Pier.
T - tide alone; S - surge alone; T` tide with interaction from surge; S` surge with interaction from tide.

Figure 7 shows some results for surge and tide elevations at Tower Pier obtained from this approach. The figure contains results from four versions of the model :

(a) a simulation of tide alone T

(b) a simulation of surge alone S

(c) simulation of tide with interaction from the surge T' } Parallel models
(d) simulation of surge with interaction from the tide S' }

One important result which emerged from this approach was that the magnitude of these cross-linkage terms, and hence the magnitude of the interaction, was shown to be proportional to a product of the tidal amplitude and surge height.

(iii) Conclusions

An analysis of results such as those shown in figure 7 revealed the following features of surge-tide interaction.

(a) The non-linear friction term is predominant in producing surge-tide interaction, it is particularly effective in reducing higher-frequency surge peaks.

(b) Shallow water terms produce a phase advance in the tidal component for positive surges and vice versa.

(c) Interaction can develop rapidly in both space and time. The occurrence of surge peaks on the rising tide in the Thames was attributed to interaction, mainly via the friction term, resulting from the large values of both tidal and surge velocities found in the near-shore region between Lowestoft and Walton.

(d) At any instant, the separate interactions of tide on surge propagation and of surge on tidal propagation tend to be in opposition. Thus the observed interaction is the small difference between two larger components. This explains why observed interaction appears to include an unexpected high frequency component and, also, why it is so difficult to relate the time series of interaction directly to either surge or tide.

7. FURTHER PROBLEMS

To conclude this summary of recent studies a brief description is included of current work and some comments included about possible future applications.

(i) Open boundary problem

In a model of limited spatial extent used to examine the impact of some change in channel geometry such as the introduction of a tidal barrier, the problem arises as to the validity of assuming fixed conditions along the open-boundary. An extreme example of this type of problem is the proposed Bay of Fundy tidal power project where it is sensibly suggested that the model might have to extend to the whole of the North Atlantic Ocean in order to overcome this problem. In this particular application, an alternative approach[1] has been developed whereby conditions at the mouth of the model are modified to reflect the combined influences of the model region and the external ocean. A similar approach,[11] employing the analogy between tidal flow and A-C electrical circuit theory, has been used to examine the influence of tidal barriers in the Thames and Bristol Channel.

(ii) Inclusion of higher-order physical mechanisms

Previous reference has been made to the inadequacy of the friction formula as used in the shallow water wave equations. Strong arguments can be expressed suggesting that some account should be taken of changing bed formations; turbulent energy levels; temperature, density and stratification of the water mass and so on. Similar arguments might be used to suggest a wind-stress coefficient varying in time and space. However good results have been obtained by the existing, perhaps crude, parameterisation of these other effects. In practical terms an extension into these higher-order effects must be justified on a number of counts. First it must be shown that present methods are inadequate and second some realistic chance of improving the modelling must be shown.

Such extensions may be necessary for applications such as the simulation of sediment transport. However an extension of the spectral range of the model will also imply an equivalent extension of the data base against which the model must be evaluated. Where the objective of this extension in spectral range is to improve the simulation in the original frequency band it is necessary to have a dynamic feed-back from the higher spectral range into the original. However, in many applications the physics in the higher frequency range are simulated less accurately than is the case for tidal propagation. Hence as the links between different scales of motion become stronger the possibility of reducing the accuracy of tidal propagation is increased.

ACKNOWLEDGEMENTS

The work described in this paper was funded by a Consortium consisting of the natural Environment Research Council, the Ministry of Agriculture, Fisheries and Food, and the Departments of Industry and Energy.

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