CHAPTER 39

WAVE HEIGHT DISTRIBUTION AROUND PERMEABLE BREAKWATERS

by

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ABSTRACT

By superimposing solutions due to Sommerfeld, a calculation was made to obtain the wave height distribution around permeable breakwaters in a constant water depth. The cases dealt with were a semi-infinite breakwater, a single relatively large gap in a long breakwater and a single detached breakwater all with incident waves normal to the breakwater. Some cases were verified through experiments in a shallow water basin.

INTRODUCTION

There are many kinds of permeable breakwater systems. In particular, as a countermeasure against beach erosion, the construction of permeable detached breakwater systems has become widely popular. However, the functioning of such breakwater systems has not yet been fully understood.

Even the wave height distribution around a permeable breakwater, which should be first considered in its construction, has received little effort not in comparison with that made for impermeable breakwater systems, because of the complicated phanomena associated with the former.

Up to data, interest in connection with permeable breakwaters has mainly concentrated on wave reflection and transmission phenomena. Only for some psecial cases in three dimensions has the study of the wave height distribution and related problems already been carried out. For instance, Ijima, Chou and Yumula(1974) studied the scattering of waves around arbitary shape permeable bodies and gave figures for the wave height distributions around circular, ellipsoid and rectangular cells. Their methods are applicable to arbitary shape breakwaters, but it seems that the method requires complicated numerical calculations. The studies above may not give satisfaction to engineers who are faced with problems in practice.

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The purpose of this study is to attempt to clarify the wave height distribution around permeable breakwater systems. However the phenomena are so complicated that applying Sommerfeld's solution for widely deployed breakwater systems was considered as a first stage in the study.

An advantage of the method used in this study, superposition of Sommerfeld's solution, is that wave heights can be calculated very easily by addition and subtraction, if we have the Fresnel Integrals. At present, even on small computers, Fresnel Integrals are prepared as a subroutine or as a function. A few second calculation gives the wave height distribution. Another advantage is that the time lag of waves transmitted through the breakwater can be easily considered in equations, although this was not done here.

Disadvantages of this method are that the boundary conditions are not satisfied and the wave height becomes discontinous on the x and y axis and on the lines which divide the regions considered. Calculated results, however, show that the discontinuity in wave height on the x and y axis and the otehr lines was small. So, we may close our eyes to this discontinuity. Experimental or field verification of the calculated results are not yet sufficient, but this method may be acceptable for engineering use to predict the wave height roughly, although further laboratory and field verification under various conditions must still be made.

Brief Summary of Sommerfeld's Solution:

For the case of an infinitely thin, vertical, rigid, impermeable, semi-infinite breakwater as drawn in Fig. 1, Penny and Price have shown that Sommerfeld's solution of the optical diffraction problem is also a solution of water wave diffraction.

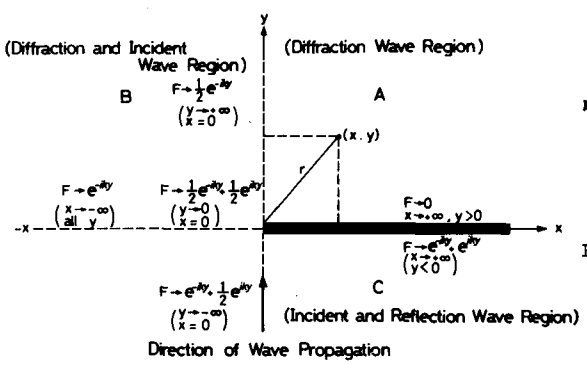

Fig. 1. Nomenclature for wave diffraction analysis at semi-infinite breakwater end.

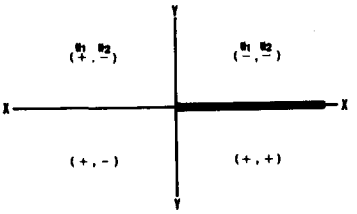

Fig. 2. Signs of u_1 and u_2

The following is a brief summary of the solution by Penny and Price(1952).

$$\phi = Ae^{ikct}\cosh k(h+z)\cdot F(x,y) \quad \cdots\cdots (1)$$

$$\eta = \frac{Aikc}{g}e^{ikct}\cosh kh\cdot F(x,y) \quad \cdots\cdots (2)$$

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + k^2 F = 0 \quad \cdots\cdots (3)$$

where

$i = \sqrt{-1}$ $\quad k = 2\pi/L$

L = wave length $\quad c$ = phase velocity

h = water depth $\quad g$ = gravity acceleration

$$F(x,y) = \frac{1}{\sqrt{2}}e^{i(\pi/4-ky)}\int_{-\infty}^{u_1} e^{-i\pi w^2/2}dw + \frac{1}{\sqrt{2}}e^{i(\pi/4+ky)}\int_{-\infty}^{u_2} e^{-i\pi w^2/2}dw = e^{-iky}f(u_1) + e^{iky}g(u_2) \quad \cdots\cdots (4)$$

$$u_1 = \pm\sqrt{\frac{4(r-y)}{L}}, \quad u_2 = \pm\sqrt{\frac{4(r+y)}{L}} \quad \cdots\cdots (5)$$

$$r = \sqrt{x^2+y^2}$$

$$f(u_1) = \frac{1}{\sqrt{2}}e^{i\pi/4}\int_{-\infty}^{u_1} e^{-i\pi w^2/2}dw \quad \cdots\cdots (6)$$

$$g(u_2) = \frac{1}{\sqrt{2}}e^{i\pi/4}\int_{-\infty}^{u_2} e^{-i\pi w^2/2}dw \quad \cdots\cdots (7)$$

