CHAPTER 23

COMPUTATIONS OF SHORT WAVES IN SHALLOW WATER

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ABSTRACT

The simulation of short wave propagation, including refraction, diffraction, partial reflection and all other such features, is made possible by a new version of the System 21, Jupiter. The System-generated models can describe any bathymetry and topography with any time-varying mean flow, correctly accounting for wave and current interactions, wave-thrusts or radiation stresses, longshore currents and other "second order" effects. The models work with irregular waves just as well as with regular waves. Radiation boundary modules are also provided so that models can be cut-out in any desired region and new features introduced without invalidating the field-study-determined boundary data. As models are system-generated, they can be constructed and run in a few hours, starting from charts, a lay-out of the proposed engineering works and the available boundary data. The development and testing of the System is briefly described, followed by an account of its applications in coastal engineering practice.

1. INTRODUCTION AND BACKGROUND

The modelling system used to compute short waves in shallow water that is the subject of this work constitutes the eight version of the System 21 "Jupiter" (Abbott, Damsgaard and Rodenhuis, 1973). This system was originally conceived for nearly-horizontal flow (negligible vertical acceleration) computations in two dimensions in plane and for vertically homogeneous (unstratified) waters. It was constructed in a system form capable of generating and running models when provided with only an elementary description of the area to be modelled, the boundary and other auxiliary conditions (e.g. barometric pressure fields or geostrophic wind fields) and resistance and similar parameters. (The difference between a model and a modelling system can be compared with the difference between a product and a factory to build products. Each

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product can be made individually more cheaply than the factory, but once the investment in the factory is made, the factory-produced products can be made more cheaply than the individually-made or "one-off" products. Thus although a modelling system costs roughly an order of magnitude more in money and time, as compared with a single one-off model, once built it can produce models at a fraction of the cost and in a fraction of the time required for one-off models). The System 21 was based upon various one-off models developed between 1960 and 1968 (e.g. Abbott, 1963; Sobey, 1970). It was started in 1969 but was only strongly backed from mid 1970. The first prototype was field tested in late 1971, pre-production versions ran in 1972 on two engineering contracts (Karachi harbour seiche analysis and Penang causeway connection) and the first production version, the Mark 5, came into use in 1973. This version used a four-stage difference scheme and was stable over a sufficiently wide range of Courant numbers and bathymetric configurations to provide a practical engineering instrument. On the basis of the success of this version, it was possible, from 1974 onwards, to design and develop an improved version, the Mark 6. This was field tested in 1975 and took-over as the main production version from 1976 onwards. These two versions have been the most widely used of all modelling systems in hydraulic and coastal engineering practice, accounting for 44 contracts in 13 countries, providing modelling services for engineering works the construction cost of which exceeds U.S.$ 10,000,000,000.

This work on nearly-horizontal flow modelling took place in the environment of institutes well known for their pioneering work in short wave theory and practice. The Danish Hydraulic Institute had pioneered the use of irregular wave generators, essential for wave disturbance testing with ships, and together with the Technical University of Denmark had engaged in much basic research on short wave phenomena. On the other hand, the numerical tools available for short wave modelling were, more or less, the usual ones of linear-harmonic analysis and geometric-optical analogy, and these appeared increasingly incongruous in this environment.

Accordingly, it was decided to produce an instrument compatible with physical modelling over the widest possible range of applications and one that would be more satisfactory from the standpoint of modern wave theory. This instrument would be a logical development of the System 21, extending it to provide the simplest description of vertical accelerations with a sufficient range of validity, corresponding to the theory and equation formulations of Boussinesq (1877).

Now it was well known from earlier work (e.g. Abbott and Rodenhuis, 1972) that solutions of the equations of Boussinesq were exceedingly sensitive to errors. On the other hand, if any such instrument were to be of practical value, it would have to work with only a small number of descriptive points per wave length (as little as 6) in order that computing costs should remain acceptable. The main problem was then to produce a numerical scheme that was of such a high accuracy that worthwhile results could be obtained with very coarse descriptions.

At first sight, the task of designing a scheme of sufficient accuracy appeared quite overwhelming, apparently involving the balancing-out of well over 100 terms in the truncation error, one against the other, each balancing necessitating the use of substitutions of formidable complexity. However, earlier work (Peregrine 1974; Abbott and Rodenhuis, 1972) had already indicated a possible simplification and on this basis of this work and its later elaborations (e.g. Bona & Smith, 1976) it was possible to develop satisfactory methodology of error elimination.

