

CHAPTER 14

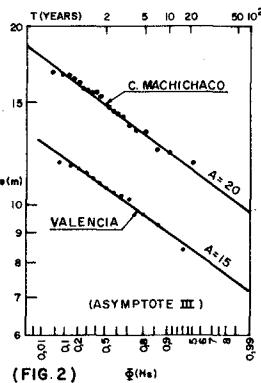
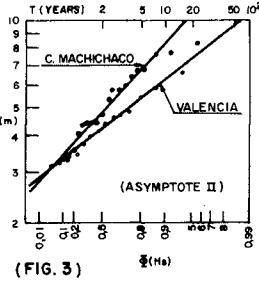
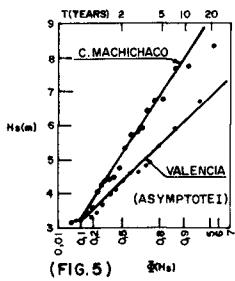
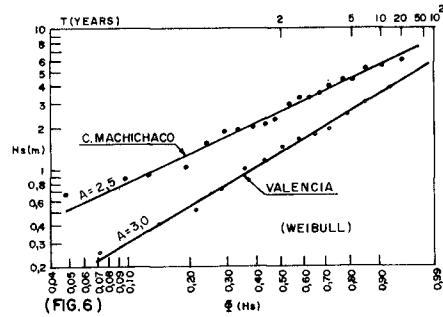
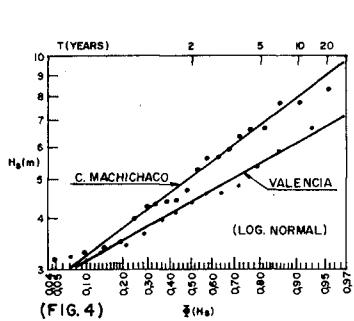
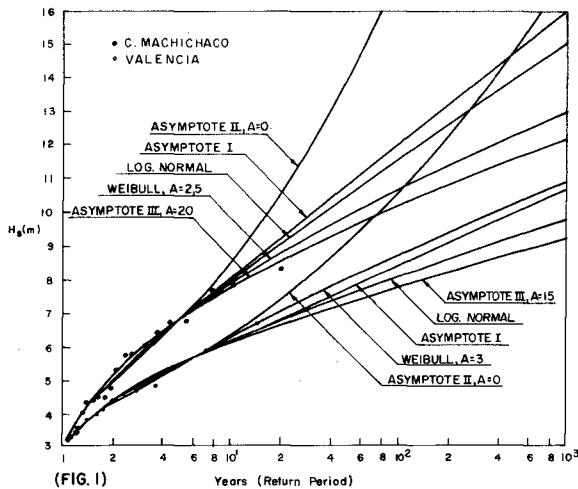
EXTREMAL PREDICTION OF SIGNIFICANT WAVE HEIGHT Enrique Copeiro*

The most generally used procedure for estimating the extremal distribution of geo physical variates consists in obtaining a sample of extreme values (for instance a number of annual maxima) and fitting to them a distribution function. One of the main problems - involved in this procedure is the choice of the type of distribution adequate in each case. No general agreement exists, to date, for any geophysical variate. This means a serious trouble because of the wide range of extrapolations which can usually be obtained by using different functions. Some of the authors who have tackled this problem have adopted a strictly empirical point of view, going as far in it as to advise to make choice for each particular case, according to the goodness-of-fit obtained when several types of distribution functions are fitted to the sample. Others have instead tried to base the choices on some theoretical foundation, placing less emphasis in the goodness of the fits and generally suggesting the use of one or other of the three well known Asymptotic Extremal Distributions.

In actual practice, the casuistic choice of function from each extremal sample -- does not provide a reliable solution to this problem as a general criterion. The methodology in use today for estimating distributions from extremal samples (or, in general, from samples of independent values) of a random variate suffers from ambiguity in several respects (1). Because of that, it is often uncertain to determine how much of the differences observed between goodness of fits is due to the methodology itself or to the different degree of adequacy of the functions which are being tried. This reduces the meaning of the differences between fits, particularly when they are small. Minor differences between fits can not be considered relevant in the choices. This is unfortunate because in most real cases the extremal sample available is not too large and usually some different functions can be used which give only minor differences between its goodness-of-fits but which diverge considerably in extrapolations. The variate significant wave height (H_s) is a good example, its extremal samples being obtained by hindcasting (or visual estimates) and resulting not only of small size generally but also of moderate (or low) accuracy. In Figs.(2) to (6) two published extremal samples of H_s (2) hindcasted for Cabo Machichaco (Bay of Biscay , Spain) and Valencia (Spain) have been fitted, with minor differences between fits, by distribution functions Asymptote-I, Asymptote-II, Asymptote-III, Weibull and Log-Normal. All of these functions have been recommended for general use (Asymptote-I, (2); Asymptote-II, (3); Asymptote-III, (4), or used in some published cases, for the variate H_s . The dispersion of extrapolations is broad (Fig.(1)): For $T=100$ years the maximum difference between results is 3 m. at Valencia and 7 m. at Cabo Machichaco. These differences would have large repercussions in the design of maritime structures, and the differences between goodness-of-fits could not provide a reliable choice criterion in both cases. It can be proved (1) that the reliability of every extremal sample (or any set of observed probabilities obtained by random sampling) is not constant but varies along the values of the variate. The approximation to the true (population) probabilities is best at the center of the distribution function $F(x)=0,5$, and diminishes towards both tails. From this point of view, a non-ambiguous fitting criterion was developed (1). When using it, the upper and lower tails of the sample points are excluded from the fits because of their low reliability. Therefore it must be realized that the effective or useful size of the samples is quite smaller than their total size. The need for very long samples is thus emphasized, the others not being capable to yield reliable extrapolations in individual analysis.

The theoretical justifications stem from the basic extremal equation for a random variate X : $\Phi(x)=(F(x))^n$, where $\Phi(x)$ is the probability of x not to be exceeded in any of n random trials, and $F(x)$ is the distribution function of the variate or probability of x not being exceeded in a single trial. For x approaching 1 and n large enough, the extremal distribution converges asymptotically towards one of the so-called First, Second and Third Asymptotic Extremal distribution functions, when one of three types of conditions is fulfilled by the tail of interest of $F(x)$. Because these conditions are quite broad, covering a very wide spectrum of distribution functions, and n is supposed to be large for most geophysical variates along one year (the basic geophysical cycle), a great number of authors have assumed (notably after the fundamental work of E.Gumbel (5))that the extremal distribution of any geophysical variate could be closely approximated by one or other of the three Asymptotes. References (5) and (1) mention a good number of published applications of the Asymptotes in extremal analysis of variates like temperature, wind speed, rainfall, river discharge, significant wave height, etc. In the justifications for the use of the Asymptotes it has generally been assumed that, for instance, a variate like the average discharge in 24 hours has in a year a value of $n=365$, a number which is fairly high and supposed to yield statistical independence at high levels of the variate (E.Gum-

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bel, (5)). This interpretation of the parameter n is erroneous. Variates like temperature, whose evolution is not discrete but continuous with time, can not be said to "occur" a certain "number of times" along a year. The variate takes an infinitude of values within a finite time interval. Therefore, it is not possible to assign directly a value to n . This, which is self-evident for an instantaneous variate like temperature, can be shown to be also true for other variates. J. Battjes (6), commenting on an extremal analysis of H_s , showed that when parameter n is given a value equal to the duration of the year divided by the duration of each H_s record, an absurd result is obtained. This author was the first, to the knowledge of the writer, to realize for a particular variate that such an assumption for n is not correct (in spite of what, some further extremal analysis of H_s with the same erroneous criterion for n have been published later). Actually the same applies to all geophysical variates which consist in averages or totals within a fixed time interval. These variates belong to a group whose evolution with time is continuous, to which most of the variates relevant in civil engineering belong: Rainfall in a time interval; Discharge in a time interval; Average wind speed in a time interval; Significant wave height; etc. The average discharge in 24 hours does not take on 365 values in a year, but an infinitude. The fact that just 365 of them are juxtaposed in time is not particularly relevant. It can be shown (1) that an assumption like $n=365$ leads to an absurd result in the extremal analysis of this variate too, and so for all the variates of the same type. It will be shown later on that, for that type of variates, n is not a constant value but a function of the variate. From this follows that the derivation of the Asymptotes, which was done with the implicit assumption $n=\text{constant}$ (see, for instance, (5)), is not valid for the continuous-evolution variates. Therefore the extensive use which has been done of the Asymptotes for those variates does not have any theoretical justification as yet. The writer (7) presented a model by which the extremal equation can be applied to continuous variates:

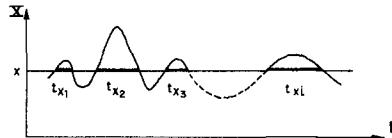
1.- EXTREMAL EQUATION FOR CONTINUOUS EVOLUTION VARIATES

The extremal equation $\Phi(x)=(F(x))^n$ was stated on a discrete basis for the occurrence of the variate. In order to apply this equation to a continuous-evolution variate, the variate itself must be prepared for a discrete analysis. This can be done when, instead of the individual values taken by the variate, the undulations described by the variate in its evolution along time are taken into consideration. The undulations can be said to have individual physical entity with finite dimensions and can therefore give support to a discrete analysis. Each entire undulation can not be assigned a certain duration. Instead, only a fixed level x of the variate will be considered. Cutting the continuous curve at the level x , a number of isolated undulations (what will be called "curves of exceedance of x " or more simply " x -exceedances") remain above the cut. Now a dichotomy can be established at each level x : The probability of occurrence of a x -exceedance, or of its non occurrence. This entails to change the continuous axis "time" into a discrete "number of times" (or statistical trials) in which an event (x -exceedance) might happen or not. For that, the "duration" of each statistical trial is taken as the average duration $t(x)$ of the x -exceedances. The "number of trials" at the level x is, in an average year of duration T_y :

$$n(x) = \frac{T_y}{t(x)} .$$

$$\text{ x -exceedances in a year or } n_x = \frac{\sum t_{xi}}{t(x)} ,$$

$$\text{the probability that in a single "trial" a -} \\ \text{ x -exceedance does not occur is } 1 - \frac{n_x}{n(x)} = 1 - \frac{\sum t_{xi}}{T_y} .$$



This is the expression for the distribution function $F(x)$ of the former continuous-evolution variate X . Therefore, the probability that in the $n(x)$ trials of the average year no x -exceedance will appear, or extremal distribution function, is:

$$\Phi(x) = [F(x)]^{n(x)}$$

The resulting expression is similar to the extremal equation for a discrete evolution variate except for the exponent, which now is not constant but a function of the variate. The type of function corresponding to $\Phi(x)$ will be determined once the types of functions $F(x)$ and $n(x)$ are found. This will be done in the following for some variates, chiefly H_s . An empirical determination of the form of $F(x)$ and $n(x)$ from several samples is far more reliable than the same direct estimate for $\Phi(x)$, due to the incomparably higher number, length and accuracy of the samples of $F(x)$ and $n(x)$ than of $\Phi(x)$. The extremal samples can best be used as control checks for the predictions made with the extremal equation. This was done in (1) for a few cases.

2.- FUNCTION $n(x)$

It will be first indicated which is the kind of relationship existing between the values of $n(x)$ corresponding to the whole population and the estimates of this parameter obtained from a limited sample or observation period. For each sample, estimates of $n(x)$ are obtained from the observed average durations $t(x)$ of the exceedances.

- The higher the number of exceedances, the more reliable (close to the population) the estimate of $t(x)$ and thus of $n(x)$. In every single observation period that number varies along the range of values of X (Fig. 48), having a sharp maximum at a certain level - (very close to the center of the distribution function, $F(x)=0.5$, (1), and steadily diminishing towards both tails. The reliability varies in the same way too.

- Assuming that along a certain "central" stretch of values of X the $n(x)$ estimates are correct or acceptable, the estimates belonging to the adjacent zones in both directions will show a random deviation from the population values due to their low reliability.

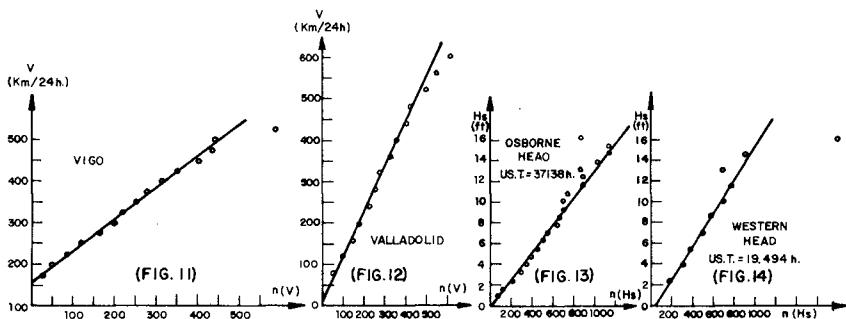
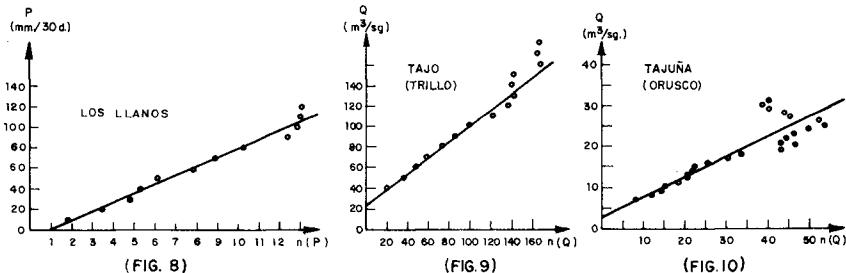
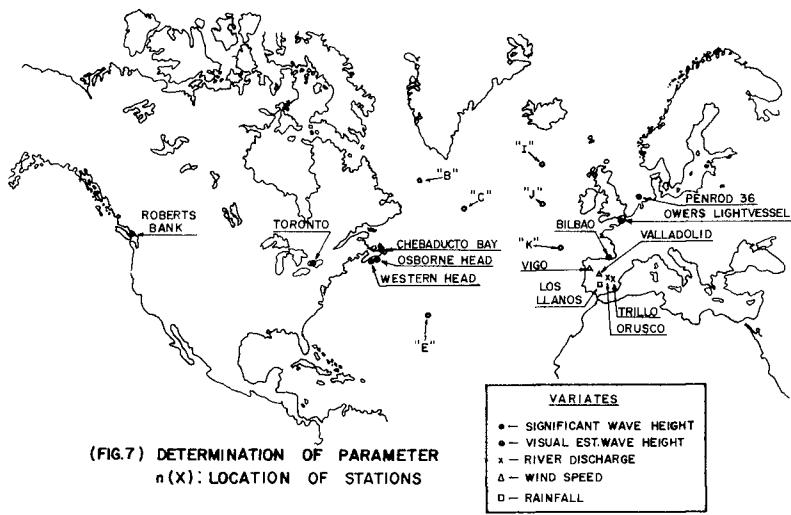
- The variate may be in general unlimited in both directions, but each sample will instead be necessarily limited by a maximum and a minimum values. The observed values of $n(x)$ will show, at both ends, systematic deviations: Towards $n(x)=0$ at the minimum observed value of X , and towards $n(x)=\infty$ at the maximum. Most geophysical variates, such as H_s , have a natural lower limit at $x=0$, and then only the upper systematic deviation exists.

