CHAPTER 36

INFLUENCE OF BREAKWATER-REFLECTION ON DIFFRACTION

by

Karl-Friedrich Daemrich and Sören Kohlhase

Franzius-Institut, Technical University of Hannover, Germany

ABSTRACT

Diffraction calculations for breakwaters are often based on SOMMERFELDs solution, which is mathematically exact for thin full-reflecting walls. For breakwaters with low reflecting frontsides, and if guidewalls are used, commonly a modified solution is applied, reducing the second term of the solution-formula proportional to the degree of reflection.

It could be shown that this approach is not sufficient in the region just behind the breakwater, especially for small angles of wave attack.

Regarding the exact solution for wedges it was possible to determine a special weighing factor for the second term of the SOMMERFELD solution, dependent on the degree of reflection and the wave direction, which leads to a better agreement between model tests and theoretical results.

INTRODUCTION

Within the scope of basic research on wave diffraction at harbour entrances with overlapping or displaced breakwaters [1] model tests with simple breakwater gaps and normal wave direction were performed, to get an impression on the comparability between theoretical results and wave heights measured in the hydraulic model.

The investigations were carried out in the wave basin of the SFB 79 (18·45 m) at the Franzius-Institut. The model-breakwaters were constructed of thin plates. Either a low reflecting rip-rap protection at the breakwater frontside, or a guidewall from the breakwater-tip to the wave generator was used, to diminish distortions by rereflected waves.

The theoretical results were calculated by the method of PENNEY and PRICE [3] , who superimposed the mathematically exact solution of SOMMERFELD [4] to a solution for simple breakwater gaps.

The SOMMERFELD solution is as follows (symbols see Fig. 1):

$$F(r,\theta)=f(\sigma)\cdot e^{-ikr\cdot\cos(\theta-\theta_o)}+f(\sigma')\cdot e^{-ikr\cdot\cos(\theta+\theta_o)}$$

with

$$\sigma = 2\cdot\sqrt{\frac{k\cdot r}{\pi}}\cdot\sin\left(\frac{\theta-\theta_o}{2}\right)$$

$$\sigma' = -2\cdot\sqrt{\frac{k\cdot r}{\pi}}\cdot\sin\left(\frac{\theta+\theta_o}{2}\right)$$

$$f(\sigma)=\frac{1+i}{2}\int_{-\infty}^{\sigma} e^{-i\pi t^2/2}\,dt$$

$$f(\sigma')=\frac{1+i}{2}\int_{-\infty}^{\sigma'} e^{-i\pi t^2/2}\,dt$$

(The modulus of F (r,θ) is equal to the diffraction coefficient K').

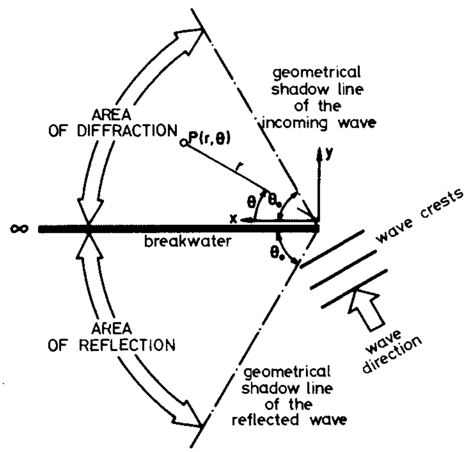

Fig. 1

Definition sketch for the SOMMERFELD solution

The solution is derived and valid for full reflecting breakwaters and cannot be used without restrictions in the case of low reflecting structures.

However, the formula consists of two terms, and because the second term includes the wave field reflected at the breakwater frontside, this term is usually related to the influence of reflection <u>even</u> in the region of diffraction.

As several authors (e.g. [5] , [6]) have suggested, the theoretical results were calculated with a modified formula with the second term reduced proportional to the degree of reflection. For the case, that guidewalls were used, consequently, the second term was neglected and the solution reduces to the socalled "simplified solution".

It shall be pointed out already here, that only the total solution fulfills the boundary condition exactly. If the second term is reduced or neglected the solution is not longer mathematically exact.

Fig. 2 shows exemplarily typical results for an opening width of two wavelenghts for the case, that guidewalls were used. Plotted are relative wave heights in lines parallel to the breakwater.

