# A NUMERICAL APPROACH FOR THE DETERMINATION OF THE WAVE HEIGHT DISTRIBUTION IN A HARBOUR

by

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## ABSTRACT

The paper deals with a mathematical method to calculate wave heights in harbours caused by diffraction and reflection. Using the complete SOMMERFELD-solution for a semi-infinite breakwater, a linear superposition method is applied. The reflections of the harbour boundary are taken into account using a geometric mirroring principle.

The computer-programme allows the phase-correct superposition of any number of wave systems and, therefore, also the consideration of re-reflections which can be important in the presence of vertical walls. For the presentation of results (K'-lines) an interpolation-plot-programme is used.

The computer-programme has been applied to the harbour Greenville extension (Liberia) (BURKHARDT, 1977). Hydraulic model tests were performed to check the programmes, especially with respect to the influence of harbour reflections and the layout of the breakwater head.

## INTRODUCTION

Hydraulic model tests are a reliable aid to investigate the wave height distribution in a harbour, but, with respect to scale effects, normally the model must get considerably large dimensions. The expenditure of construction and operation-time is often high because normally different wave directions have to be checked and the wave heights have to be measured in a narrow grid to allow an objective comparison of results for different wave and harbour-layout conditions.

Because of the simple geometric conditions of the Harbour of Greenville and to save costs a numerical model was used to investigate the wave height distribution in the harbour. The programmes adopted were developed within a special research project (SFB 79) of the GERMAN RESEARCH COUNCIL (DFG).

Fig. 1 shows a plan-view of the Harbour of Greenville which has been built in the 50th and which is now going to be extended. It is planned to elongate the existing breakwater