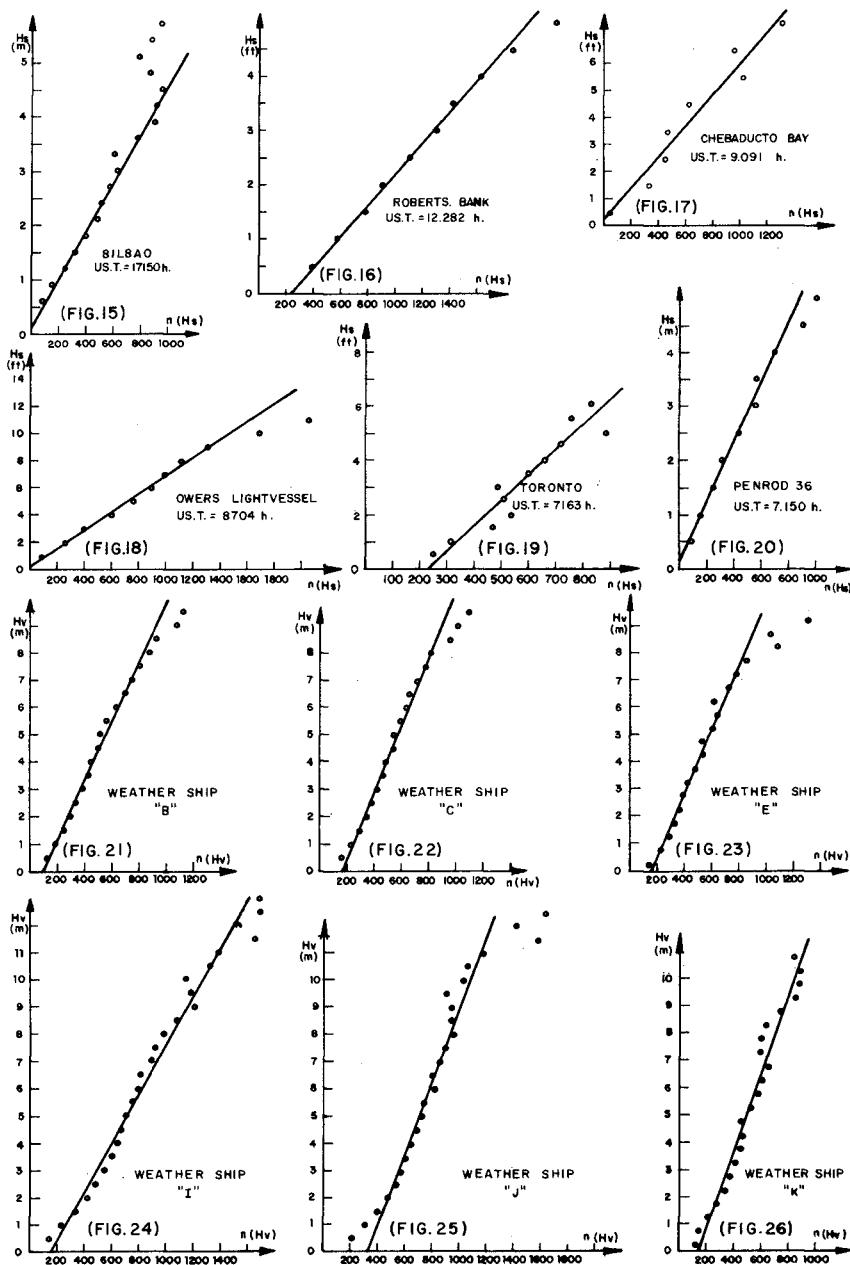
STATION	SITUATION	VARIABLE	OBSERVATION PER.	USEFUL TIME	INTERVAL MET. OBSERV.	DEPTH (m.)	SOURCE
LOS LLANOS	CUENCA (SPAIN)	RAINFALL IN 30 DAYS	I-X 1940 - I-X 1970		30 DAYS		INTECSA (MADRID)
VALLADOLID PEÑADOR	VALLADOLID (SPAIN) VIGO (SPAIN)	AVERAGE WIND SPEED IN 24 HOURS	I-X 1970 - I-X 1973 I-II 1970 - I-X 1972		12-24 HOURS		INSTITUTO METEOROLOGICO NACIONAL (MADRID)
TRILLO ORUSCO	RIVER TAJO (SPAIN) RIVER TAJURÁ (SPAIN)	AVERAGE DISCHARGE IN 24 HOURS	I-X 1963 - I-X 1973 I-X 1952 - I-X 1963		24 HOURS		J. CIRUJEDA (CENTRO DE ESTUDIOS HIDROGRÁFICOS)
OSBORNE HEAD WESTERN HEAD CNEBADUO BAY ROBERTS BANK TOKTO OWERS LIGHTVESSEL PENROO 36 BILBAO, P. LUCERO	NORTH ATLANTIC (W.) NORTH ATLANTIC (W.) NORTH ATLANTIC (W.) STRaits of GEORGIA LAKE ONTARIO ENGLISH CHANNEL NORTH SEA BAY OF BISCAY	SIGNIFICANT WAVE HEIGHT (INSTRUMENTALLY RECORDED)	15-XII 1970 - 1-I 1976 15-IV 1970 - 5-V 1973 24-X 1974 - 4-III 1976 7-II 1974 - 3-IV 1976 15-IV 1972 - 18-VI 1973 1-X 1979 - 1-X 1979 1-III 1975 - 1-II 1974 2-IV 1976 - 2-V 1976	37.138 H. (70.1%) 19.494 H. (74.2%) 9.091 N. (81.5%) 12.282 H. (70.0%) 7.165 H. (81.2%) 8.704 H. (99.4%) 7.150 H. (81.9%) 17.355 H. (98.6%)	30.3 40 - 45 26.7 139 108 13 - 15 26 40		DON BURRELL (MARINE INFORMATION DIRECTORATE, CANADA)
WEATHER SHIP "B" WEATHER SHIP "C" WEATHER SHIP "E" WEATHER SHIP "I" WEATHER SHIP "J" WEATHER SHIP "K"	NORTH ATLANTIC (E.)	VISUALLY ESTIMATED WAVE HEIGHT		1949 - 1972 1952 - 1972 1952 - 1972 1949 - 1971 1949 - 1971 1949 - 1969	91 % 92 % 89 % 88 % 88 % 93 %	1-3 HOURS	ENVIRONMENTAL DATA SERVICE (U.S.A)

TABLE-1

These conditions were satisfied by the linear relationship $n(x)=A(x-B)$, in the analysis of 13 sets of data (see Table 1) belonging to the following continuous evolution variates: Instrumentally recorded significant wave height (8 cases); average wind speed in 24 hours (2 cases); average river discharge in 24 hours (2 cases); and total rainfall in 30 days (1 case). In Fig. 7 the location of all observation stations is shown. Figs. 8 to 20 show the linear fits. In this figures, the points for whose computation less than 10 exceedances were available have been excluded. That has limited considerably the deviations which could be seen. The fact that in the cases analyzed the upper deviation next to the central stretch starts more often towards large $n(x)$ values than towards small ones, goes in well with the skewness which was appreciated in the distributions of the durations at each level of X . The skewness indicates a higher frequency of durations below the average than above it. The proper fit of the function $n(x)$ to the data calls for the use of "accuracy intervals" as defined in (1). However, visual fits are acceptable for extremal analysis provided that the "central" stretch of reliable estimate is long enough. Later on some interesting results concerning H_s are obtained.

In Figs. 21 to 26 six long duration sets of visually estimated wave height are analyzed. These are not intended to be a check of the linear function, since the reliability of the visual estimates is as yet less clear than should be. However, they have been included here because of the practical importance of visual wave observations. The central and lower points lie in a peculiar sinuous pattern, which only becomes straight on a log-log paper (1). This seems to give support to the power relationship between H_s and H_v suggested by the theoretical analysis of the extreme value theory.





ted by Nordstrom (9). But the log-log plots yield a quick systematic deviation of the upper tail, which might indicate that the power relationship is not applicable to the higher waves. It is possible that no single simple relationship is valid for the whole range of wave heights. This point still needs clarification, and in the mean time the linear fits of Figs. 21 to 26 will be used (in following sections) as an approximation which probably (in view of - the acceptable behaviour of the upper points) is reasonably accurate for extrapolations.

3.- FUNCTION $F(x)$

Only the variate significant wave height will be tackled in this section. The 8 sets of instrumental data used in the preceding section (Table 1) will be analyzed, together with other 12 sets selected from the technical literature (Table 2). The aim of the selection was to choose the longest possible durations (and complete annual series when duration consists in a small number of years) and waters not too shallow at the site. The result is a compromise which seems acceptable as a whole. Fig. 28 shows the location of the 20 stations.

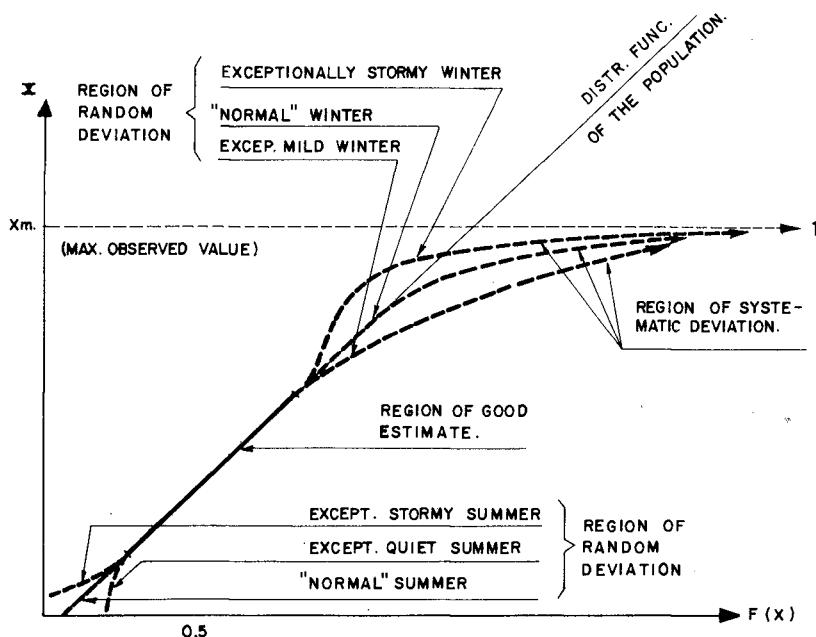
A comparison has been done between the distribution functions Exponential, Log-Normal, Weibull and Double Exponential (Asymptote-I) which are the most widely used today for H_s . No general agreement exists today in this respect, what is unfortunate because of the large differences which can be obtained in the extrapolations when one or another of those functions are used. The comparative study is set up under the initial hypothesis that for - any geophysical variate a single type of distribution is valid, at least within each type - of homogeneous climate which can be discriminated in the behaviour of that variate. The results obtained prove that the hypothesis works in the case of H_s . The relative amount of swell existing within the waves recorded at each site has been chosen as an operative criterion to discriminate between different "wave climates". The situation of the stations relative to prevailing winds, size and limitations of available fetches and local shelters, and tables of coincidence wave height-wave period, have been evaluated in order to assign each station to a certain group. According to this, four different groups have been defined: In both extremes are the groups denominated "Very low swell" (Toronto, Roberts Bay, Morecambe Bay, Mersey Bar, Nice, Benghazi, Chausey Sud, Dunkerque), and "Heavy swell" (important relative weight of swell reaching relatively high values of the variate: Sevenstones, Bilbao, Cattlewash, Camp Pendleton). As intermediate groups, "Low swell" (Chebaducto Bay, St. John Deep, Smith's Knoll) and "Moderate swell" (Osborne Head, Western Head, Penrod 36, Varne, - Owers Lightvessel). It must be admitted that the border between the two intermediate groups is not altogether clear, but that is not a serious trouble in the comparative study.