$$f(u) = g(u) = \frac{1}{\sqrt{2}}e^{i\pi/4}\int_{-\infty}^{u} e^{-i\pi w^2/2}dw = \frac{1}{2}\left[\{1+C(u)+S(u)\} + i\{C(u)-S(u)\}\right] \quad \cdots\cdots (8)$$

$$f(-u) = g(-u) = \frac{1}{\sqrt{2}}e^{i\pi/4}\int_{-\infty}^{-u} e^{-i\pi w^2/2}dw = \frac{1}{2}\left[\{1-C(u)-S(u)\} + i\{S(u)-C(u)\}\right] = S + iW \quad \cdots\cdots (9)$$

$$C(u) = \int_0^u \cos\frac{\pi}{2}w^2 dw, \quad S(u) = \int_0^u \sin\frac{\pi}{2}w^2 dw \quad \cdots\cdots (10)$$

$$S = \frac{1}{2}\{1 - C(u) - S(u)\}$$

$$W = \frac{1}{2}\{S(u) - C(u)\} \quad \cdots\cdots (11)$$

The diffraction coefficient, K, is defined as the ratio of wave height in the area affected by diffraction to the wave height in the area unaffected by diffraction, and given by the modulus of F(x, y) for the diffracted wave: taht is

$$K = |F(x, y)| = \sqrt{R^2 + I^2} \quad (12)$$

R: Real component of F(x, y)

I: Imaginary component of F(x, y)

The final forms of F(x, y) for numerical calculation for a breakwater located on the + x axis as shown in Fig. 1 are:

Region A, x > 0, y > 0

$$F_0^A(x,y) = e^{-iky} f(-u_1) + e^{iky} g(-u_2)$$
$$= (S_1 + S_2)\cos ky + (W_1 - W_2)\sin ky$$
$$+ i[(W_1 + W_2)\cos ky + (-S_1 + S_2)\sin ky] \text{ -------- } (13)$$

Region B, x < 0,

$$F_0^B(x,y) = e^{-iky} f(u_1) + e^{iky} g(-u_2)$$
$$= (1 - S_1 + S_2)\cos ky + (-W_1 - W_2)\sin ky$$
$$+ i[(-W_1 + W_2)\cos ky + (-1 + S_1 + S_2)\sin ky] \text{ ------- } (14)$$

Region C, x > 0, y < 0

$$F_0^C(x,y) = e^{-iky} f(u_1) + e^{iky} g(u_2)$$
$$= (2 - S_1 - S_2)\cos ky + (-W_1 + W_2)\sin ky$$
$$+ i[(-W_1 - W_2)\cos ky + (S_1 - S_2)\sin ky] \text{ -------- } (15)$$

As a matter of convenience later, we shall give the final forms of F(x,y) for numerical calculation for a breakwater located on the - x axis as shown in Fig. 3.

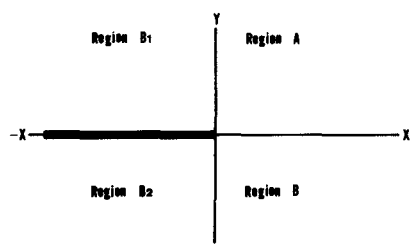

Fig. 3. Nomenclature for wave diffraction analysis at breakwater when located on -x axis.

Region A and C

$$F_1^A(x,y) = e^{-iky} f(u_1) + e^{iky} g(-u_2)$$
$$= (1 - S_1 + S_2)\cos ky + (-W_1 - W_2)\sin ky$$
$$+ i[(-W_1 + W_2)\cos ky + (-1 + S_1 + S_2)\sin ky] \text{ ------ } (16)$$

Region B1

$$F_1^{B1}(x,y) = e^{-iky} f(-u_1) + e^{iky} g(-u_2)$$
$$= (S_1 + S_2)\cos ky + (W_1 - W_2)\sin ky$$
$$+ i[(W_1 + W_2)\cos ky + (-S_1 + S_2)\sin ky] \text{ ------- } (17)$$

Region B2

$$F_1^{B2}(x,y) = e^{-iky} f(u_1) + e^{iky} g(u_2)$$
$$= (2 - S_1 - S_2)\cos ky + (-W_1 + W_2)\sin ky$$
$$+ i[(-W_1 - W_2)\cos ky + (S_1 - S_2)\sin ky] \text{ ------- } (18)$$

NUMERICAL CALCULATIONS OF WAVE HEIGHT DISTRIBUTION FOR PERMEABLE BREAKWATER

(1). A single small gap in a long permeable breakwater as shown in Fig. 4.

The heights of waves which have passed through a single gap small compared with the wave length of a long impermeable breakwater have been already given by Lamb (1945).

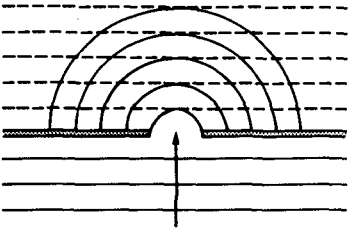

Fig. 4. Diffraction at small gap

The solution is:

$$F_b(x,y) = \frac{\sqrt{\frac{L}{4r}}}{\left(\log\frac{kb}{4} + \gamma + \frac{\pi}{2}i\right)} e^{-i(\frac{\pi}{4}+kr)} \quad \cdots\cdots (19)$$

γ: Constant of Euler

$$F_a = \alpha e^{-iky} \quad \cdots\cdots (20)$$

This equation shows a wave which concentrically spreads from a gap. When the breakwater is permeable we assume that the wave heights inside the breakwater can be express of by superimposing Equations (19) and (20). That is:

$$F = F_a + F_b$$
$$= \alpha e^{-iky} + \frac{\sqrt{\frac{L}{4r}}}{\left(\log\frac{kb}{4} + \gamma + \frac{\pi}{2}i\right)} e^{-i(\frac{\pi}{4}+kr)}$$