2. CONTINUUM AND DISCRETE DESCRIPTION

The present system solves difference equations that are consistent with the following differential equations

$$n \frac{\partial \zeta}{\partial t} + \frac{\partial p}{\partial x} + \frac{\partial q}{\partial y} = 0 \tag{1}$$

$$\begin{aligned} n \frac{\partial p}{\partial t} &+ \frac{\partial}{\partial x}\left(\frac{p^2}{h}\right) + \frac{\partial}{\partial y}\left(\frac{pq}{h}\right) + n^2 gh \frac{\partial \zeta}{\partial x} \\ &= n \frac{Dh}{2}\left[\frac{\partial^3}{\partial x^2 \partial t}\left(\frac{Dp}{h}\right) + \frac{\partial^3}{\partial x \partial y \partial t}\left(\frac{Dq}{h}\right)\right] \\ &-n \frac{D^2 h}{6}\left[\frac{\partial^3}{\partial x^2 \partial t}\left(\frac{p}{h}\right) + \frac{\partial^3}{\partial x \partial y \partial t}\left(\frac{q}{h}\right)\right] \\ &-(1-n)^3 \frac{\alpha \nu}{d^2} p - \frac{(1-n)}{n} \frac{\beta}{d} p \sqrt{\left(\frac{p}{h}\right)^2 + \left(\frac{q}{h}\right)^2} \end{aligned} \tag{2}$$

$$\begin{aligned} n \frac{\partial q}{\partial t} &+ \frac{\partial}{\partial y}\left(\frac{q^2}{h}\right) + \frac{\partial}{\partial x}\left(\frac{pq}{h}\right) + n^2 gh \frac{\partial \zeta}{\partial y} \\ &= n \frac{Dh}{2}\left[\frac{\partial^3}{\partial y^2 \partial t}\left(\frac{Dq}{h}\right) + \frac{\partial^3}{\partial x \partial y \partial t}\left(\frac{Dp}{h}\right)\right] \\ &-n \frac{D^2 h}{6}\left[\frac{\partial^3}{\partial y^2 \partial t}\left(\frac{q}{h}\right) + \frac{\partial^3}{\partial x \partial y \partial t}\left(\frac{p}{h}\right)\right] \\ &-(1-n)^3 \frac{\alpha \nu}{d^2} q - \frac{(1-n)}{n} \frac{\beta}{d} q \sqrt{\left(\frac{p}{h}\right)^2 + \left(\frac{q}{h}\right)^2} \end{aligned} \tag{3}$$

In open water the pore volume n is set to 1 while in permeable breakwater and similar porous structures is it set to the prevolume.

The equations are solved using a difference scheme that is brought to third order accuracy in all its essential terms, by use of the elimination method described by Abbott, Petersen and Skovgaard (1978). The range of application of the equation system (1, 2, 3) is also given in that work, together with experimentally derived phase portraits and other more theoretical information.

3. FIELD TESTS AND APPLICATIONS

In order to prove the System, simulations were first made of one-dimentional situations. Fig. 1 shows results of a simulation of shoaling cnoidal waves, the computed results being compared with the experiments of Madsen and Mei (1969). The agreement is seen to be very satisfactory. Another simulation is that of wave reflection and transmission at a permeable breakwater. In this case, illustrated in fig. 2, comparisons were made with experimental results reported with Keulegan (1973). In this case it is shown that by setting n to its physical value, values of the laminar and turbulent resistance parameters α and β can be found such that the experimental results are recovered to an acceptable accuracy. It should be added that the α and β values so found were entirely reasonable on physical grounds.

Results of a two dimensional simulation with small-amplitude periodic waves is shown in fig. 3 together with the results of the corresponding classical Sommerfeld diffraction theory. The agreement is again striking. Fig. 4 shows a perspective plot of the wave pattern of fig. 3, providing a more visually familiar picture.

Field testing of the Mark 8 was rounded-off by simulations of regular and irregular wave behaviour in Hanstholm harbour, Denmark. The prototype is illustrated in fig. 5 while fig. 6 shows the physical model that provided the results used for the comparison. Fig. 7 shows the Hanstholm physical model bathymetry(a) and how well this bathymetry is resolved in the numerical model(b). A perspective plot of fig. 7b is provided in fig. 8, for comparison with the photographs.

The first stages in a simulation, when a model is "cold started", are pictured by contours in fig. 9 for periodic waves. The first diffractions and refractions are clearly seen. Fig. 10 shows how steady a state can be attained with periodic waves after a suitable number of time steps have been run (here about 300-400 are necessary). The corresponding comparison between physical model and numerical model results for periodic waves is shown in fig. 11. The agreement is satisfactory taking account of the over-sensitivity of periodic wave amplification factors to measuring station positions and periods (Sørensen, 1973; Abbott, Petersen and Skovgaard, 1978). The reproducibility of phenomena with irregular waves, exemplified in plan in fig. 12, is generally more satisfactory and the comparison between physical model and numerical model results (fig. 13) is correspondingly much better. By way of visual verification, fig. 14 shows a perspective plot of waves in the outer harbour of Hanstholm.