STATION	SITUATION	DEPTH (m)	OBSERVATION PERIOD	USEFUL TIME	INTERVAL BET. OBSER.	SOURCE
DUNKERQUE	NORTH SEA (S)		12-IV-1960-17-VIII-1966	706 DAYS		R. BONNEFILLE ET AL. (1967) ($H_{max}, \sim H_s$)
CHAUSEY SUD	ENGLISH CHANNEL	9.5	27-VI-1956-4-IV-1960	204 DAYS		M. ALLEN (1970)
NICE	MEDIT. SEA (N)	9-14	17-IX-1954-27-V-1960	1006 DAYS		H. POWERS ET AL. (1968)
CAMP PENDLETON	NORTH PACIFIC (W)	9.8	1.954 AN-1.956		6 H.	M. SINGH ET AL. (1968)
BENGHAZI	MEDIT. SEA (S)	12.8-14.6	1961-1965 (IRREGULAR)			J. KHANNA ET AL. (1974)
ST. JOHN DEEP	NORTH ATLANT. (W)	38.6	1-III-1972-26-II-1973			C. DEANE (1974)
CATTLEWASH	CARIB. SEA (E)		XII-1972-XI-1973			
MORECAMBE BAY	IRISH SEA	21.9	XI-1966-X-1967	8760 HOURS	3 H.	
MERSEY BAR	IRISH SEA	17.6	IX-1966-VIII-1966	8760 HOURS	3 H.	
SEVENSTONES	NORTH ATLANT. (W)	60.4	I-1962-XII-1962	8760 HOURS	3 H.	J. BATTJES (1970)
VARNE	ENGLISH CHANNEL	27.5	II-1965-I-1966	8760 HOURS	3 H.	
SMITH'S KNOB	NORTH SEA	49.4	III-1969-II-1970	8760 HOURS	3 H.	

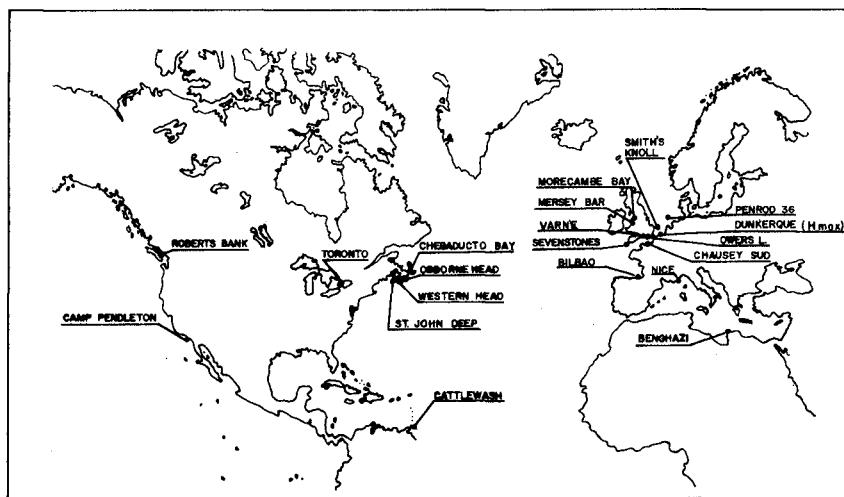
TABLE-2

Similarly to what was done for parameter $n(x)$, it will be indicated which kind of - relationship exists between the values of $F(x)$ corresponding to the population of the variate and the values observed in a sample of limited size (observed distribution):

SAMPLE OF INDEPENDENT OBSERVATIONS: In a sample of N independent observations of a random event, the probability of the observed probability of success being $\frac{m}{N}$ can be computed from the binomial distribution, $P(m) = \binom{N}{m} p^m (1-p)^{N-m}$ where p is the true (population) probability of success in a single trial. Now, the event "success" is the occurrence of a value higher than x in each observation of a random variate X with a distribution function $F(x)$. The probability that in N independent observations the observed probability at the level \underline{x} (the observed frequency of x being exceeded) is $\frac{m}{N}$, can be computed as: $P(x, m) = \binom{N}{m} q(x)^m (1-q(x))^{N-m}$ where $q(x) = 1-F(x)$. The reliability of a sample estimate of $q(x)$ - can be measured as the probability of the observed probability $\frac{m}{N}$ to fall inside a certain - (arbitrary) interval around the true probability $q(x)$: $((1-k) q(x) < q(x) < (1+k) q(x))$. It can be shown (1) that: a) The convenient intervals ("accuracy intervals") should be defined in relative terms with respect to $q(x)$, so that k is a certain percentage of $q(x)$ to be deter-



(FIG. 27).— OBSERVED DISTRIBUTION CURVES: CHARACTERISTIC REGIONS.

(FIG. 28).— DETERMINATION OF $F(H_s)$: LOCATION OF STATIONS

mined accordingly to the accuracy required by the user; b) the appropriate "true" probability to be used as reference for these intervals is $q(x)$ for the upper half of the distribution function ($q(x) < 0,5$) and $F(x)$ for its lower half ($F(x) < 0,5$). The use of the binomial distribution as indicated above shows that, for any fixed width of the accuracy intervals (any constante value of k), the reliability of the observed probability (probability of that probability to fall inside the intervals) is maximum at the center of the distribution function $F(x)=q(x)=0,5$, and diminishes towards both tails of the distribution. Thus, for any sample of independent values of a random variate X , provided it is large enough, two characteristic regions can be discriminated in the set of observed probability points:

- A central region of good estimate, where the observed probabilities are close to the true (population) probabilities.

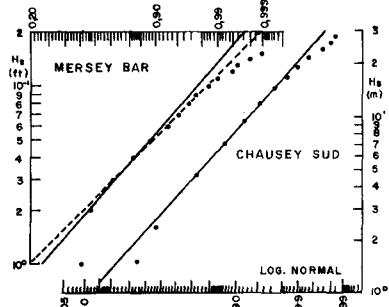
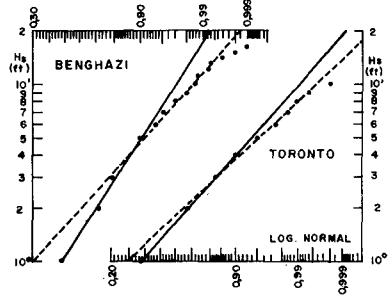
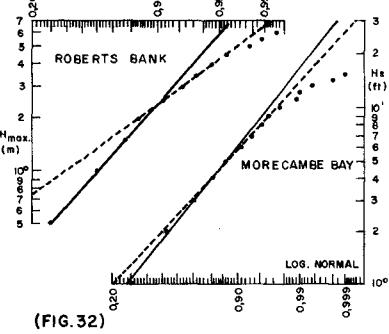
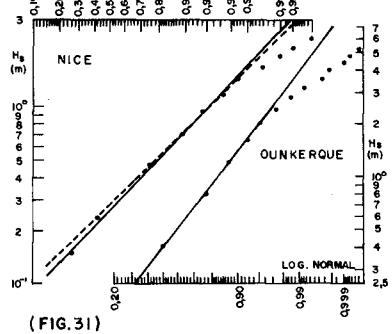
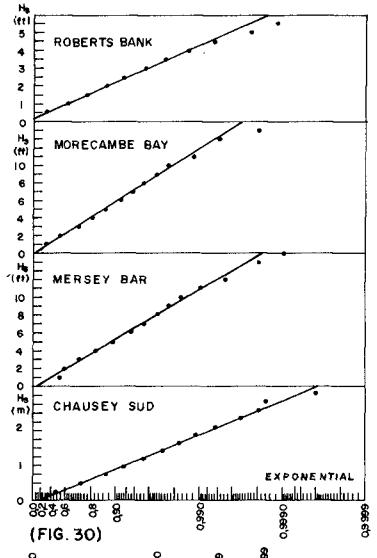
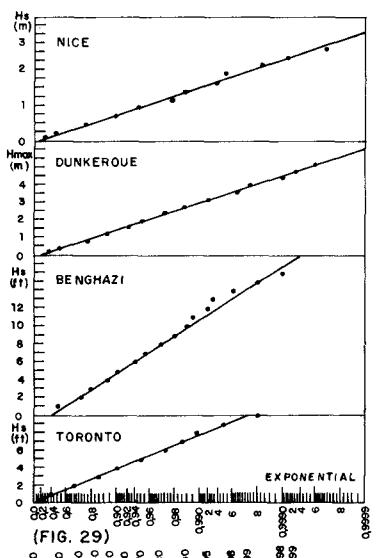
- Regions of poor estimates at both tails, where the observed probabilities are expected to show wide random deviations from the population probabilities. Limits between both regions can be set by use of accuracy intervals, as discussed in (1) (Accuracy intervals can be defined in terms of the probability or of the value of the variate, and different procedures for estimating their values from the sample itself can be used).