The superimposed result is:

$$F = \alpha\cos ky + \sqrt{\frac{\pi}{2kr}}\sqrt{C_r^2 + C_i^2}\cos\left(\frac{\pi}{4} + kr - \theta\right)$$
$$-i\left\{\alpha\sin ky + \sqrt{\frac{\pi}{2kr}}\sqrt{C_r^2 + C_i^2}\sin\left(\frac{\pi}{4} + kr - \theta\right)\right\} \quad \cdots\cdots (21)$$

where

$$C_r = \frac{\log\frac{kb}{4} + \gamma}{\left(\log\frac{kb}{4} + \gamma\right)^2 + \frac{\pi^2}{4}}$$

$$C_i = \frac{-\frac{\pi}{2}}{\left(\log\frac{kb}{4} + \gamma\right)^2 + \frac{\pi^2}{4}}$$

$$\theta = \tan^{-1}\frac{C_i}{C_r}$$

The wave heights are given by:

$$R = \alpha\cos ky + \sqrt{\frac{\pi}{2kr}}\sqrt{C_r^2 + C_i^2}\cos\left(\frac{\pi}{4} + kr - \theta\right)$$
$$I = -\left\{\alpha\sin ky + \sqrt{\frac{\pi}{2kr}}\sqrt{C_r^2 + C_i^2}\sin\left(\frac{\pi}{4} + kr - \theta\right)\right\}$$
$$K = \sqrt{R^2 + I^2}$$

Diagrams for a gap width b, b/L ratios of 0.1 and 0.2 and transmission coefficients, α, from 0.1 to 0.5 are given in Appendix 1.

(2) A semi-infinite permeable breakwater

For the waves on the shore side of the breakwater, regions A and B1 ($y > 0$) as shown Fig 5-(3), superposition of waves as shown in Fig 5-(1) and the waves as shown in Fig 5-(2) is assumed.

On the other hand, for the waves on the offshore side of the permeable breakwater, regions B2 and C ($y < 0$), it is assumed that the waves can be expressed by where the reflection coefficient, β, multiplies a free wave propagating in the - y direction, e^{iky}.

$$e^{-iky}f(u_1) + \beta e^{iky}g(u_2) \text{----------} (22)$$

Eqations based on those assumptions in their final forms are:

Region A $\quad x>0\ ,\ y>0$

$$F_0(x,y) = e^{-iky}f(-u_1) + e^{iky}g(-u_2)$$
$$F_1(x,y) = e^{-iky}f(u_1) + e^{iky}g(-u_2)$$

$$F = F_0 + \alpha F_1$$

$$F_2^A = \{\alpha + (1-\alpha)S_1 + (1+\alpha)S_2\}\cos ky + \{(1-\alpha)W_1 - (1+\alpha)W_2\}\sin ky$$
$$+ i\left[\{(1-\alpha)W_1 + (1+\alpha)W_2\}\cos ky - \{\alpha + (1-\alpha)S_1 - (1+\alpha)S_2\}\sin ky\right] \text{----------} (23)$$

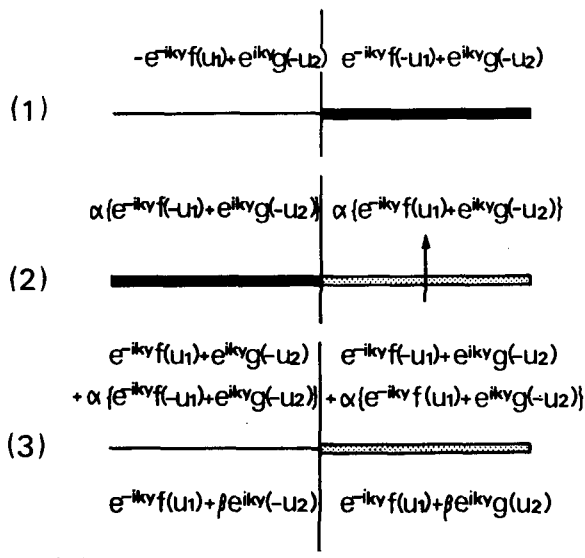

Fig. 5. Superposition of waves for semiinfinite permeable breakwater.

Region B_1 $\quad x<0\ ,\ y>0$

$$F_0(x,y) = e^{-iky}f(u_1) + e^{iky}g(-u_2)$$
$$F_1(x,y) = e^{-iky}f(-u_1) + e^{iky}g(-u_2)$$

$$F = F_0 + \alpha F_1$$

$$F_2^{B_1} = (1-(1-\alpha)S_1 + (1+\alpha)S_2)\cos ky + \{-(1-\alpha)W_1 - (1+\alpha)W_2\}\sin ky$$
$$+ i\left[\{-(1-\alpha)W_1 + (1+\alpha)W_2\}\cos ky - \{1-(1-\alpha)S_1 - (1+\alpha)S_2\}\sin ky\right] \quad \text{----------(24)}$$

Region B_2 $\quad x<0\ ,\ y<0$

$$F = e^{-iky}f(u_1) + \beta e^{-iky}g(-u_2)$$

$$F_2^{B_2} = (1 - S_1 + \beta S_2)\cos ky + (-W_1 - \beta W_2)\sin ky$$
$$+ i\left[(-W_1 + \beta W_2)\cos ky + (-1 + S_1 + \beta S_2)\sin ky\right] \quad \text{----------(25)}$$