When preparing a system model it is usual to set the charts of the region on a plotting table and follow contours with the plotter follower. The sequence of positions of the follower are translated by

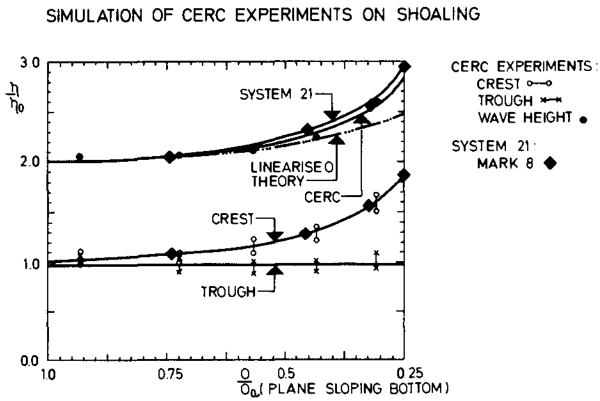

Fig. 1.

Comparison of numerical computations of shoaling waves obtained using the System in one-dimensional mode, as compared with the experimental results of Madsen and Mei (1969).

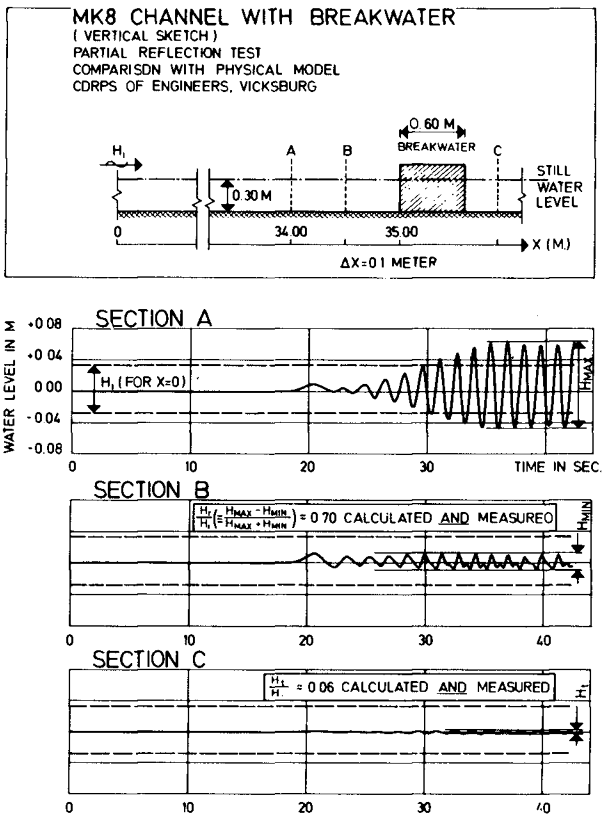

Fig. 2.

Water elevation for wave transmission through a permeable breakwater, with partial reflection, obtained using the System in one-dimensional mode.

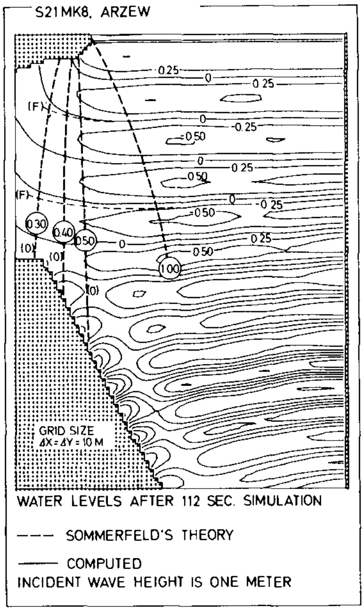

Fig. 3. Comparison of two-dimensional computations of pure diffraction with the analytical results of Sommerfeld. Plan view of contours.

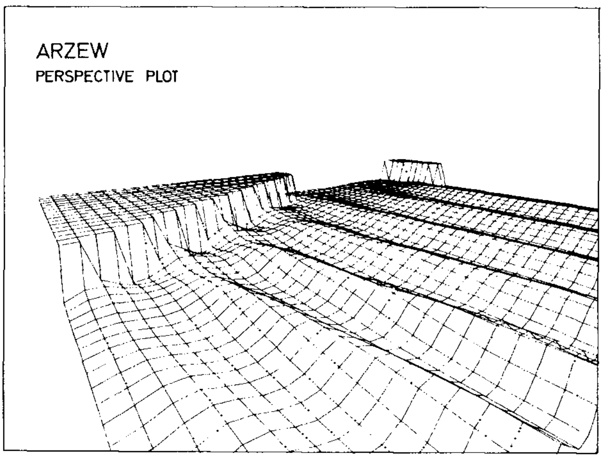

Fig. 4.

Perspective plot of the situation shown in fig. 3

Fig. 5. Aerial photograph of harbour at Hanstholm.

Fig. 6. Photograph of physical model of harbour at Hanstholm.