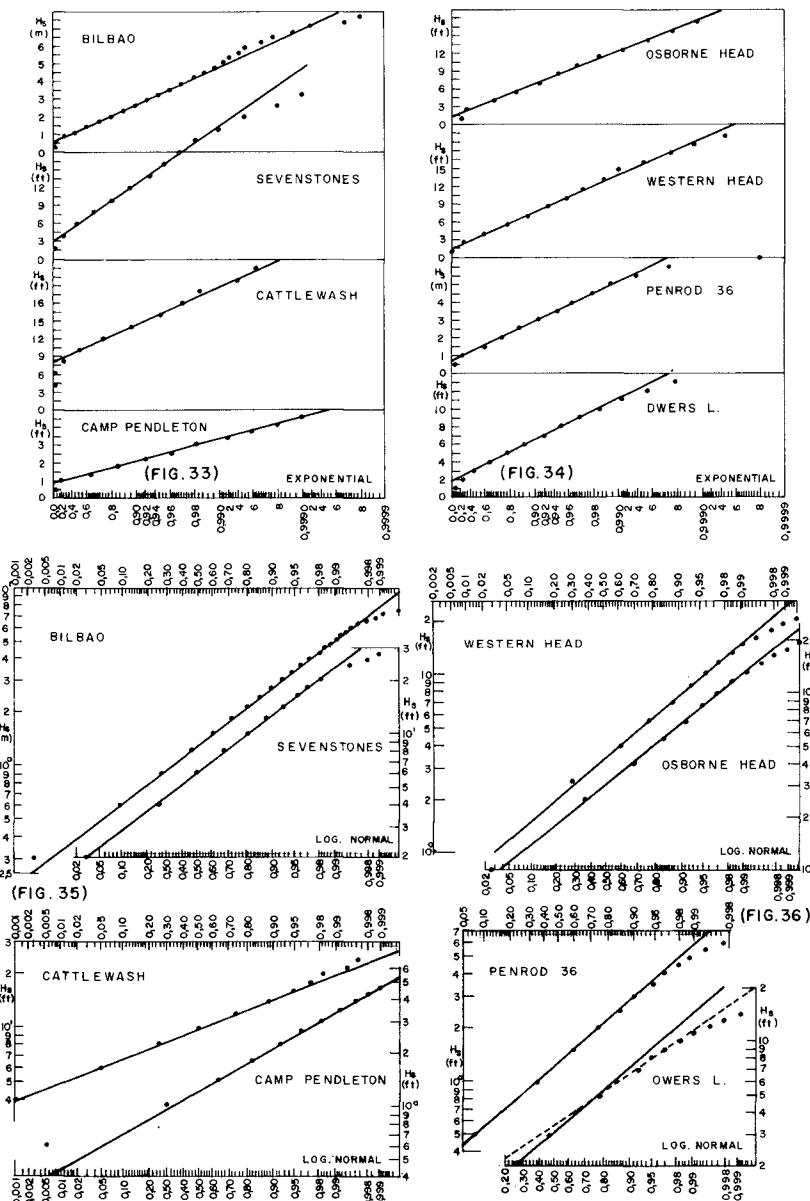
SAMPLE OF CONTINUOUS OBSERVATIONS: If a continuous-evolution variate is observed systematically with short time intervals between records (for instance H_s , with typical observation intervals of 2-3-4 hours), statistical independence between observations can not be assumed. The high density of observations allows the whole curve of evolution of the variate to be drawn, and from it a complete observed distribution (with probabilities from 0 to 1) can be obtained. It can be shown (1) that also for these samples the observed probabilities have a maximum reliability at the center of the distribution function and lower reliability towards both tails. For a sampling period long enough, three characteristic regions can be distinguished in the observed distribution curve: Two of them are the same as indicated above for independent observations, and the third is a final systematic deviation of the uppermost and lowermost tails, which converge asymptotically towards cumulative probabilities 1 and 0 at the maximum and minimum values of the variate observed in the sampling period. This is a natural consequence of any finite sample having a maximum and a minimum observed values, in contrast with the population whose values are in general unlimited. In Fig. (27) the three characteristic regions of an observed distribution of H_s are shown. The lower systematic deviation does not appear, since this variate has a natural lower bound at $H_s=0$. When estimating the distribution function of the population from one of such observed distributions, only the central region of expected good estimate should be considered for the fit.

Comparative studies published in previous years have adopted the criterion of choosing the function which would give the best fit to the whole set of sample points, and specially to its upper tail if extrapolation is the final goal. Such a criterion is erroneous, particularly for continuous or almost-continuous observations, as Fig. 27 makes it evident. The aim of fitting the complete observed distribution and the aim of estimating from it the distribution of the population are not only two different purposes but in fact incompatible. In particular, to place the emphasis of the fit in following the uppermost tail of the observed distributions means to follow the points with the poorest reliability, belonging to the region of expected random deviation or, worse, to the final systematic deviation. This can only lead us away from the expected behaviour of the population, in extrapolations.

In order to compare the behaviour of the 4 functions, it will be checked whether the deviations of the observed values in the regions of "random deviation" are actually random in the set of fits, or whether they are systematic. This criterion suffices to solve satisfactorily the comparison. Although the strictly correct method would be a simultaneous use of accuracy intervals to make the fits and confidence intervals to evaluate the deviations quantitatively, such laborious procedure does not prove necessary in this case. The Exponential and Log-Normal functions will be compared first and later on the Weibull and Double Exponential which are closely related to the Exponential.

EXPONENTIAL - LOG.NORMAL.- In Figs. 29, 30, 31, 32, the fits corresponding to the group "very low swell" can be seen. The Exponential fits are uniformly satisfactory, with deviations which are not systematic and start at reasonable levels of the probability. The longer the observation period, the longer is the central region of good fit (Nice, Dunkerque, Chausey Sud). Instead, the Log-Normal fits are uniformly poor. The upper tails show systematic deviation towards low values of the variate, deviation which begins to show very quickly. In Figs. 33, 35, the group "heavy swell" is fitted, showing quite a different behaviour. The Log-Normal fits are good from the lower region to high levels of





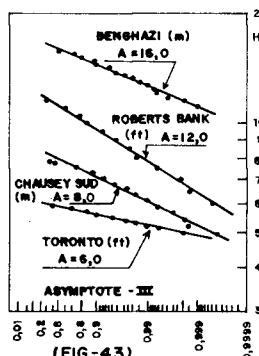
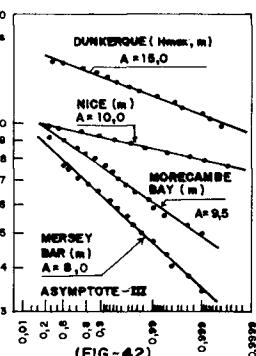
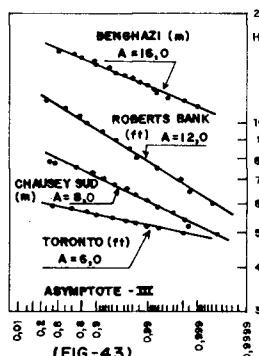
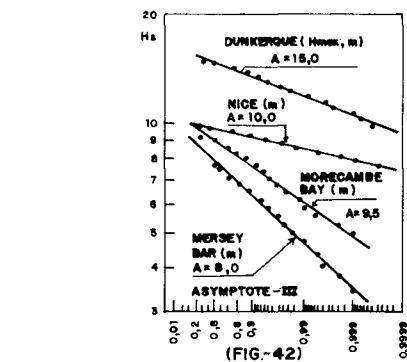
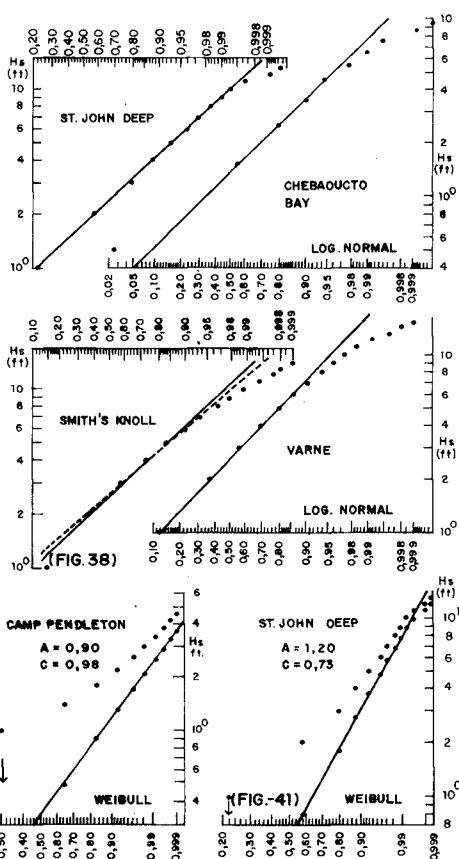
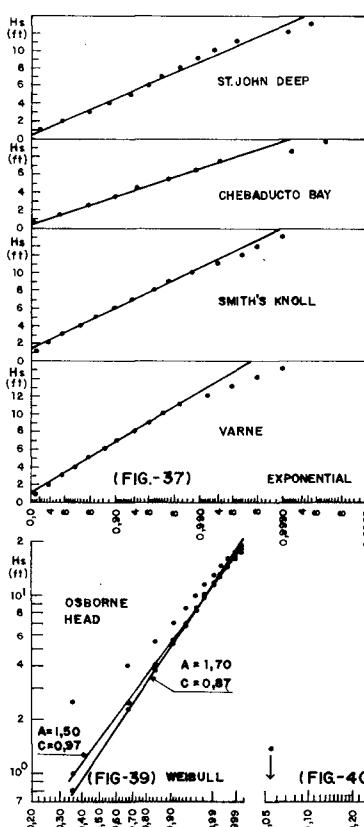
the probability. The Exponential function behaves well in the upper region, but shows a sharp systematic deviation in the lower tail. The number of stations included in this group is small, but the inspection of the two remaining groups will clarify the behaviour of both functions. In Figs. 34, 36, 37, 38, the "low swell" and "moderate swell" groups are plotted. The fits show characteristics which are intermediate between those seen in the former groups. As a rule the Log-Normal behaves better in the lower part of the central region, but in the upper zone shows a quick, systematic deviation. At Penrod 36, Owers L., Varne and Smith's Knoll, the deviations start at probabilities from 0,8 to 0,9, which means a total exceedance time of 73 to 36 days (their observation period is 1 year). This is certainly excessive, since a good number of exceedances are included in that time for those levels of H_s . Even if only the upper part of the central region is fitted, neglecting the lower part^s (discontinuous lines in the figures), the Log-Normal still deviates systematically in its upper tail. Therefore this function should be rejected for extrapolations. The Exponential law shows for this two groups a systematic deviation at its lower tail. This deviation starts at low values of H_s for low levels of swell, and at higher points for high levels of swell. At the upper half of the distributions, the Exponential function behaves uniformly well for both groups and hence for all the 4 groups. Therefore this function appears to be in principle acceptable for extrapolations and, thus, for extremal analysis, although its lower tail can not be used when swell has any relative importance.