Region C $\quad x>0\ ,\ y<0$

$$F = e^{-iky}f(u_1) + \beta e^{iky}g(u_2)$$

$$F_2^{C} = (1+\beta - S_1 - \beta S_2)\cos ky - (W_1 - \beta W_2)\sin ky$$
$$- i\left[(1-\beta - S_1 + \beta S_2)\sin ky + (W_1 + \beta W_2)\cos ky\right] \quad \text{----------(26)}$$

$$\alpha + \beta = 1$$

$$S_1 = 1 - C(u_1) - S(u_1)\ , \qquad W_1 = C(u_1) - S(u_1)$$
$$S_2 = 1 - C(u_2) - S(u_2)\ , \qquad W_2 = C(u_2) - S(u_2)$$

where

$$C(u) = \int_0^u \cos\frac{\pi}{2}w^2\,dw\ , \qquad S(u) = \int_0^u \sin\frac{\pi}{2}w^2\,dw$$

If we ignore the energy loss when waves pass the breakwater, α the transmission coefficient plus β, the reflection coefficient equal unity. Taking α equal to zero, the final equations agree with the solution of a impermeable breakwater. However, if α is not zero, the waves becomes discontinuous on the x-axis. Calculated results with $\alpha = 0.0$ through $\alpha = 0.5$ are given in Appendix 2.

(3) A single relatively large gap in a long permeable breakwater.

For the case of an impermeable breakwater, Penny and Price(1952) have laready given the solution by superimposing Eqs. (13), (14) and (15) and Eqs. (16), (17) and (18). Johnson(1952; 1953), Morihira and Okuyama(1966) and others have given detailed diagrams for engineering practice.

We shall write again here the final forms of the solution for numerical calculation because they will be needed the in succeeding sections. Suffxs r and ℓ, indicating the cases when the breakwater is located at the + x and − x regions(Fig. 1 and Fig. 2), will be added.

The final forms are:

Region A1, 0 < x < b/2, y > 0

$$\begin{aligned} F_3^{A1}(x,y) &= e^{-iky}f_r(u_1) + e^{iky}g_r(-u_2) + e^{-iky}f_\ell(u_1) + e^{iky}g_\ell(-u_2) - e^{-iky} \\ &= (1 - S_{r1} - S_{\ell 1} + S_{r2} + S_{\ell 2})\cos ky + (-W_{r1} - W_{\ell 1} - W_{r2} - W_{\ell 2})\sin ky \\ &\quad + i[(-W_{r1} - W_{\ell 1} + W_{r2} + W_{\ell 2})\cos ky \\ &\quad + (-1 + S_{r1} + S_{\ell 1} + S_{r2} + S_{\ell 2})\sin ky] \quad \text{---------- (27)} \end{aligned}$$

Region A2, b/2 < x, y > 0

$$\begin{aligned} F_3^{A2}(x,y) &= e^{-iky}f_r(-u_1) + e^{iky}g_r(-u_2) + e^{-iky}f_\ell(u_1) + e^{iky}g_\ell(-u_2) - e^{-iky} \\ &= (S_{r1} - S_{\ell 1} + S_{r2} + S_{\ell 2})\cos ky + (W_{r1} - W_{\ell 1} - W_{r2} - W_{\ell 2})\sin ky \\ &\quad + i[(W_{r1} - W_{\ell 1} + W_{r2} + W_{\ell 2})\cos ky \\ &\quad + (-S_{r1} + S_{\ell 1} + S_{r2} + S_{\ell 2})\sin ky] \quad \text{---------- (28)} \end{aligned}$$

Region C2, x > b/2, y < 0

$$
\begin{aligned}
F_3^{C2}(x,y) &= e^{-iky} f_r(u_1) + e^{iky} g_r(u_2) + e^{-iky} f_\ell(u_1) + e^{iky} g_\ell(-u_2) - e^{-iky} \\
&= (2 - S_{r1} - S_{\ell 1} - S_{r2} + S_{\ell 2}) \cos ky + (-W_{r1} - W_{\ell 1} + W_{r2} - W_{\ell 2}) \sin ky \\
&\quad + i[(-W_{r1} - W_{\ell 1} - W_{r2} + W_{\ell 2}) \cos ky \\
&\quad + (S_{r1} + S_{\ell 1} - S_{r2} + S_{\ell 2}) \sin ky] \quad \text{----------} (29)
\end{aligned}
$$

In the case of a permeable breakwater, the same idea described in the preceding section (2) is assumed. To express the waves on the shore side as show in Fig. 6-(4), the waves in Figs. 6-(2) and 6-(3) are superimposed on the waves as shown Fig. 6-(1). On the offshore side of the breakwater, a reflection coefficient, β, multiplies all terms, e^{iky}.

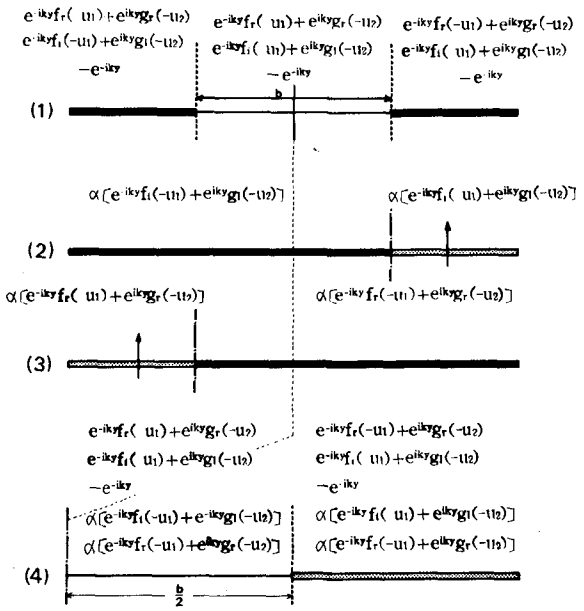

Fig. 6. Superposition of waves for a relatively large gap in permeable breakwater.