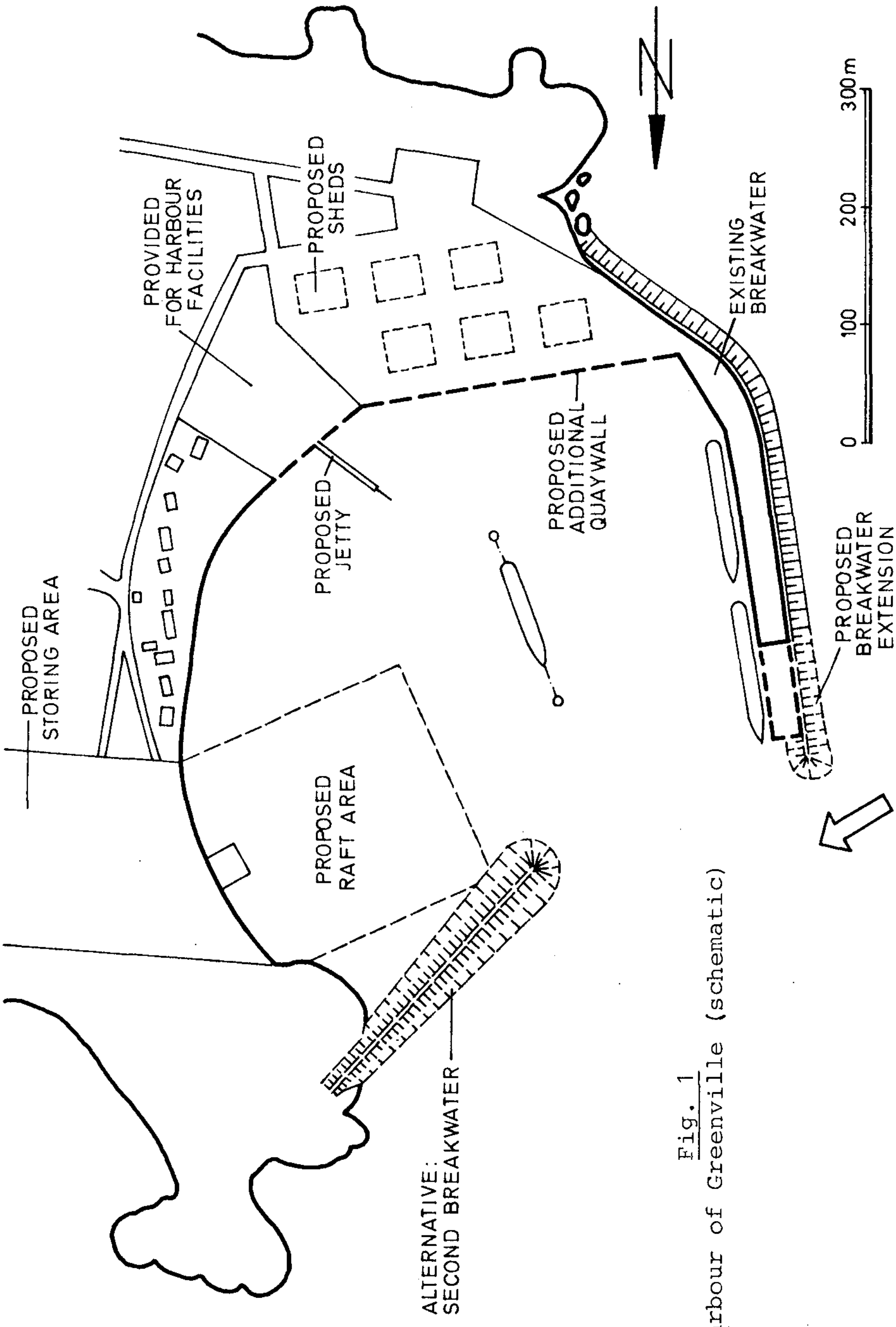



Fig. 1
Harbour of Greenville (schematic)

of the harbour to provide an additional cargo berth. The construction of a second breakwater is considered (RRI, 1976). The breakwater is built as a vertical wall with a rip-rap protected frontside. For the harbour bundaries different alternatives have been discussed.

The directions of the incoming waves are - of course - variable as well as wave heights and periods. The arrow corresponds approximately to the main wave direction.

## PRINCIPLE OF CALCULATION

The numerical procedure is based on a simple geometrical mirroring which has been published already by CARR (1952) generally.

The principle of the method used is as follows (Fig. 2): The incoming waves are diffracted at the breakwater head. Diminished by diffraction the waves are propagating into the harbour area and will be reflected at the harbour boundaries.