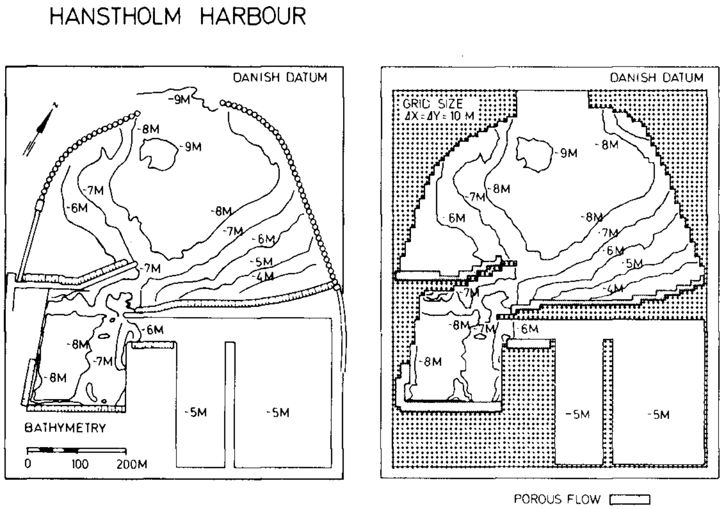

Fig. 7. Contours of the Hanstholm harbour as
a. (left) used in the physical model tests, and
b. (right) used in the numerical model tests.
The porous areas are shown in both a and b.

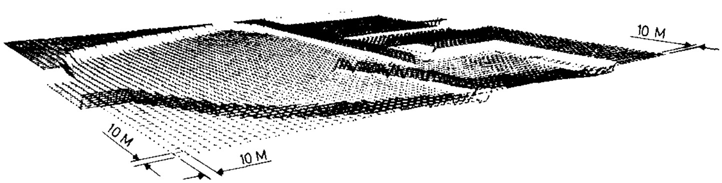

Fig. 8. Perspective plot of the numerical model bathymetry
(Hanstholm harbour).

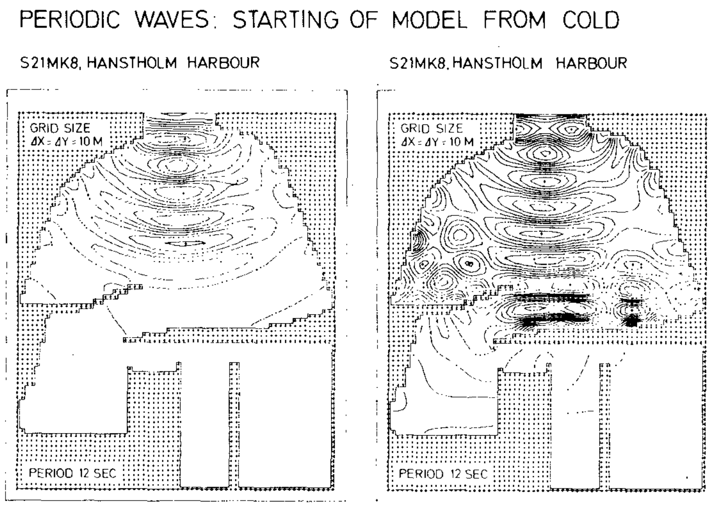

Fig. 9. Surface elevation contours with regular waves.
Left: After 44 sec. simulation.
Right: After 88 sec. simulation.

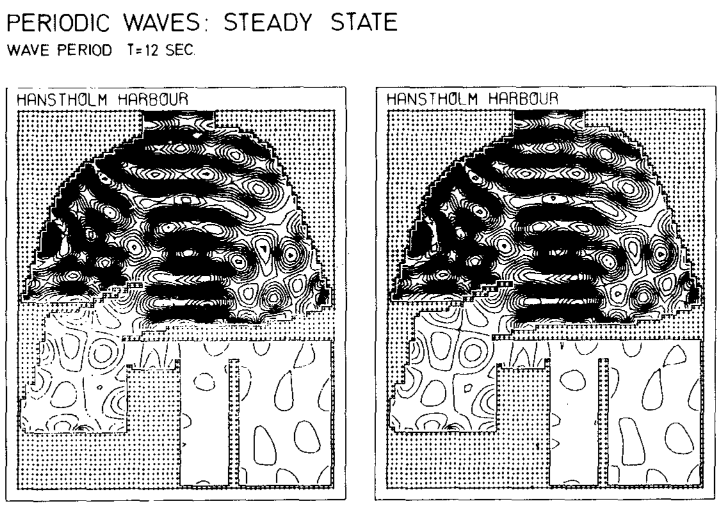

Fig. 10. Surface elevation contours with regular waves compared for one wave period time difference to illustrate the steady state attained.
Left: After 468 sec. simulation.
Right: After 480 sec. simulation.