According to this assumption. The F(x,y) which should be added to $F_3(x, y)$ because the breakwater is permeable are:

Region A1, $0 < x < b/2$, $y > 0$

$$F(x,y) = \alpha\left[e^{-iky} f_\ell(-u_1) + e^{iky} g_\ell(-u_2)\right] + \alpha\left[e^{-iky} f_r(-u_1) + e^{iky} g_r(-u_2)\right]$$
$$= \alpha\left[F_1^{B1}(x,y) + F_0^A(x,y)\right] \quad \text{----------} (30)$$

Region A2, $b/2 < x$, $y > 0$

$$F(x,y) = \alpha\left[e^{-iky} f_\ell(u_1) + e^{iky} g_\ell(-u_2)\right] + \alpha\left[e^{-iky} f_r(-u_1) + e^{iky} g_r(-u_2)\right.$$
$$= \alpha\left[F_1^A(x,y) + F_0^A(x,y)\right] \quad \text{----------} (31)$$

We shall rewrite the function F_0^A, F_1^A, and F_1^{B1}, adding the suffixes, r, ℓ.

$$F_0^A(x,y) = (S_{r1} + S_{r2})\cos ky + (W_{r1} - W_{r2})\sin ky$$
$$+ i\left[(W_{r1} + W_{\ell 2})\cos ky + (-S_{r1} + S_{r2})\sin ky\right] \quad \text{-----} (14)'$$

$$F_1^A(x,y) = (1 - S_{\ell 1} + S_{\ell 2})\cos ky + (-W_{\ell 1} - W_{\ell 2})\sin ky$$
$$+ i\left[(-W_{\ell 1} + W_{\ell 2})\cos ky + (-1 + S_{\ell 1} + S_{\ell 2})\sin ky\right] \quad \text{----} (16)'$$

$$F_1^{B1}(x,y) = (S_{\ell 1} + S_{\ell 2})\cos ky + (W_{\ell 1} - W_{\ell 2})\sin ky$$
$$+ i[(W_{\ell 1} + W_{\ell 2})\cos ky + (-S_{\ell 1} + S_{\ell 2})\sin ky] \text{-----} (17)'$$

Now, we shall consider the coordinate system.
For Eq. (28), (29) and (27)

$$r_r = \sqrt{(x - b/2)^2 + y^2}$$
$$r_\ell = \sqrt{(x + b/2)^2 + y^2}$$

On the other hand, for Eq. (14)', (16)' and (17)'

$$r_\ell = \sqrt{(x - b/2)^2 + y^2}$$
$$r_r = \sqrt{(x + b/2)^2 + y^2}$$

If we exchange the suffixes r and ℓ in Eqs. (14)', (16)' and (17)' we can use the same coordinate system in Eqs. (28), (29), (27), (14)', (16)' and (17)'.

Finally, we have:

Region A1, $0 < x < b/2$, $y > 0$

$$F_{3P}^{A1}(x,y) = F_3^{A1}(x,y) + \alpha[F_1^{B1}(x,y) + F_0^{A}(x,y)]$$
$$= \{1-(1-\alpha)S_{r1} - (1-\alpha)S_{\ell 1} + (1+\alpha)S_{r2} + (1+\alpha)S_{\ell 2}\}\cos ky$$
$$+\{-(1-\alpha)W_{r1} - (1-\alpha)W_{\ell 1} - (1+\alpha)W_{r2} - (1+\alpha)W_{\ell 2}\}\sin ky$$
$$+ i[\{-(1-\alpha)W_{r1} - (1-\alpha)W_{\ell 1} + (1+\alpha)W_{r2} + (1+\alpha)W_{\ell 2}\}\cos ky$$
$$+\{-1+(1-\alpha)S_{r1} + (1-\alpha)S_{\ell 1} + (1+\alpha)S_{r1} + (1+\alpha)S_{\ell 2}\}\sin ky]$$
$$\text{----------} (32)$$

Region A2, x > b/2, y > 0

$$F_{3P}^{A2} = F_3^{A2} + \alpha\,[\,F_1^A + F_0^A\,]$$

$$= \{\alpha + (1-\alpha)S_{r1} - (1-\alpha)S_{\ell 1} + (1+\alpha)S_{r2} + (1+\alpha)S_{\ell 2}\}\cos ky$$

$$+ \{(1-\alpha)W_{r1} - (1-\alpha)W_{\ell 1} - (1+\alpha)W_{r2} - (1+\alpha)W_{\ell 2}\}\sin ky$$

$$+ i[\{(1-\alpha)W_{r1} - (1-\alpha)W_{\ell 1} + (1+\alpha)W_{r2} + (1+\alpha)W_{\ell 2}\}\cos ky$$

$$+ \{-\alpha - (1-\alpha)S_{r1} + (1-\alpha)S_{\ell 1} + (1+\alpha)S_{r2} + (1+\alpha)S_{\ell 2}\}\sin ky] \qquad (33)$$

Region C2, x > b/2, y < 0

$$F_{3P}^{C2} = e^{-iky} f_r(u_1) + \beta e^{iky} g_r(u_2) + e^{-iky} f_\ell(u_1) + \beta e^{iky} g_\ell(-u_2) - e^{-iky}$$

$$= \{1 + \beta - S_{r1} - S_{\ell 1} + \beta(-S_{r2} + S_{\ell 2})\}\cos ky$$

$$+ \{-W_{r1} - W_{\ell 1} + \beta(W_{r2} - W_{\ell 2})\}\sin ky$$

$$+ i[\{-W_{r1} - W_{\ell 1} + \beta(-W_{r2} + W_{\ell 2})\}\cos ky$$

$$+ \{-1 + \beta + S_{r1} + S_{\ell 1} + \beta(-S_{r2} + S_{\ell 2})\}\sin ky] \qquad (34)$$

(4) A single impermeable detached breakwater.