The diffracted waves are superimposed by the reflected waves, which will be re-reflected at the harbour quays again.

To calculate the influence of reflection and re-reflection the harbour boundaries are idealized as a polygonal course of n elements.

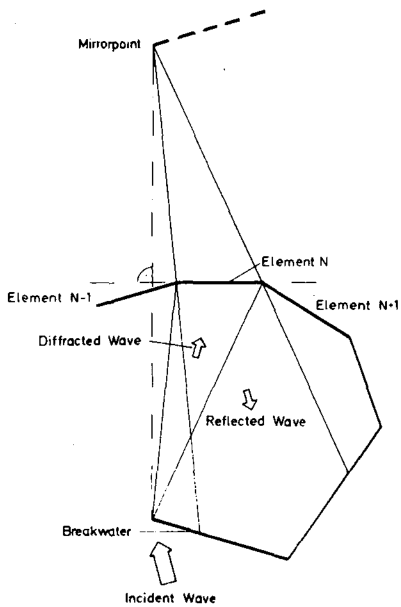



Fig. 2

Explanation of mirroring principle

According to the rules of the geometric optics the wave reflected at the boundary element $n = N$ can be seen as a diffraction wave starting from the geometric mirror point of the breakwater tip.

The consideration of a reflected wave as a diffraction wave is equivalent to the consideration of the reflecting boundary element as an opening, which implies, that additionally scattered wave systems will be formed at the joints to the neighbouring elements N-1 and N+1, which have to be superimposed to the other wave systems. These scattered waves provide for a balancing of wave height differences in the lines of geometric shadows.