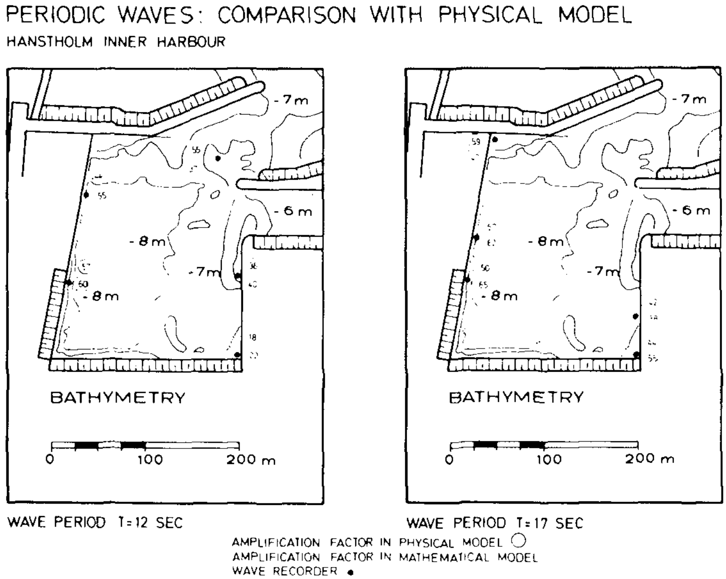

Fig. 11. Comparisons between amplification factors obtained in the first inner harbour in the physical model with those computed using the System, for 12s and 17s periodic waves of 10 cm amplitude applied at the harbour entrance.

suitable software into a digital description of the region and from this the grid points of a model of any size, orientation and degree of resolution can be generated. The results usually contain a considerable amount of 'noise' (2-4 points per resolved wave length), as illustrated by the bathymetry contours of a model resulting from this process, shown in Fig. 15a. Since this 'noise' is irrelevant to the essential behaviour of the model and it is improperly resolved in the model, it is usual to filter the bathymetry during the grid-generating process. By way of contrast with Fig. 15a, a very heavily smoothed model bathymetry is shown in Fig. 15b. Considerably less smoothing is used in System 21 models.

This process of automatic model generation and model operation allows the use of many different model areas in a single job. Fig. 16 illustrate the use of this facility in a study of a large harbour in the Persian Gulf. The objective here was to determine wave agitation in the area at different stages of the harbour construction. Fig. 17 shows the irregular wave train used as input while Fig. 18 shows (with a greatly exaggerated vertical-horizontal scale) a perspective plot of wave action around a 200 m long obstruction placed in the area. Fig. 19 shows the corresponding isolines of significant wave heights. (Figs. 16-19 courtesy Stevin Dredging B.V.).

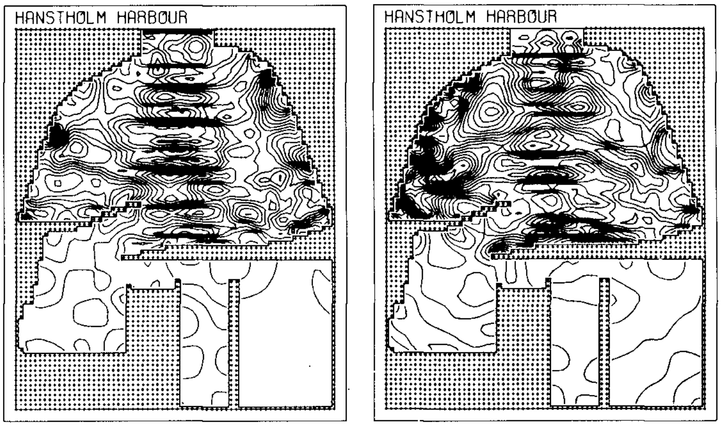

Fig. 12. Surface elevation contours with irregular waves (Significant wave height ≈ 1.5 m). Left: After 400 sec. simulation. Right: After 500 sec. simulation.

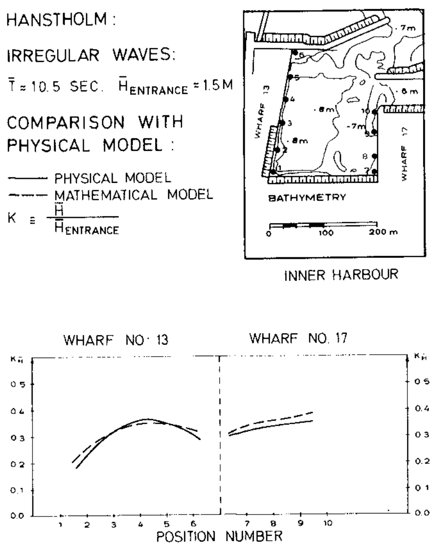

Fig. 13. Comparison between amplification factors obtained in the first inner harbour in the physical model with those computed using the System model, for tests with irregular, field-measured waves, applied at the harbour entrance.