For a single detached breakwater or an insular breakwater that allows waves to diffract around both ends as shown in Fig. 7, Morse and Rubenstein (1938) and Goda, Yoshimura, Ito(1971) developed an exact theory. Numerical calculations for the theory can be done in terms of Mathieu functions. Detailed figures for this have been prepared by Goda et al.(1971). However, if the length of the breakwater is relatively large compared with the wave length, it could be considered that the each end of the breakwater is the end of a semi-infinite breakwater.

The following are the final forms arrived at from this assumption. Detailed diagrams are not given here because of limited space. However, such diagrams may easily be reproduced from the equations.

Region A1, $0 < x < b/2$, $y > 0$

$$F_4^{A1} = (S_{r1} + S_{\ell 1} + S_{r2} + S_{\ell 2})\cos ky + (W_{r1} + W_{\ell 1} - W_{r2} - W_{\ell 2})\sin ky$$
$$+ i[(W_{r1} + W_{\ell 1} + W_{r2} + W_{\ell 2})\cos ky + (-S_{r1} - S_{\ell 1} + S_{r2} + S_{\ell 2})\sin ky] \quad \text{----------(35)}$$

Region A2, $x > b/2$, $y > 0$

$$F_4^{A2} = (1 + S_{r1} - S_{\ell 1} + S_{r2} + S_{\ell 2})\cos ky + (W_{r1} - W_{\ell 1} - W_{r2} - W_{\ell 2})\sin ky$$
$$+ i[(W_{r1} - W_{\ell 1} + W_{r2} + W_{\ell 2})\cos ky + (-1 - S_{r1} + S_{\ell 1} + S_{r2} + S_{\ell 2})\sin ky] \quad \text{----------(36)}$$

Region C1, $0 < x < b/2$, $y < 0$

$$F_4^{C1} = (2 - S_{r1} - S_{\ell 1} - S_{r2} - S_{\ell 2})\cos ky + (-W_{r1} - W_{\ell 1} + W_{r2} + W_{\ell 2})\sin ky$$
$$+ i[(-W_{r1} - W_{\ell 1} - W_{r2} - W_{\ell 2})\cos ky + (S_{r1} + S_{\ell 1} - S_{r2} - S_{\ell 2})\sin ky] \quad \text{----------(37)}$$

Region C2, $x > b/2$, $y < 0$

$$F_4^{C2} = (1 - S_{r1} - S_{\ell 1} - S_{r2} + S_{\ell 2})\cos ky + (-W_{r1} - W_{\ell 1} + W_{r2} - W_{\ell 2})\sin ky$$
$$+ i[(-W_{r1} - W_{\ell 1} - W_{r2} + W_{\ell 2})\cos ky + (-1 + S_{r1} + S_{\ell 1} - S_{r2} + S_{\ell 2})\sin ky] \quad \text{----------(38)}$$

When the breakwater is permeable. We assume that the wave in regions A and B1 ($y > 0$) are the superimposed waves arising from an impermeable breakwater as discribed above, and the waves multiplied by α passing through a gap as described in (3) [Fig. 8].

In region $y < 0$, the idea is the same in the proceeding sections.

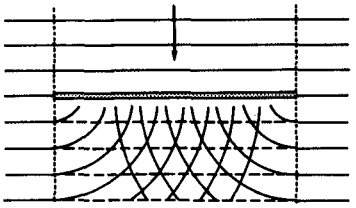

Fig. 7. Diffraction around detached breakwater.

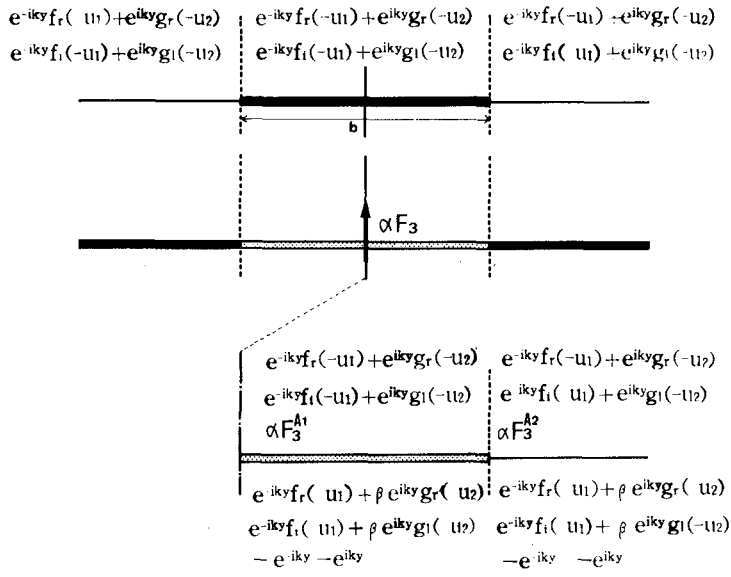

Fig. 8. Superposition of waves for permeable detached breakwater.

We again review the coordinate system.

In Eqs. (36), (37), (38) and (35)

$$r_\ell = \sqrt{(x - b/2)^2 + y^2} \qquad r_r = \sqrt{(x + b/2)^2 + y^2}$$

In Eqs. (27), (28) and (29)

$$r_\ell = \sqrt{(x + b/2)^2 + y^2} \qquad r_r = \sqrt{(x - b/2)^2 + y^2}$$

To use the same coordinate system, we must exchange the suffixes r and ℓ in Eqs. (27), (28) and (29). Exchanging suffixes r and ℓ in Eqs. (27) and (28) and adding Eq. (36) and Eq. (28) to Eq. (37).