In the region just behind the breakwater typical differences between theoretical results and wave heights measured in the hydraulic model can be seen. The wave heights should be about zero theoretically, but they reach considerable values in the model. (Similar deviations were observed in the tests with a rip-rap-protected breakwater frontside).

These differences between theory and experiment have been the reason to deal more intensively with the diffraction theories under the special consideration of the influence of reflection.

SOLUTION FOR A BREAKWATER WITH GUIDEWALL

For the case, that guidewalls or wave splitters are used, an appropriate solution is available from the exact solution for semi-infinite vertical wedges.

MITSUI and MURAKAMI [2] have derived solutions for different wedge angles and all wave directions. For an example, the solution for a rectangular wedge is given (symbols acc. to Fig. 3):

$$F(\rho,\theta)_{\nu=\frac{3}{2}\pi} = \frac{4}{3} J_o(\rho) + \frac{8}{3} \sum_{n=1}^{\infty} e^{in\pi/3} \cdot J_{2n/3}(\rho) \cdot \cos\frac{2}{3}n\alpha \cdot \cos\frac{2}{3}n\theta$$

with

$J_o(\rho)$, $J_{2n/3}$ = BESSEL functions, first kind

$\rho = k \cdot r = \frac{2\pi}{L} \cdot r$

(The solution has to be halved for wave directions parallel to one wall)