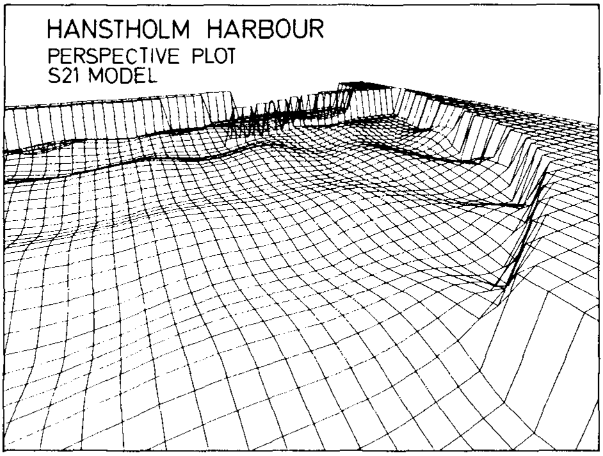

Fig. 14. A perspective plot of regular waves in the outer harbour (view from the harbour entrance).

Another application is illustrated by the contours of a Mediterranean harbour in Fig. 20. Fig. 21 shows 10s waves penetrating the harbour as viewed from two different elevations. Fig. 22 shows 14s waves, the upper picture corresponding to 143 time steps (200 sec simulation), and the bottom picture to 200 time steps (280 sec simulation) after starting from cold. The time step here was 1.4s. A corresponding chart of amplification factors is shown in Fig. 23.

It is possible to include ship motions simultaneously with wave motions in a new version of the system. At each time step the pressure field around the ship is computed simultaneously with the wave field, the resultant force and thence motion of the ship is determined, the ship moved into the corresponding new position. Then at the next time step this process is repeated.

A typical resulting movement in pure (heave) of a dredger is illustrated in Fig. 24.

4. ECONOMIC FEASIBILITY

The results obtained with this latest version of the System 21 show that it is capable of solving a wide range of short wave problems of engineering interest. Models can be constructed from charts and run under

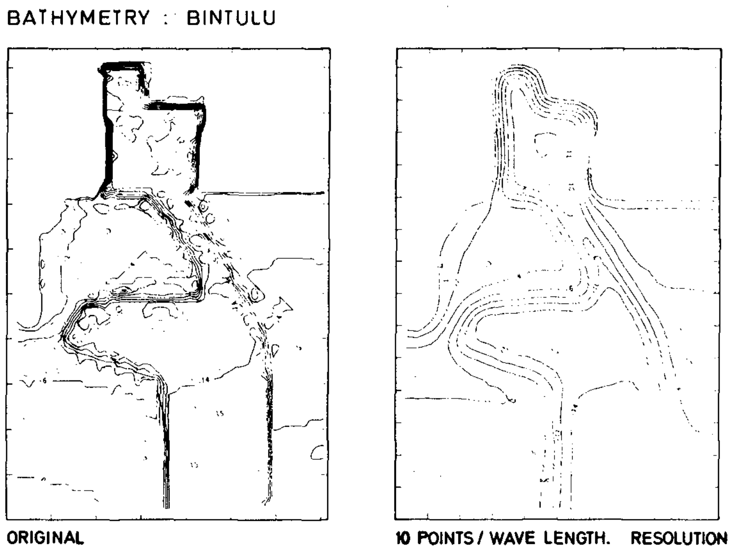

Fig. 15. Contours of the Bintulu Deep water harbour (Borneo) as
a. (left) generated by the grid process (no smoothing)
b. (right) generated by the grid process, when great smoothing are requested.

a wide range of incident wave climates in a few hours, or at most days and a large number of alternative engineering solutions can be investigated. The results are compatible with the input data used to steer irregular wave generators for physical model tests, so that a working interaction between numerical and physical models is practical.

However, it must be accepted that short wave modelling is more demanding in the level of resolution required, as compared with the now established nearly-horizontal flow modelling of tides, storm surges, seiches and similar "long-period" phenomena. This need for a much finer resolution implies the use of many more grid points in both space and time in a model and although the operational speed of the code is but little reduced by the higher accuracy and vertical acceleration resolving capacity, the cost of running a model for a given physical time becomes important. This cost must, of course, be acceptable to engineering practice if the work is to have any other than an academic interest. For example, the cost of running the simulation shown in Fig. 22 is about U.S. $ 24 per physical minute simulation of the 3 km^2 harbour. It is considered necessary, however, that this price be reduced to U.S. $ 6

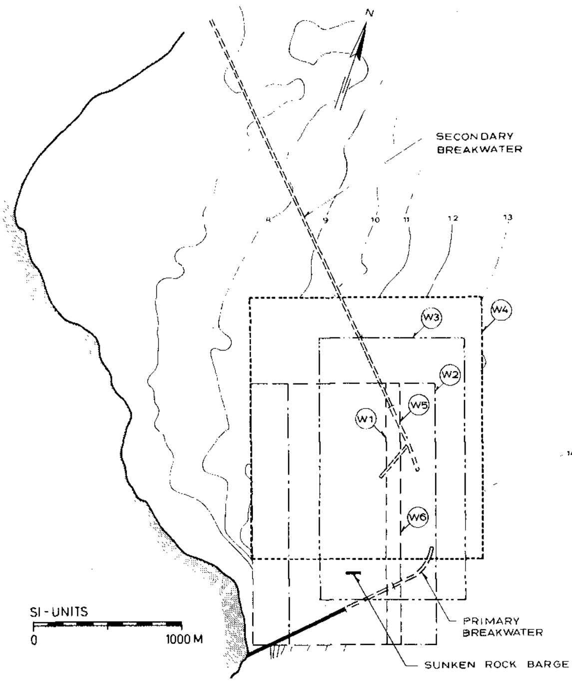

Fig. 16. Mathematical model grids used in a large harbour in the Persian Gulf.