We have finally:

$$\begin{aligned} F_5^{A1} &= F_4^{A1} + \alpha F_3^{A1} \\ &= \cos ky \{\alpha + (1-\alpha) S_{r1} + (1-\alpha) S_{\ell 1} + (1+\alpha) S_{r1} + (1+\alpha) S_{\ell 2}\} \\ &\quad + \sin ky \{(1-\alpha) W_{r1} + (1-\alpha) W_{\ell 1} - (1+\alpha) W_{r2} - (1+\alpha) W_{\ell 2}\} \\ &\quad + i[\cos ky \{(1-\alpha) W_{r1} + (1-\alpha) W_{\ell 1} + (1+\alpha) W_{r2} + (1+\alpha) W_{\ell 2}\} \\ &\quad + \sin ky \{-\alpha - (1-\alpha) S_{r1} - (1-\alpha) S_{\ell 1} + (1+\alpha) S_{r2} + (1+\alpha) S_{\ell 2}\}] \end{aligned} \qquad \text{----------(39)}$$

$$\begin{aligned} F_5^{A2} &= F_4^{A2} + \alpha F_3^{A2} \\ &= \cos ky \{1 + (1-\alpha) S_{r1} - (1-\alpha) S_{\ell 1} + (1+\alpha) S_{r2} + (1+\alpha) S_{\ell 2}\} \\ &\quad + \sin ky \{(1-\alpha) W_{r1} - (1-\alpha) W_{\ell 1} - (1+\alpha) W_{r2} - (1+\alpha) W_{\ell 2}\} \\ &\quad + i[\cos ky \{(1-\alpha) W_{r1} - (1-\alpha) W_{\ell 1} + (1+\alpha) W_{r2} + (1+\alpha) W_{\ell 2}\} \\ &\quad + \sin ky \{-1 - (1-\alpha) S_{r1} + (1-\alpha) S_{\ell 1} + (1+\alpha) S_{r2} + (1+\alpha) S_{\ell 2}\}] \end{aligned} \qquad \text{----------(40)}$$

$$F_5^{c1} = e^{-iky} f_r(u_1) + \beta e^{iky} g_r(u_2) + e^{-iky} f_\ell(u_1) + \beta e^{iky} g_\ell(u_2) - e^{-iky} - \beta e^{iky}$$
$$= \cos ky\{1+\beta - S_{r1} - S_{\ell 1} - \beta(S_{r2}+S_{\ell 2})\} + \sin ky\{-W_{r1} - W_{\ell 1} + \beta(W_{r2}+W_{\ell 2})\}$$
$$+ i[\cos ky\{-W_{r1} - W_{\ell 1} - \beta(W_{r2}+W_{\ell 2})\} + \sin ky\{-1 + S_{r1} + S_{\ell 1} + \beta(1 - S_{r2} - S_{\ell 2})\}] \qquad (41)$$

Preliminary experiments

Preliminary experiments were carried out in a small water basin 0.5 m deep, 1.0 m wide and 11 m long. Vertical homogeneous crib-style walls of 8 cm thickness filled with glass ball in diameter were used as the model breakwater.

Because of the limited basin width, standing waves appeared on the shore side of the breakwater and the results were not considered adequate for examining the theoretical development. The following are some general experimental results.

(1) Eq. (21) is acceptable for situations where the ratio between the gap width and wave length is less than 0.2.

(2) After the distance of one wave length, calculated diffraction patterns by Eqs. (23) and (24) were similar to the experimental patterns.

(3) Wave heights at the gaps were not constant when the widths of gaps were less than one wave length.

(4) Energy transmitted to the shore side region of the breakwater was proportional to the ratio of the gap width, to the wave length, within the range less than a ratio of 0.5.