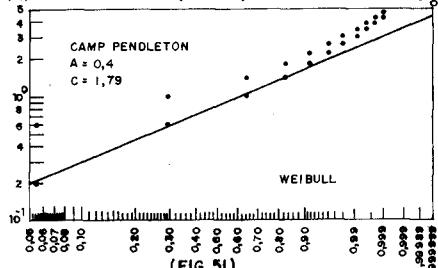
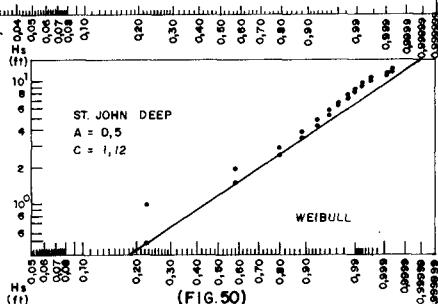
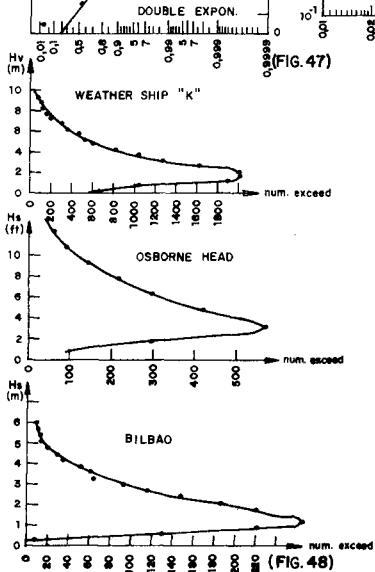
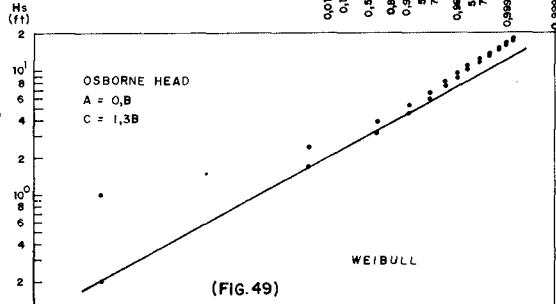
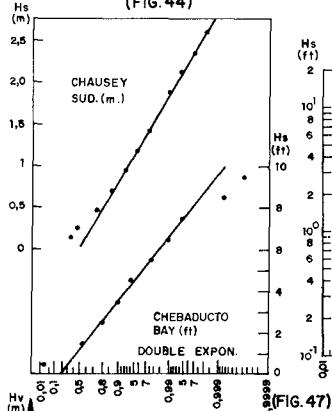
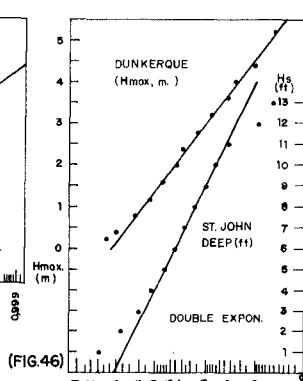
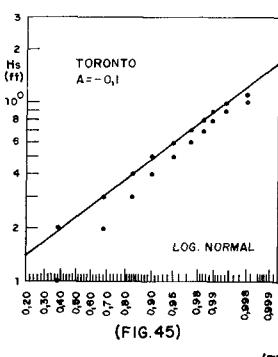
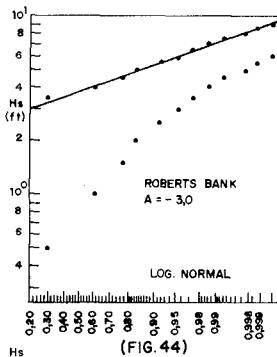
$$\text{EXPONENTIAL - WEIBULL. - Exponential: } F(x) = 1 - e^{-\frac{x-A}{B}} ; \text{ Weibull: } F(x) = 1 - e^{-\frac{(x-A)^C}{B}}$$

The difference between both expressions is only the exponent C which appears in the Weibull function. The aim of this comparison is thus to investigate whether the inclusion of that third parameter in the Exponential function leads to better fits. The individual fits of the 20 stations are not shown here because of space limitations (they can be seen in (1)), but in the following table the values obtained for A and C in the fits are listed. Its three parameters give to the Weibull function a higher flexibility, but also a higher degree of indetermination for estimating the values of the parameters themselves. This is especially true for observations of not long duration, whose central region of good estimate is short. As an example, in Fig. 39 the fit corresponding to Osborne Head is shown: When parameter A is increased in 0,2 feet (≈ 6 cms.), parameter C (which is a certain measure of the slope of the line) changes from 0,97 to 0,87. Should the data of this station belong to a short observation period, there would be no clear way to make a choice between both fits. Due to its high sensitivity to small changes in A (parameter which arranges the points for the fit), the particular values obtained for C must be understood with some amplitude. In the evaluation of the table, the really meaningful values are those corresponding to the stations with relatively long observation periods (Osborne Head, Chausey Sud, Nice, Bilbao, Camp Pendleton, Western Head, Dunkerque, Roberts Bank). Their central region of good estimate is the longest, and thus the influence of the upper and lower sample peculiarities in the fit is reduced. In all these stations the C values are very close to 1. According to what was indicated above about the accuracy of C estimates, it can be assumed $C=1$ for all, in practice. Therefore it has not been found justified to include this third parameter and the choice is still the 2-parameters Exponential function. In the table, the cases in which C takes on values not close to 1 belong to samples with the shortest durations (1 year), where it can be supposed that the peculiar flexibility of the Weibull function leads the one who performs the fit to follow to some extent the deviations of the tails since the central region of good estimate is too short for an unequivocal choice of A . The Weibull function has, in Exponential paper, a curvature concave upwards for $C<1$ and concave downwards for $C>1$, but not always that flexibility allows it to fit the deviations of both tails simultaneously in short samples: In Figs. 49, 50, 51 three cases are shown where the fit of the lower tail leads to an ill behaviour of the rest of the distribution (compare with Figs. 39, 40, 41, where the lower tail was not fitted). In these kind of fits, the values given to A makes the lower points "disappear" from the probability paper, giving an outward look of perfect fit throughout.

	A (m.)	C
Camp Pend.	0,27	0,98
Bilbao	0,60	1,02
Cattlew.	1,52	1,24
Sevens.	0,61	1,20
Osborne H.	0,46	0,97
Western H.	0,52	0,93
Penrod 36	0,50	1,05
Owers L.	0,12	1,41
Varne	0,15	1,11
Smith's K.	0,06	1,31
St.John D.	0,37	0,73
Cheb.Bay	0,15	0,99
Dunker.	0	0,97
Chausey S.	0	0,96
Nice	0	1,02
Rob. Bay	0	1,09
Benghazi	0	0,81
Morec.Bay	0	1,05
Mers. Bar	0	1,02
Toronto	0	1,03

Other 3-parameter functions do also have a high flexibility. For instance, in Figs. 42, 43 the group of stations "very low swell" has been fitted with Asymptote-III functions. The Log-Normal function may include a third parameter too (change $H_s = H_s' - A$): see in Figs. 44, 45, how in this way the fits obtained in Figs. 29, 30 with two parameters are improved with 3.





Even more flexibility would be attained with 4 parameters (Log-Normal with the change $H_s = \frac{H_s - A}{B - H_s}$), and so on. But, as it has been shown, the proper use of functions with a high number of parameters would only be feasible had we a better knowledge of the statistical properties of geophysical variates, than is available today. Otherwise their flexibility is more of a trouble than an advantage. More detailed statistical studies including large numbers of samples are needed. In the mean time we have to be content with the usual simple functions which, if proved well-behaved (like the Exponential herein), are able to give reasonable approximations.

EXPONENTIAL - DOUBLE EXPONENTIAL.- Both functions converge quite quickly for large probabilities:

$$\text{Exponential: } F(x) = 1 - e^{-\frac{x-A}{B}} \quad \text{Convergence: } F(x) \xrightarrow{F(x) \approx 1} e^{-(F(x)-1)} = e^{-e^{-\frac{x-A}{B}}} \quad (\text{Double Exponential}).$$

The speed of the convergence can be visually appreciated by comparing both probability papers. The difference between their probability scales is that, for the lower probabilities, the Double Exponential scale is "stretched" with respect to the other. However this feature is not able to improve, in general, the fits in the lower part of the distributions (which were found to be unsatisfactory with the Exponential). Some of the fits can be seen in Figs. 46, 47 (the rest can be seen in (1)) showing a systematic deviation, this time upwards instead of downwards: The "stretching" of the scale was excessive. Therefore the use of this function is not advisable.