The calculation of wave systems to be superimposed is based on the SOMMERFELD-solution of the diffraction at a semi-infinite breakwater. The solution-function is as follows (Fig. 3):

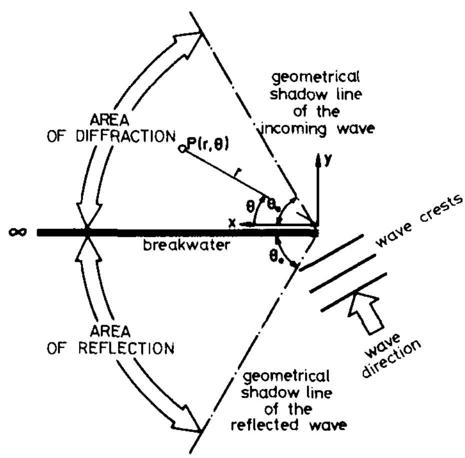



Fig. 3

Co-ordinate system and notations

$$F(r,\Theta) = f(\sigma) \cdot e^{-ikr\cdot\cos(\Theta-\Theta_o)} + f(\sigma') \cdot e^{-ikr\cdot\cos(\Theta+\Theta_o)}$$

with

$$\sigma = 2\sqrt{\frac{k\cdot r}{\pi}} \cdot \sin\left(\frac{\Theta-\Theta_o}{2}\right)$$

$$\sigma' = -2\sqrt{\frac{k\cdot r}{\pi}} \cdot \sin\left(\frac{\Theta+\Theta_o}{2}\right)$$

$$f(\sigma) = \frac{1+i}{2} \int_{-\infty}^{\sigma} e^{-i\pi t^2/2}\, dt$$

$$f(\sigma') = \frac{1+i}{2} \int_{-\infty}^{\sigma'} e^{-i\pi t^2/2}\, dt$$

Details of calculation are outlined in DAEMRICH (1978) and BERGER (1976) respectively.

The programme allows the phase correct, linear superposition of any number of partial wave systems, but to save computer-time for the calculations of the Greenville harbour only 5 elements and not more than about 15 partial wave systems were used. The degrees of reflection, which must be determined before, are variable.

K'-values are calculated in a cartesic grid. For the graphic representation of the wave pattern a plot programme was used, which plots lines of constant K'-values by means of interpolation.

## SOME RESULTS OF CALCULATION

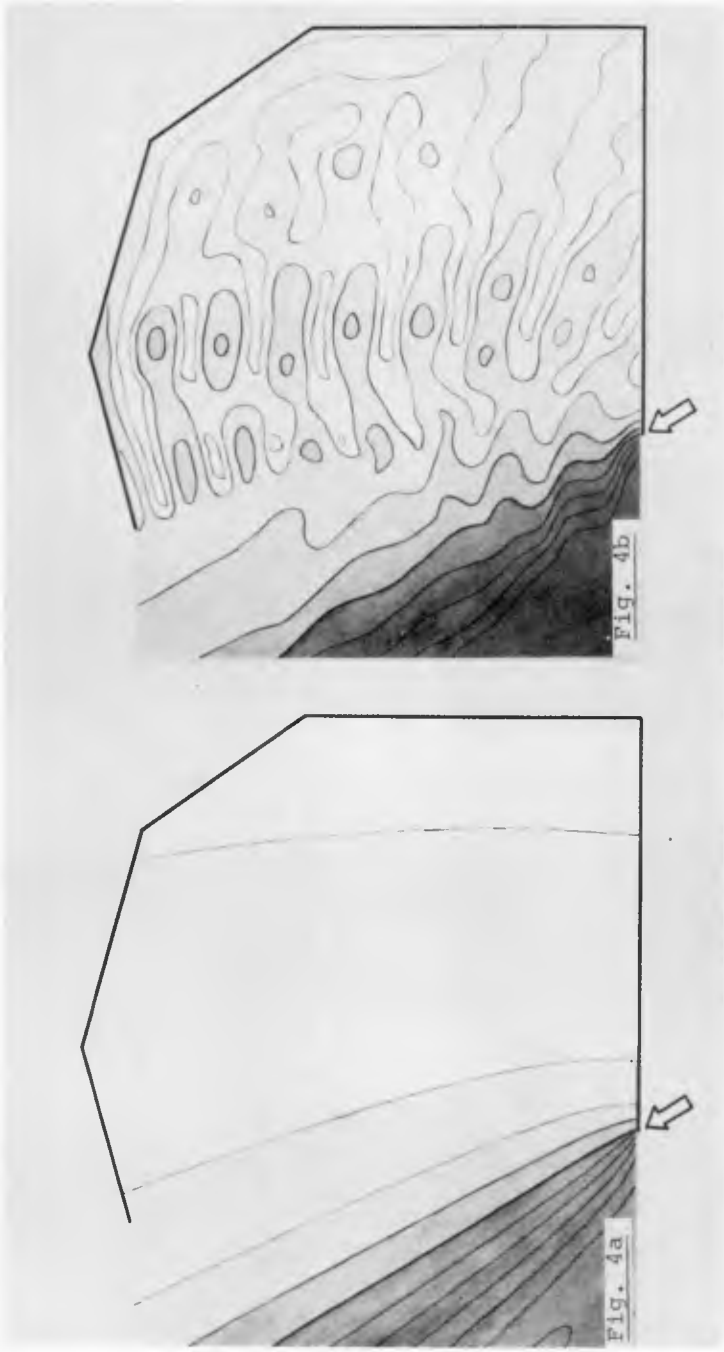
Calculations have been carried out for different harbour configurations and wave conditions. But special results for the harbour of Greenville shall not be reported here, rather it shall be demonstrated in which way the resulting wave system is influenced by reflection and diffraction effects.