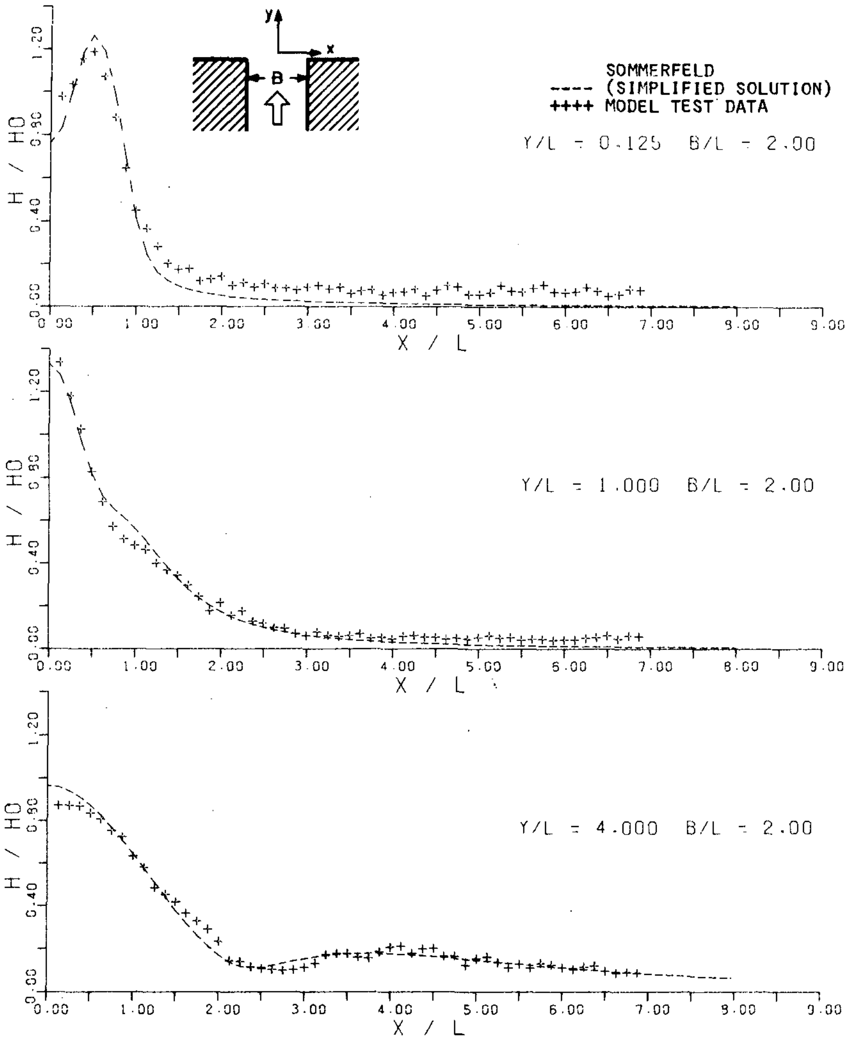

Fig. 2 Comparison of theoretical results with hydraulic model tests

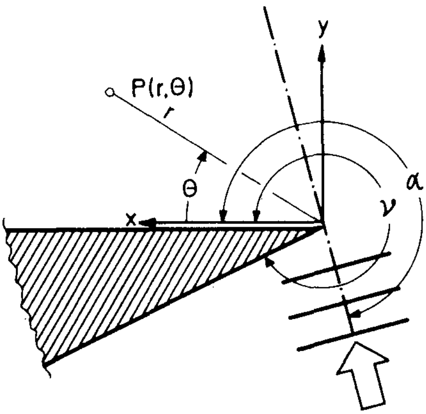

Fig. 3
Definition sketch for the MITSUI solution

Fig. 4 shows exemplarily the wave heights in the region of diffraction for perpendicular wave approach for different theories.

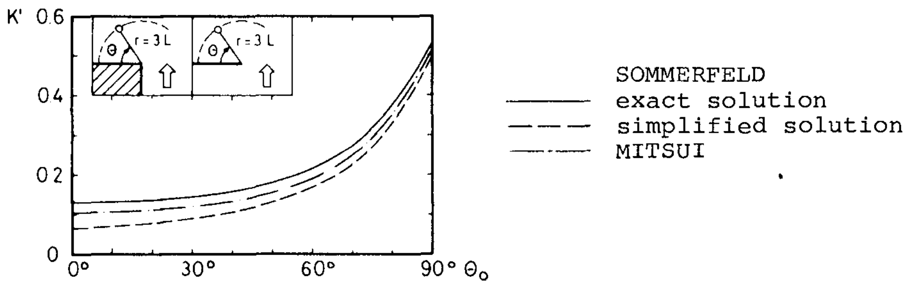

Fig. 4 Comparison of the diffraction coefficient K' for different theories

It can be seen, that the results according to the MITSUI solution are typically heigher than the results according to the simplified SOMMERFELD solution.

Furthermore, it can be seen that the MITSUI solution may be approximated by a modified SOMMERFELD solution. The difference between total solution and simplified solution corresponds to the influence of the second term of the SOMMERFELD solution and therefore a weighing of this second term may be a good approximation.

Fig. 5 gives a comparison of experimental results with results according to the MITSUI solution. The characteristic better agreement confirms the validity of this solution.