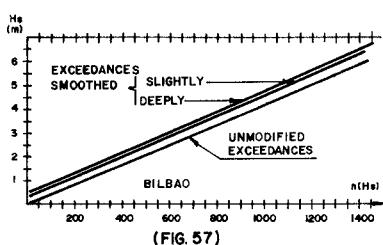
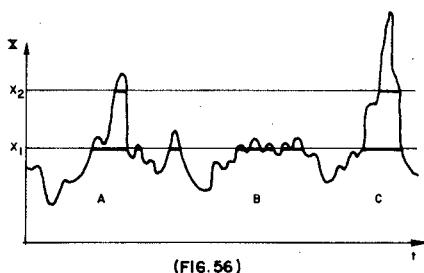
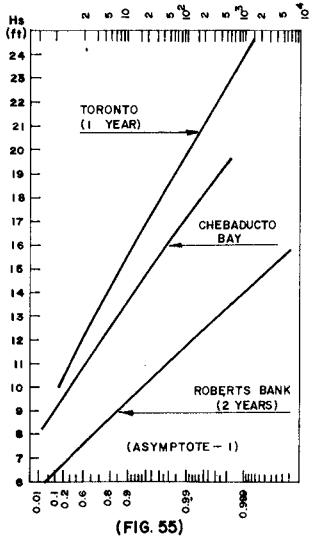
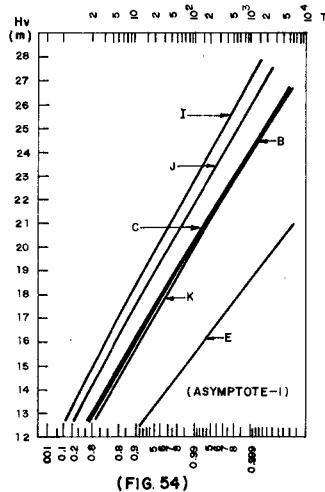
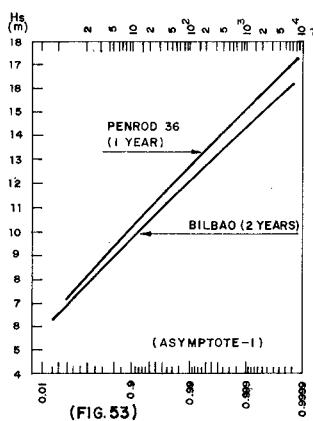
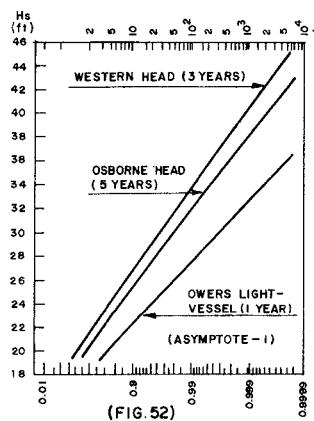
CONCLUSIONS.- None of the 4 functions tried is adequate for the whole extent of the distributions in all the stations studied. The Exponential function has been found to behave satisfactorily in the upper part of the distributions, and thus apt for extrapolations. In the lower part its fits are good when swell is negligible, but as the importance of swell reaches higher levels of H_s so grows a lower region with a systematic deviation. Log-Normal function behaves reciprocally: its fits are good in the lower region, reaching higher levels of H_s as the importance of swell also reaches higher H_s values. In the upper region, the fits show a systematic deviation. This result suggests that the statistical heterogeneity found in each station corresponds with different physical properties of swell -- and sea, each of which is predominant within a certain range of H_s values (with a region of overlapping). The Exponential function could then describe statistically the growth of the sea caused by wind fields reaching the observation site, originated by differential pressure centers (typically low pressure centers). The highest waves are almost everywhere formed in this way; thus the adequacy of the Exponential law for extrapolations. On the other hand, the Log-Normal law seems to fit correctly the region where waves are a mixture, in space and time, of a good proportion of swell, low waves formed by the local microclimate, and sea that is being generated by larger wind fields. It is curious to notice that -- the Log-Normal distribution is theoretically correct for natural variates formed by a high number of random factors which join their individual effects in a multiplicative way (V. - Chow, 1955). In (1) further speculations along the same line are indicated to reason this hypothesis, which is able to explain why the deviation of the observed points in the lower tail of the Exponential fits takes place in the form of a quite sudden drop. Summing up, at every single station a discrimination between different wave "climates" can be made, according to the different relative importance of swell along the H_s levels. This discrimination has proved effective in selecting distribution functions with general applicability.

EXTREMAL DISTRIBUTION $\Phi(H_s)$

From the results obtained before for $n(H_s)$ and $F(H_s)$, the following expression is reached for $\Phi(H_s)$:

$$\text{Convergence: } \Phi(H_s) = (F(H_s))^{\frac{n(H_s)}{n(H_s) - 1}} \xrightarrow{F(H_s) \approx 1} e^{-(1-F(H_s))n(H_s)} ; \quad F(H_s) = 1 - e^{-\frac{H_s - k_1}{k_2}} ; \\ n(H_s) = k_3(H_s - k_4) ; \quad \Phi(H_s) \xrightarrow{n(H_s) \rightarrow \infty} e^{-(H_s - C)e^{-\left(\frac{H_s - A}{B}\right)}} .$$

It is a 3-parameters distribution function. Should $n(H_s)$ have an exponential form, the expression for $\Phi(H_s)$ would be the double exponential or Asymptote-I. With a linear growth for $n(H_s)$, $\Phi(H_s)$ has a quicker growth than the Asymptote-I has. In Figs. 52, 53, 54, - 55, the extremal distributions computed for the 14 wave observation stations listed in Table 1 have been plotted in Asymptote-I paper. From the curvature of the distributions it turns out that the use of Asymptote-I for fitting extremal samples of H_s consistently overestimates the real values in the extrapolations. However the overestimations are small,



and acceptable for most applications, within the range of return periods usual in practice. Instead, the Asymptote-II would cause large (unacceptable) overestimates.

The good performance showed by the linear and exponential functions with the sets of data analyzed in the two previous sections makes one confident that this simple laws will yield sufficiently accurate results for the extremal analysis of H_s . In some meaningful cases, a good correspondence between both functions has been noticed^s. In station Bilbao, for instance, where exceptionally rough winters took place within the observation period, the sharp deviation of the sample points above the exponential fit for $F(H_s)$ (Fig. 33) starts almost exactly at the same point where the (also sharp) deviation appears above the linear fit for $n(H_s)$ (Fig. 15). This kind of correspondence is to be expected only in case of very exceptional winter seasons (abnormally rough or mild), since not only the durations but also the number of exceedances play a role in $F(H_s)$.

Some aspects of the use of the extremal equation for significant wave height predictions will be commented in the following sections.

5.- STATISTICAL INDEPENDENCE

One of the basic hypothesis on which the extremal model that is being used here was built, is the independence between "statistical trials". This means randomness in the presentation of exceedances. The practical adequacy of this hypothesis must be inquired.

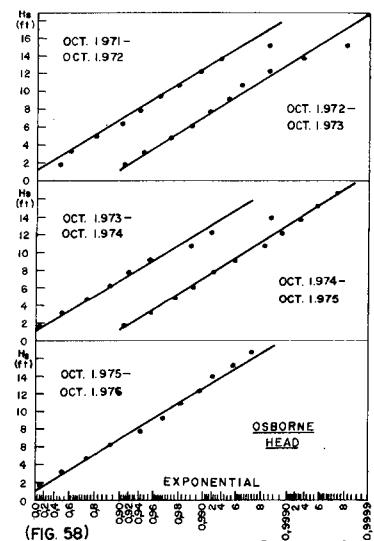
When the crest of an exceedance is long, it usually follows a sinuous pattern rather than a smooth curve (Fig. 56, case B). These secondary peaks are obviously inter-dependent. Should this feature be dominant along the entire range of values of the variate, the use of the model would become rather difficult. Fortunately the crests of the exceedances which reach high levels of the variate are typically pointed, leaving little room for secondary oscillations. In order to check in a real case the quantitative influence of this effect, one year of H_s observations in Bilbao (April 1976-April 1977) has been used to compute 3 different estimates of $n(H_s)$: a) using the entire H_s -curve unmodified (analyzed by computer); b) using the H_s -curve where secondary peaks have been suppressed by a slight -- smoothing (hand made) of the exceedances; c) deep smoothing of the exceedances. The result (Fig. 57) are three straight lines which run quite parallel. Thus the relative difference between the three estimates steadily diminishes with increasing levels of the variate. At H_s levels relevant for extremal predictions, the predictions calculated with any of these $n(H_s)$ estimates are practically identical.

The distinction between these secondary peaks and proper exceedance courses is not always neat. Sometimes (Fig. 56, case A) crests with good sizes lie close to each other in a way that suggests some kind of inter-dependence. In this respect, it can be remembered - that P.Rijkort and J. Hemelrijck (1957) found proof of statistical dependence between storms in the North Sea. Again it can be argued that this dependence loses importance with increasing levels of the variate. At medium and long return periods the exceedances appear typically as isolated, well-spaced peaks. This seems to be the general behaviour of geophysical variates. In (1) some real records are shown which illustrate his statement, and two comparisons between extremal predictions and extremal samples for the variates average wind speed and total rainfall in an interval show good agreement even for low return periods. It is - unfortunate that accurate (instrumental) extremal samples of H_s are not available with enough length to make similar checks, but in principle there are no reasons to assume a too - different behaviour for this variate.