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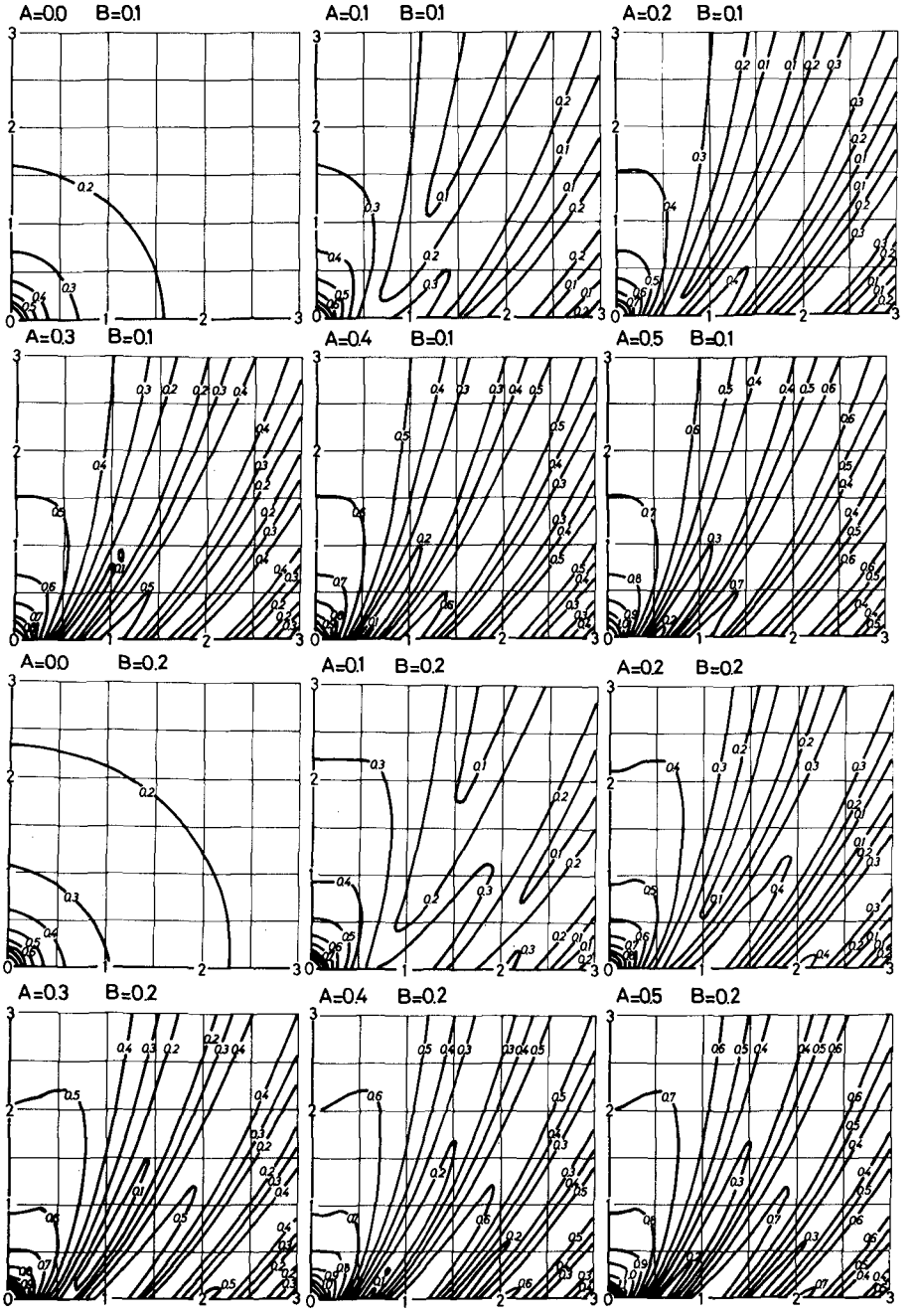
APPENDIX 1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APPENDIX 2

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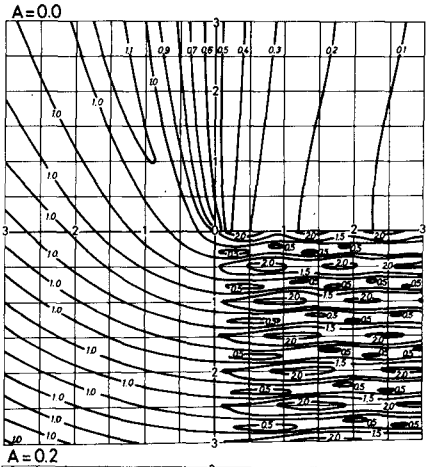

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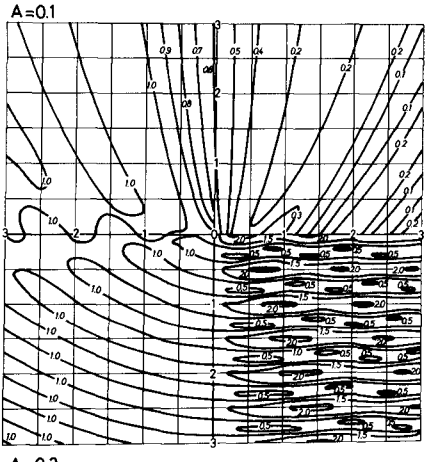

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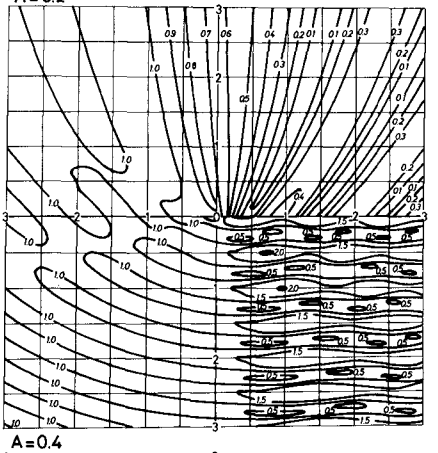

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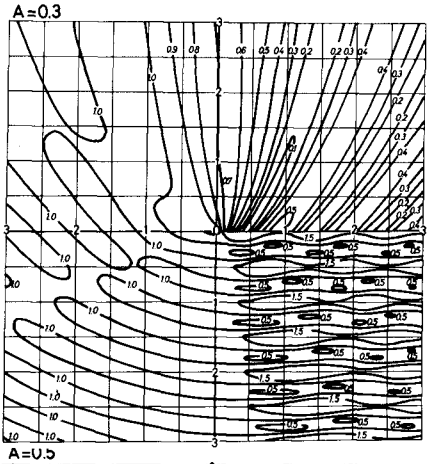

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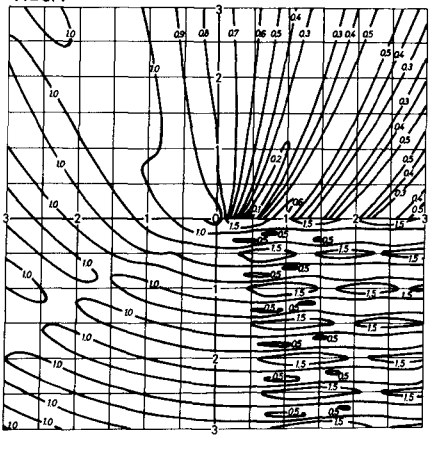

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