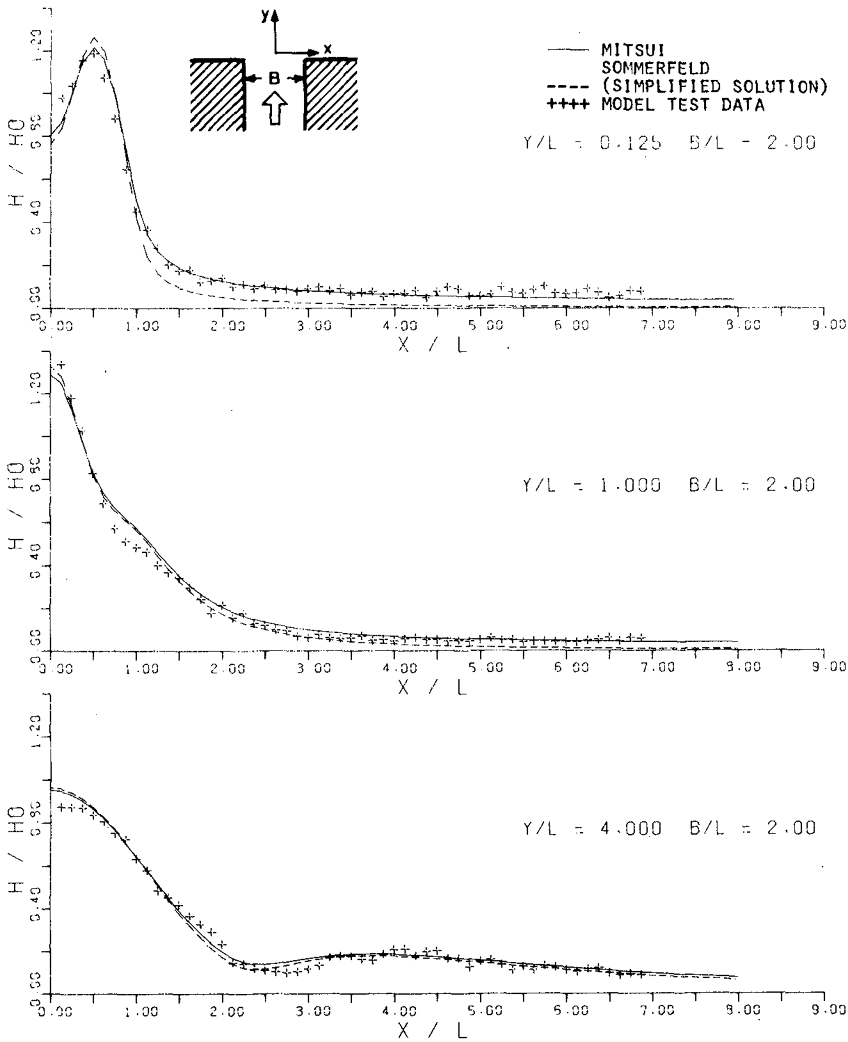

Fig. 5 Model tests in comparison with theoretical results

SOLUTION FOR A BREAKWATER WITH NON-REFLECTING FRONTSIDE

As a basis, the SOMMERFELD solution and the MITSUI solution were used for the theoretical considerations. For illustration, the SOMMERFELD solution shall be briefly discussed.

As mentioned before it consists of two terms. Each term represents a part of the wave field around the breakwater and can be divided mathematically into a straight-crested wavefield, according to the laws of geometrical optics, and a nearly circular scattered wave field. This wave fields, represented by their wave crests, are shown schematically in Fig. 6.

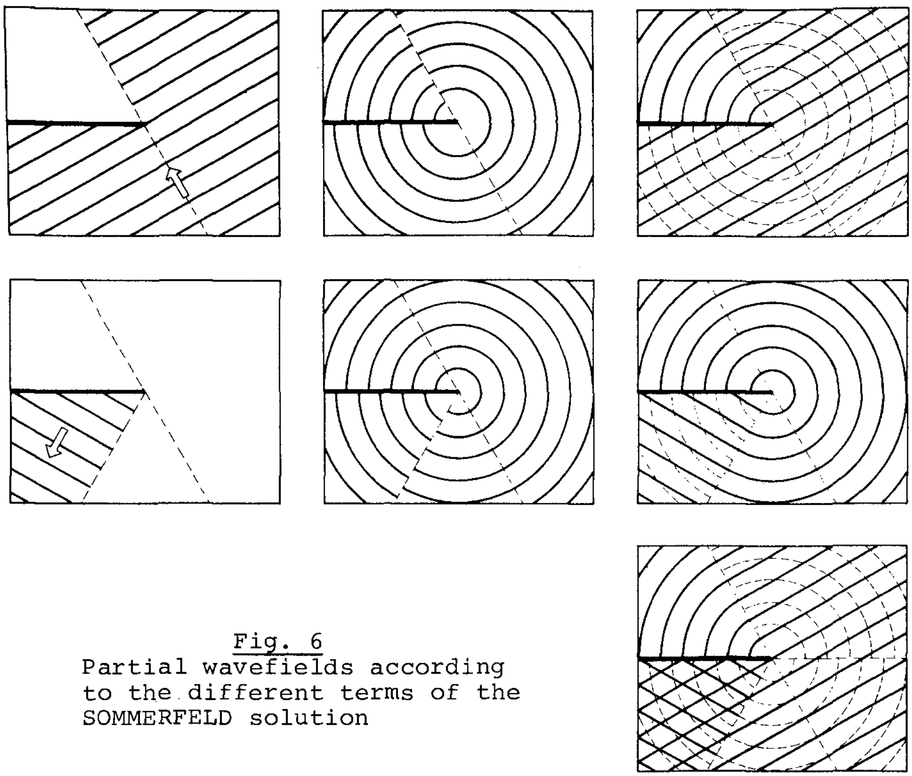

Fig. 6
Partial wavefields according to the different terms of the SOMMERFELD solution

The incoming wave field and the pertinent scattered wave field together are represented by the first term of the solution, the reflected wave field and the pertinent scattered wave field by the second term. The characteristics of the scattered wave fields are influenced as well by the boundary condition "breakwater" as by the distribution of the wave heights of the generating straight crested wave field in its geometric shadow line.