Fig. 4a shows the pure diffraction at the breakwater and - in this initial stage - reflections and higher order reflections were not considered. In the line of the geometric shadow the wave height is about 50 % of the incident wave height; the wave heights are decreasing rapidly in the area of the geometric shadow and are in the order of 10 to 20 % of the incident wave height.

The influence of reflection at the harbour boundaries is shown exemplarily in Fig. 4b. The degree of reflection was chosen constant with $\kappa_R = 0.3$. Not considered are the additional scattered wave systems at the corners of the harbour boundaries caused by diffraction as outlined before. Also re-reflections at the harbour side of the breakwater have been neglected.

The comparison of the calculations given by the examples of Fig. 4 shows that the wave pattern - as was to be expected - will become very irregular even with small degrees of reflections. The wave heights become throughout higher within the harbour area. The influence of reflection reaches even into the area outside the geometric shadow line.

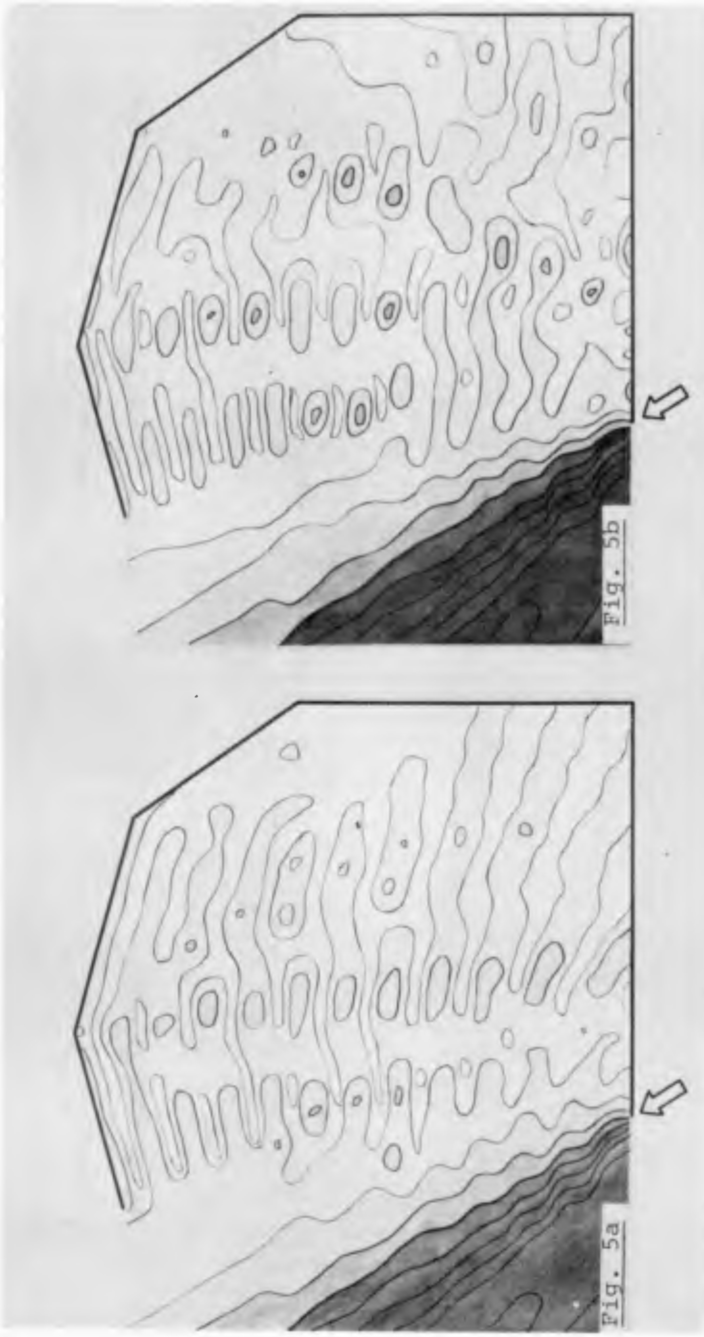
The influence of re-reflection at the breakwater can be seen exemplarily in Fig. 5.