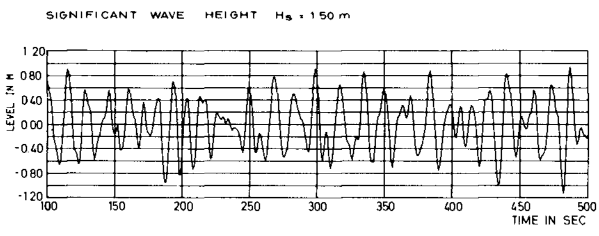

Fig. 17. The irregular wave train which was used as input to the S21 models in Fig. 16.

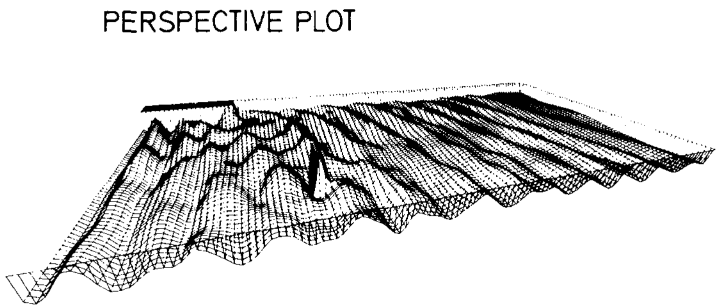

Fig. 18. A perspective plot of the wave action around the sunken rock barge in fig. 16.

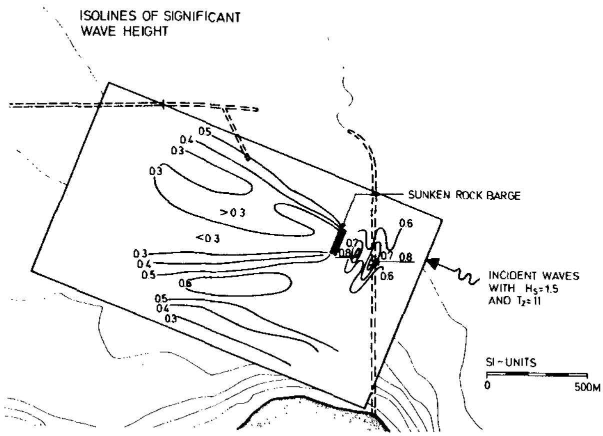

Fig. 19. Isolines of significant wave height H_s around the sunken rock barge in fig. 16.

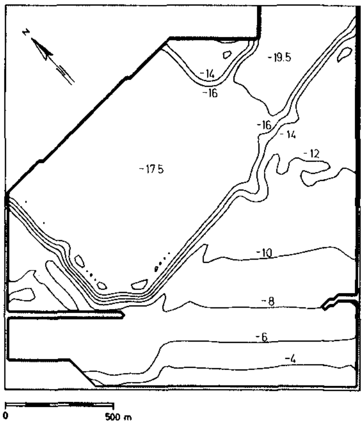

Fig. 20. Bathymetry (depths in meters) of the planned Misurata Harbour, used in the numerical model tests in figs. 21-23.

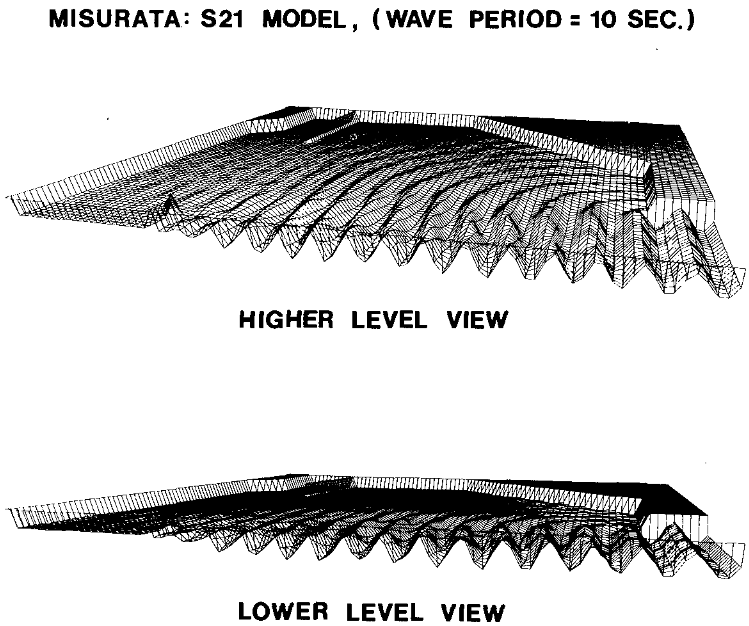

Fig. 21. Perspective plots of a regular wave field in the planned Misurata Harbour (wave period T = 10 sec.).