Going back to the problem and regarding the wave systems in the upper part of Fig. 6, which correspond to the first term of the SOMMERFELD solution, apparently this case is physically in accordance with the case of a non-reflecting breakwater. There are only incoming waves, together with a pertinent scattered wave system.

However, it must be noticed, that the two scattered wave systems of the two terms of the SOMMERFELD solution fulfill the boundary condition at the breakwater only combined. Each scattered wave field alone does not fulfill the boundary condition, except for a wave direction $\theta_o = 180^o$.

In the following, a similar solution shall be derived for the condition, that the incoming waves are not reflected at the (non-reflecting) breakwater frontside, whilst the pertinent scattered wave system is in accordance to the boundary condition at an impermeable full-reflecting breakwater.

Exemplarily for a given wave direction three similar constellations shall be compared (Fig. 7)

- a non-reflecting thin breakwater according to the above definition, where a solution is searched for (Fig. 7a)
- a breakwater with wave direction parallel to the breakwater, where an exact solution exists, and no reflection can occur (Fig. 7b). (The SOMMERFELD solution has to be halved in this case)
- and a rectangular wedge with wave direction parallel to one wall, where also an exact solution, the MITSUI solution, is known (Fig. 7c)

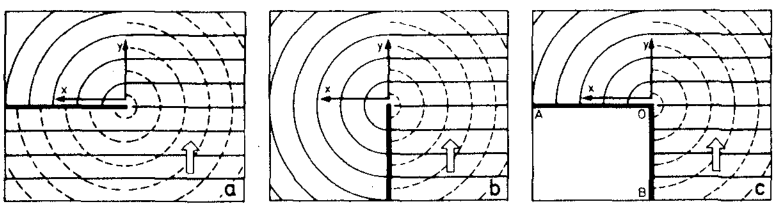

Fig. 7 Wave crests for different breakwater constellations

By comparing the wave fields according to Fig. 7b and c it is possible to determine the alteration of the scattered wave field for the case that the wall $\overline{AO}$ is moved towards the wall $\overline{BO}$, i.e. if we substract the wave system at the right from the wave system in the middle we get a "difference scattered wave system", which represents the effect of the spreading or diffraction of the scattered wave field at the wedge into the region of the wedge.

In the same manner, as the solution for the case "wave direction parallel to the breakwater"(Fig. 7b) can be derived from the case "wedge" (Fig. 7c) by adding the difference scattered wave field to the scattered wave field at the wedge, it is possible to derive the searched solution for the non-reflecting breakwater (Fig. 7a) by adding a difference scattered wave field,representing the spreading of the scattered wave field into the region of the wedge at the line $\overline{OB}$.

From the fact that the wave heights of the scattered wave field in the line $\overline{OB}$ are relatively exact one third of the corresponding wave heights in the line $\overline{OA}$, we can conclude, that the characteristic form of the difference scattered wave field is the same as mentioned before. The wave heights, however, are one third only for this special wedge angle.

In Fig. 8 the development and the wave height distribution of the difference scattered wave field is shown.