MSPIEG = 0 CASAB = 0,3 APPAB 6/13=0.0
L = 112m 230°

REINE DIFFRAKTION
L = 112m 230°

Fig. 4




MSPIEG=0 CASAB=0,3 APPAB 6/13=0,0
L=75m 230°

MSPIEG=1 CASAB=0,3 APPAB 6/13=0,0
L=75m 230°

Fig. 5

On the left (Fig. 5a) the condition without re-reflection, on the right (Fig. 5b) with re-reflection is shown. The degree of reflection has been taken as $\kappa_R = 1.0$ at the breakwater. In other respects, the boundary conditions are the same on both Fig. 5a and Fig. 5b , i.e. same wave lengths and periods and uniform degrees of reflection at the harbour boundaries ($\kappa_R = 0.3$). Additionally occuring scattered waves are not considered.

The effect of re-reflected waves running back into the inner part of the harbour can be clearly seen. Lines of equal K'-values which are relatively smoothed in the left part of Fig. 5 are broken up more and more. This break up is specially marked in the breakwater area due to the geometrics of the harbour.

Only in exeptional cases comparable forms on the left and the right side of Fig. 5 can be found.

The influence of additional occuring scattered waves can be seen exemplarily from Fig. 6.

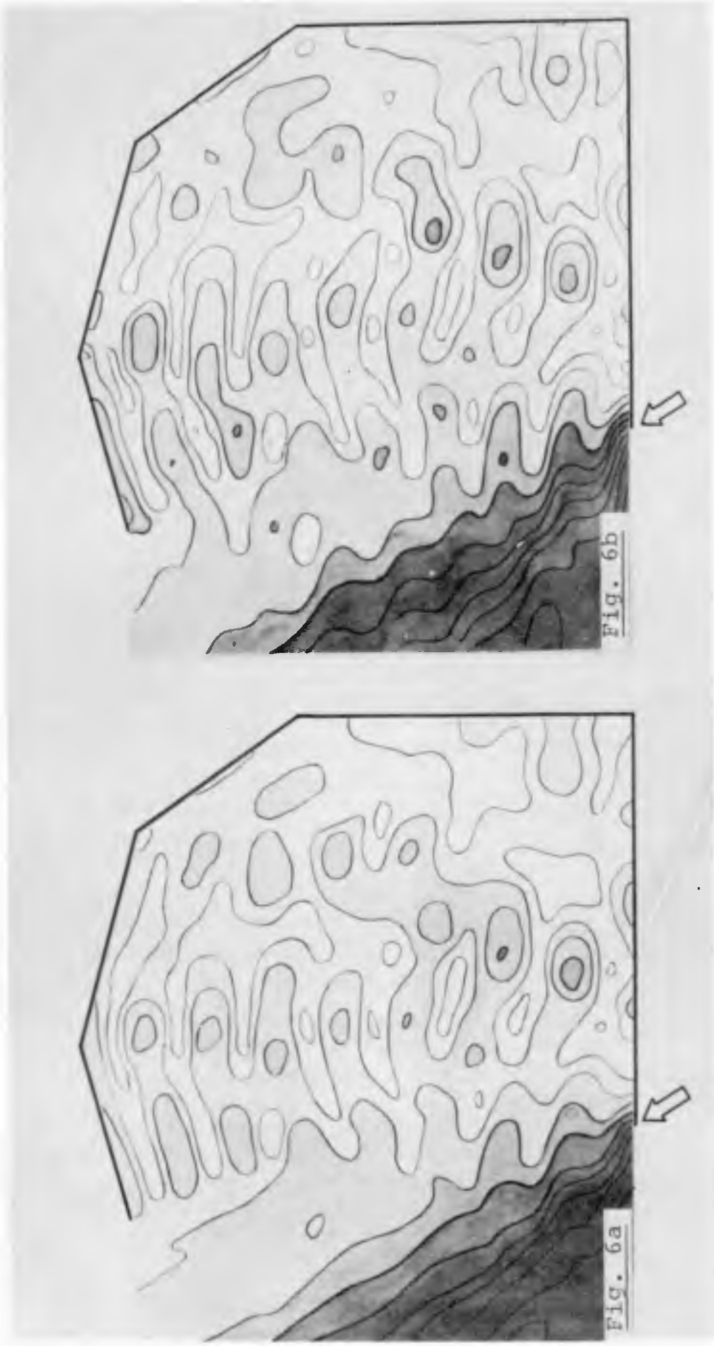
On the left we have an example without scattered waves caused at the boundary corners, on the right with scattered waves.

Differences of the wave height distribution can be seen quite well, although single forms of K'-lines can be found. As shown in the preceding Figures the consideration of any additional influence causes local increases of the wave heights in the harbour area.

Two more exemples of the calculations may show qualitatively the significant influence of the wave parameters period and direction.

In Fig. 7 the wave-direction has been changed by $30^{\circ}$. In other respects the conditions are the same left (Fig. 7a) and right (Fig. 7b) and as well re-reflections at the breakwater as additional scattered waves were considered for calculations.

In this example it is nearly impossible to find again single forms of K'-lines just as in the example of Fig. 8 which shows the influence of the wave-length or period. It can be seen once more, how complicated the wave height distribution can be even in the case of a very simple harbour.