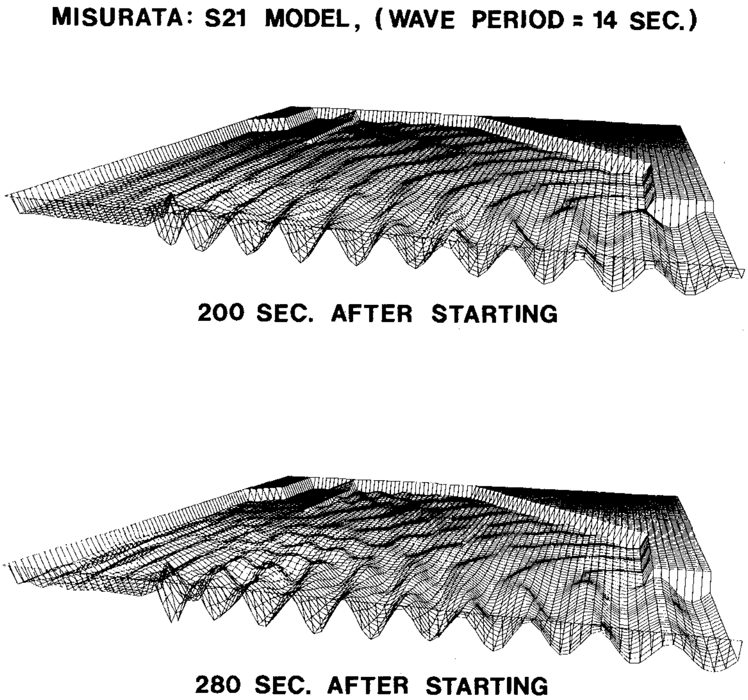

Fig. 22. Perspective plots of a regular wave field in the planned Misurata Harbour (wave period T = 14 sec.).

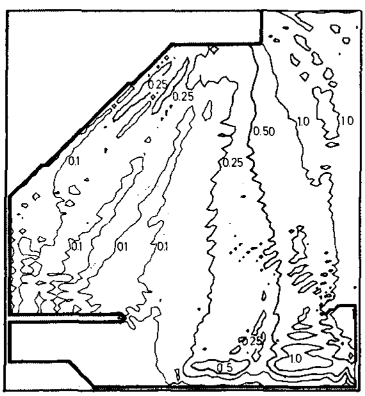

Fig. 23. Amplification factors for a regular wave field in the planned Misurata Harbour (wave period T = 10 sec.).

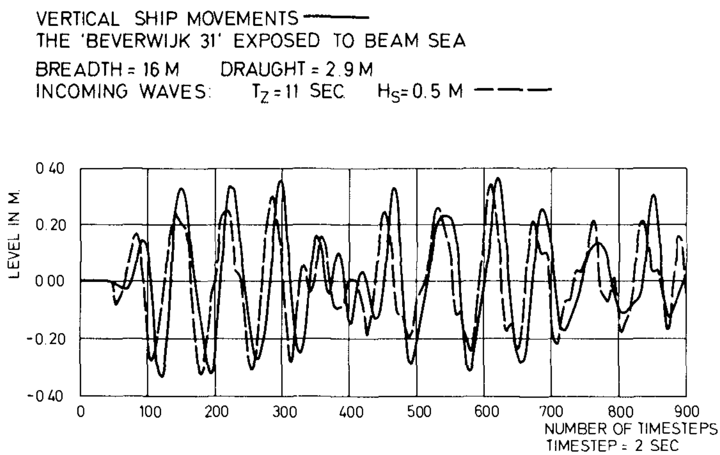

Fig. 24. Simulation of heave motion of a dredger exposed to beam sea.

per physical minute by mid 1979 in order that the system may have a satisfactory range of practical applications. It is expected that this objective will be met. This new System 21 version obviously has a wide range of applications, to ship motions, sediment transport and other problems, as outlined by Abbott, Petersen and Skovgaard (1978).

ACKNOWLEDGEMENT

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Notation

d	Rubble diameter (m)
D	Mean water depth (m)
h	Water depth, $\equiv D + \xi$ (m)
n	Pore volume
p	x-volume flux, or alternatively x-horizontal momentum level per unit density, ($m^2 s^{-1}$)
q	y-volume flux, or alternatively y-horizontal momentum level per unit density, ($m^2 s^{-1}$)
t	Time (s)
x	Horizontal coordinate (m)
y	Horizontal coordinate, direction orthogonal to that of x (m)
z	Bed elevation (m)
α	Laminar flow resistance coefficient for a permeable breakwater
β	Turbulent flow resistance coefficient for a permeable breakwater
ξ	Water surface elevation (m)
ν	Kinematic viscosity ($m^2 s^{-1}$)

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