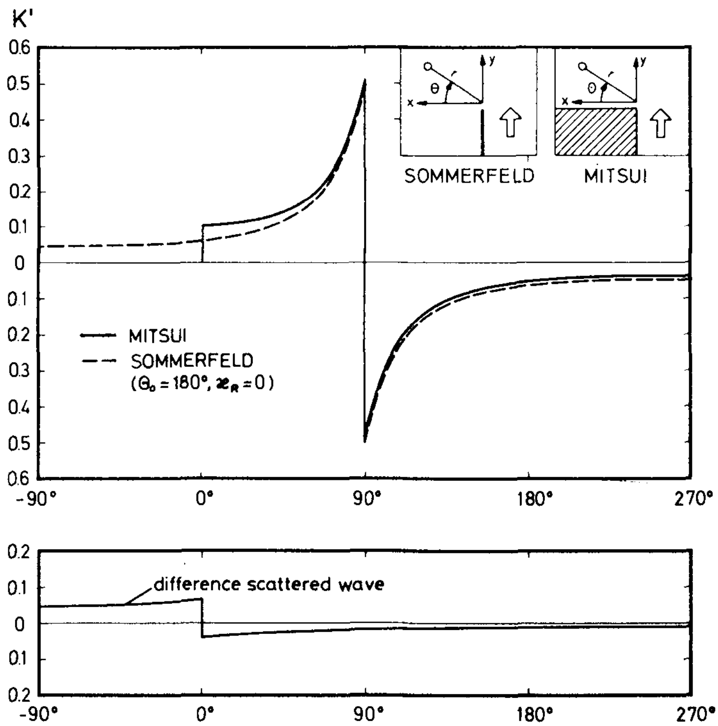

Fig.8 Wave heights of the scattered waves according to the constellations b and c in Fig. 7, and pertinent "difference scattered wave" (exemplarily for r = 3·L)

In the upper part the wave heights of the both exact solutions are shown, in the lower part the difference scattered wave heights.

Fig. 9 shows finally the wave height distribution of the scattered wave field for the non-reflecting breakwater, and a remarkable difference can be seen in the region of diffraction in comparison to the up to now recommended simplified SOMMERFELD solution with neglected second term.

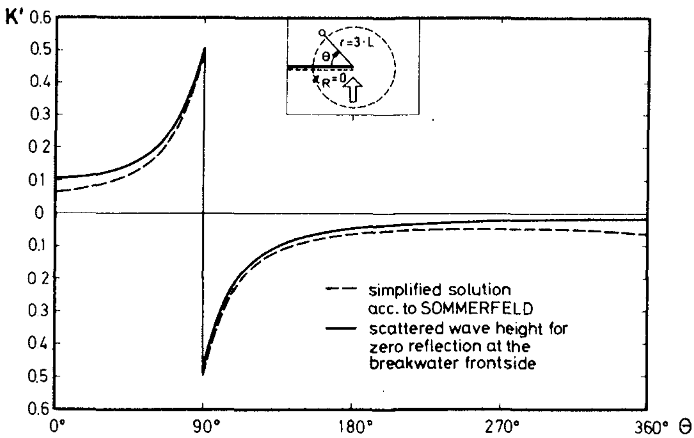

Fig. 9 Diffraction coefficient K' of the scattered wave for zero-reflection in comparison to results from the "simplified solution"

APPROXIMATE METHODS BASED ON THE SOLUTIONS OF MITSUI AND SOMMERFELD

The new solution for non-reflecting breakwaters is very similar to the corresponding solution of MITSUI in the region of diffraction. Therefore, the MITSUI solution for wedges with wave direction parallel to one wall can be used as a good approximation.

(The condition - "breakwater with guidewalls or wave splitters" - is equivalent to the condition - "breakwater with non-reflecting frontside" - in the region of diffraction, as it was assumed by different authors before).

Furthermore comparative calculations have shown, that the difficult solutions according to MITSUI can be approximated well in the region of diffraction, by an adequate modified SOMMERFELD solution, which needs only a half percent of the computer time. This modification consists in a weighing of the second term of the SOMMERFELD solution depending on the wave direction θ_o (Fig. 10).

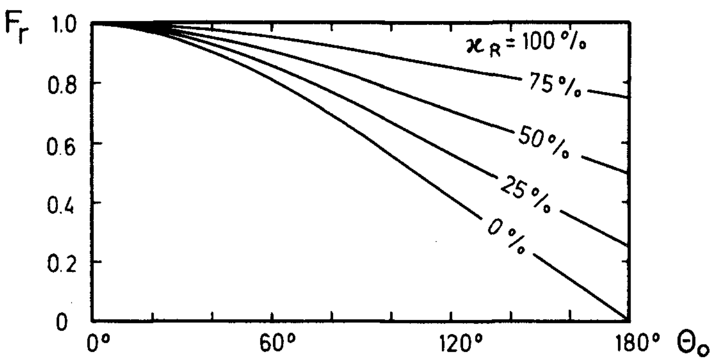

Fig. 10 Weighing factor F_r for the second term of the SOMMERFELD solution for various degrees of reflection

The lowest line in Fig. 10 shows this weighing factor for zero-reflection as a function of the wave direction which has been determined by comparative calculations. It can be readily seen the difference to the up to now recommended method, taking this factor for zero for a non-reflecting breakwater without regarding the wave direction.

This modification, weighing the second term of the SOMMERFELD solution, has furthermore the advantage, that partial reflections can be considered easily by linear interpolation between the weighing factors for zero reflection and total reflection.