MSPIEG=1 CASAB=0,3 APPAB 6/13=0,0
L=112m 230°

MSPIEG=1 CASAB=0,3
L=112m 230°

Fig. 6

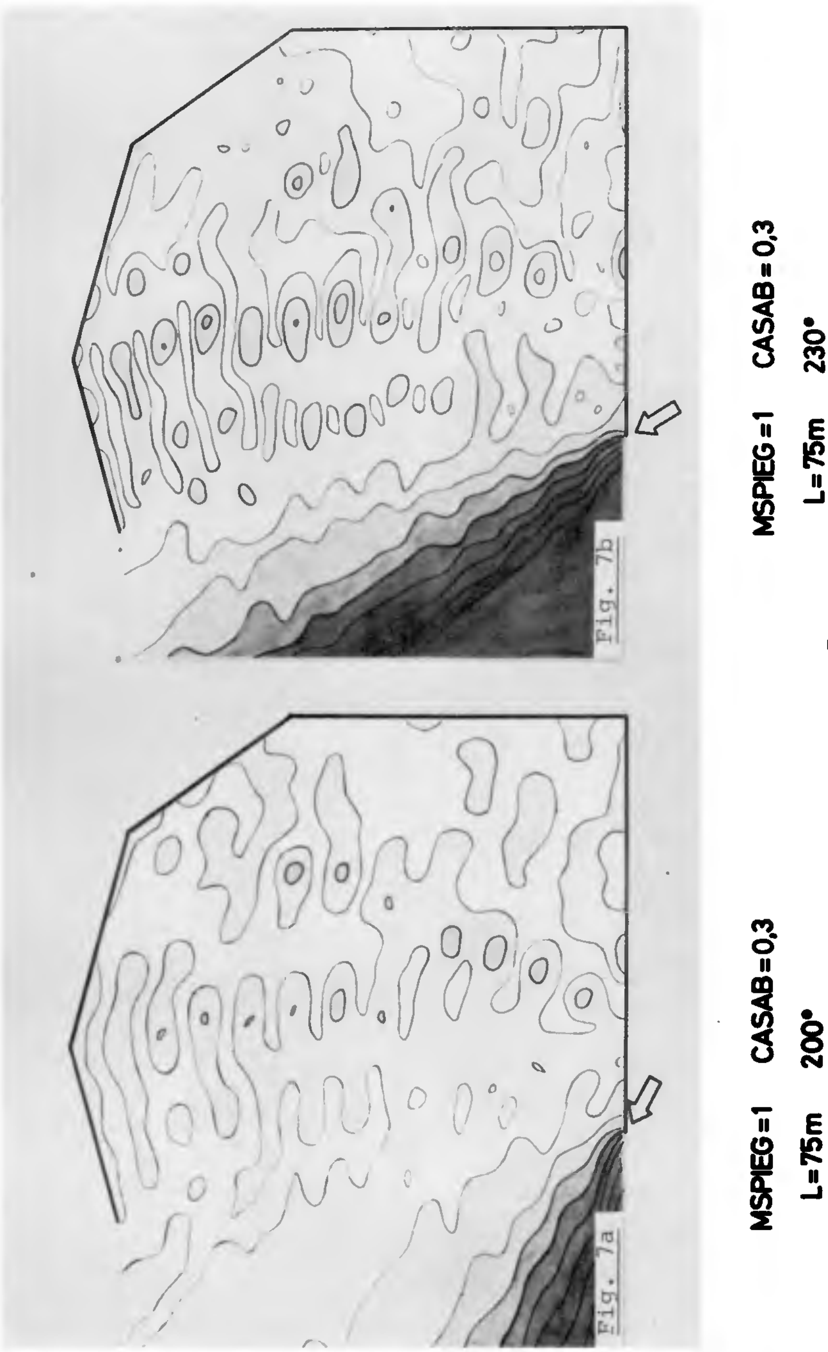


Fig. 7a
MSPIEG=1 CASAB=0,3
L=75m 200°
Fig. 7b
MSPIEG=1 CASAB=0,3
L=75m 230°

Fig. 7

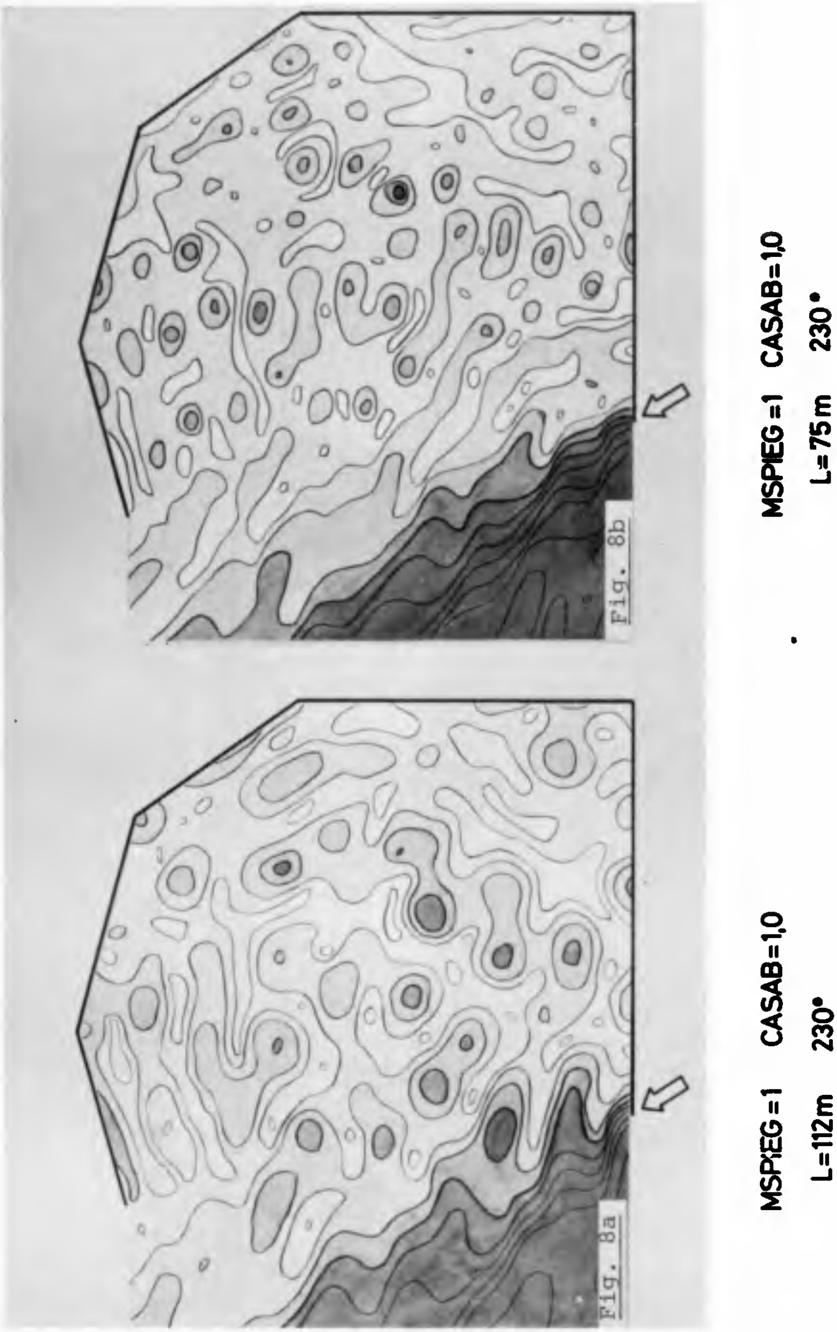


Fig. 8a
MSPIEG=1 CASAB=1,0
L=112m 230°
Fig. 8b
MSPIEG=1 CASAB=1,0
L=75m 230°

Fig. 8

## CONCLUDING REMARKS

A numerical procedure to calculate the wave height distribution in a harbour is presented. The method has been applied to the harbour of Greenville. It has been tested using a schematic hydraulic model showing a fair agreement with the theoretical results. But some general restrictions of the method should be mentioned:

- The method requires a constant water depth and does not include refraction effects within the harbour. This simplification is often but not always permissible.

- The influence of the degree of reflection of the breakwater is considered but not the influence of the breakwater head. The design of the breakwater head can have an influence on the wave heights in the harbour, which - in certain cases - will not be negligible as shown by some comparative model investigations.

- The method allows the superposition of any number of wave systems but the computer time is increasing over-proportional with the number of harbour boundary elements, because of the necessary consideration of re-reflections and scattered waves.

  Therefore, the number of elements should be restricted which includes some subjectivity and requires some experiences.

The main advantage of the method is its flexibility, i.e. wave directions, periods, degrees of reflection in connection with or structural changes of quays berth etc. can be varied in the model easily.

In this respect the numerical method can also be a good help to select the test conditions of a hydraulic model.

Finally some remarks on the performance of hydraulic model investigations should be given.

The calculations habe shown, that the wave height distribution reacts sensitively on small modifications of the boundary conditions of the numerical model. A hydraulic model will do this too but this sensitivity is not always realized.

To compare the results of a hydraulic model fortuities of measurements must be excluded.

Measurements at a few fixed points in a threedimensional model are therefore useless for comparing purposes, because the position of nodal points of the wave oscillation is changing.

For the performance of hydraulic model test this signifies that the wave data have to be measured in a appropriate narrow grid, which is often ignored.

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