Finally, Fig. 11 shows a comparison of experimental results with rip-rap protected breakwater frontsides and theoretical results according to MITSUI and to this modified solution.

The weighing factor for the modified solution was choosen to 0.65 for a wave direction of 90° and a degree of reflection of approx. 10 %. (The results of the MITSUI solution are given in addition for comparison only)

CONCLUSION

To consider the effect of a low-reflecting breakwater frontside on the wave heights in the diffraction area, it is usually recommended to reduce the second term of the SOMMERFELD solution proportional to the degree of reflection.

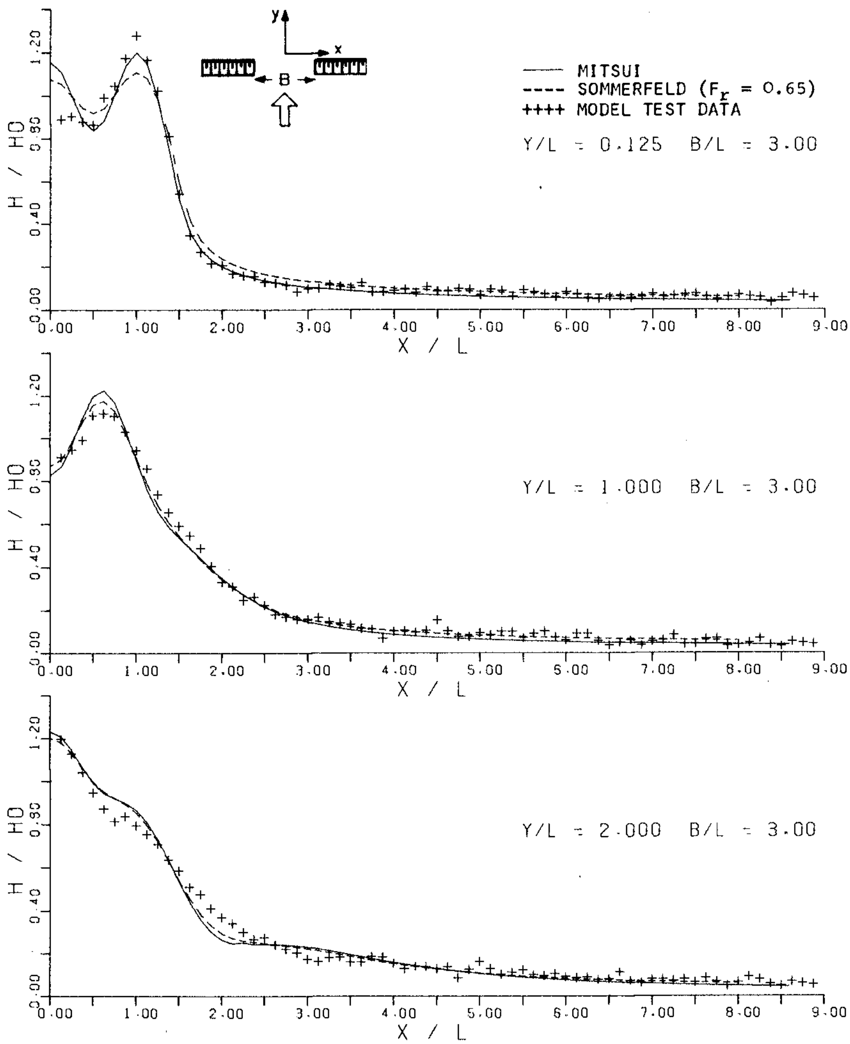

Fig. 11 Model tests in comparison with theoretical results

Within model tests with low-reflecting breakwaters and breakwaters with guidewalls, typical differences between this theoretical approach and the experimental results were observed, specially in the region just behind the breakwater.

It was shown, that the theoretical results according to the MITSUI solution and the developed solution for non-reflecting thin breakwaters are in a better agreement with the experimental results.

Although the commonly used theoretical approach is not acceptable in the region of diffraction just behind the breakwater, it can be used as an appropriate solution introducing a <u>special</u> weighing factor for the second term.

This weighing factor has to be determined with regard to the wave direction and the degree of reflection.

A diagramm with recommended weighing factors is presented (Fig. 10).

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