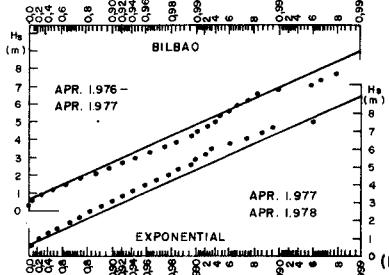
6.- HYPERANNUAL CYCLICITY

The extremal distribution has been stated here on a yearly basis (probability of - not-exceedance in the average year). The use of this probability in terms of return periods implies the assumption of randomness in the intensity of the variate in different years. - The adequacy of this hypothesis should be questioned.

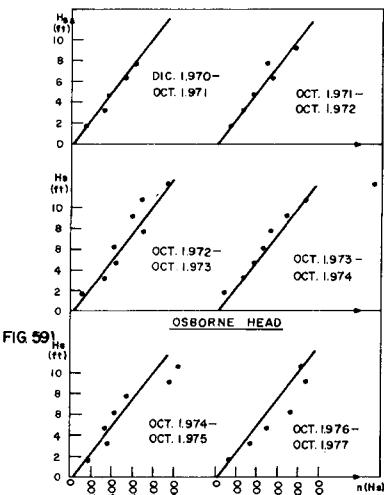
Numerous authors have found significant evidences of some kind of periodicity (more properly called "pulsations") in several geophysical variates. These pulsations are generally thought of as being connected with the 11-years cycles found in the solar activity. Pulsations of about of 11(or 22) years in the maxima of the variate should not appreciably influence the practical use of predictions made on an average-year basis, since usual design return periods have an order of magnitude of hundreds of years. However, it can still be - questioned whether the estimates of $n(x)$ and $F(x)$ (with which the extremal prediction is - calculated) are random from year to year. Should they be subject to pulsations, a minimum



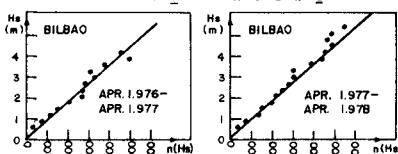
(FIG. 58)



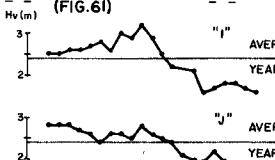
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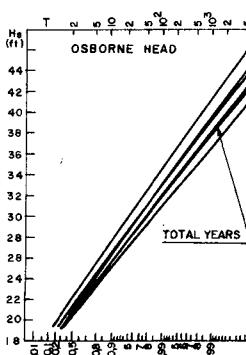
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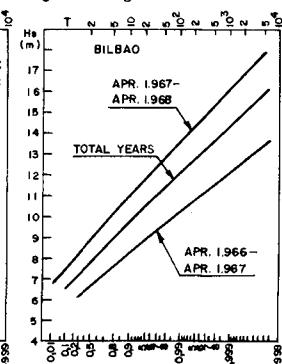
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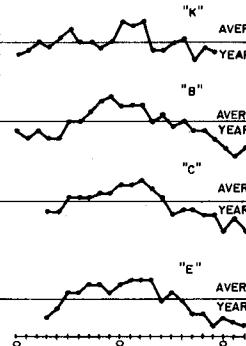
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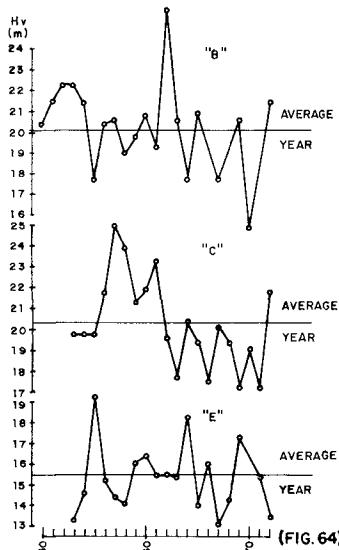
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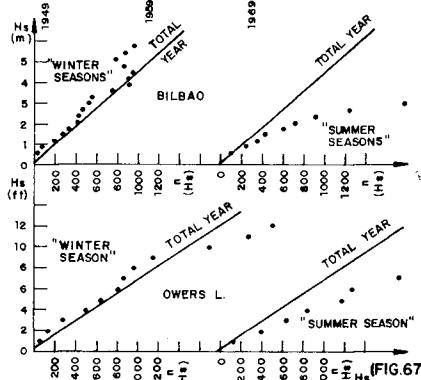
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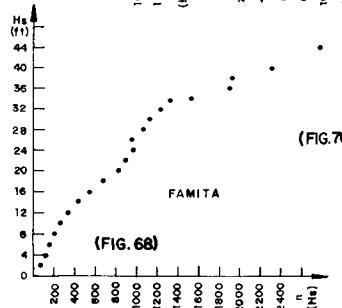
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(FIG. 64)

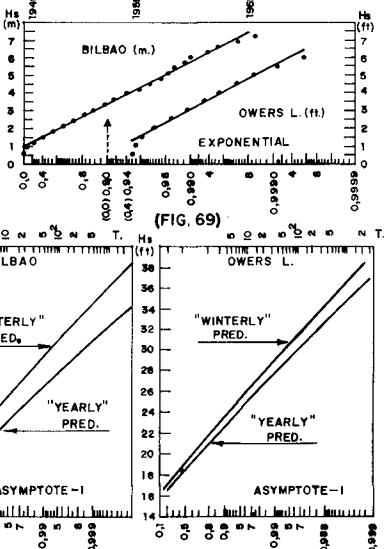


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observation period of those 11 or 22 years would be needed for a proper characterization of the "average year". Long duration instrumental records are still missing, but the visual observations of wave height in the Weather Ships (more than 20 years) will be used to get some information. In Fig. 63, the wave height corresponding to probability $F(H_s) = 0.5$ in the distribution observed each year (October–October) are plotted for six Weather Ships. Clear grouping of higher and lower heights appears in most of them, with semi-periods curiously near to the 11 years above mentioned. Extreme distributions were computed for the six W.S. with each yearly estimates of $F(H_s)$, $n(H_s)$ (1). In Figs. 64, 65, the points corresponding to $T=100$ years in each prediction are plotted, showing that in some Ships there is no trace of grouping and where grouping may seem to be detected it does not keep in correspondence with the groupings observed in Fig. 63. Although the quantitative value of these estimates does not seem to be high (next section), the qualitative pattern obtained suggests that cycl city does not have a significant influence in the extremal predictions of H_s .

7.- SUFFICIENT ESTIMATES OF $n(H_s)$ AND $F(H_s)$

Even assuming that the hypothesis of annual randomness is reasonable in practice, still remains the problem of determining which is the minimum observation time necessary to obtain acceptably accurate estimates of the extremal distribution, i.e. how many years are enough to get a good estimate of $n(H_s)$. Recent works assume that one single year of observation is enough to yield the average year in terms of $F(H_s)$. However this hypothesis has never been sufficiently checked, and the main reason to keep it is probably the tight time limits which are customary in actual projects. In Figs. 58, 60, the observed distributions for each year in Osborne Head (5 years) and Bilbao (2 years) are compared with the distributions fitted to the complete sets of observations. In Figs. 59 and 61 the same comparison is carried out with the parameter $n(H_s)$. In Fig. 62 the extremal predictions computed from each 1-year estimates are plotted. For $T=500$ years (10% risk of exceedance in 50 years), the difference between the higher and lower predictions is 1.3 m. at Osborne Head ($\pm 5.7\%$ of the intermediate value), and 3.7 m. at Bilbao ($\pm 13.6\%$). Although the number of years worked out is too small for stating general conclusions, the results might be indicative of two climates with different degrees of homogeneity. Anyway the dispersion of results obtained for Bilbao represents a heavy influence on the design of maritime structures and shows the need for wider comparative studies of this kind in various ocean areas.

The comparisons made for Bilbao and Osborne Head show reasonable agreement between the yearly estimates of $n(H_s)$. The differences obtained in the extremal predictions are almost exclusively due to the different yearly estimates of $F(H_s)$. Furthermore, it can be easily showed (1) that $n(H_s)$ is far more sensitive to variations of $F(x)$ than of $n(H_s)$.

Figs. 64, 65 show a large variability of the yearly estimates of $n(H_s)$, much in excess than what was observed in Fig. 62. Moreover the yearly estimates of $n(H_s)$ (showed in (1)) also show a high dispersion, in contrast with the behaviour of Figs. 59, 61. This suggests a deficiency in the visual observations of wave height. It seems that a higher number of $n(H_s)$ values than a year includes is necessary to get an acceptable approximation of $n(H_s)$, and that a higher number of observations of H_s is needed than what is usually performed in a year in order to get an acceptable estimate of $F(H_s)$, in that year (aside from the variability of both parameters from year to year). It can be concluded that 1-year of visual observations do not suffice to obtain useful extremal predictions of wave height.

With the aim of reducing as much as possible the time of observation, some published works use only a "winter season" (often the roughest 6 months of the year) of measurements in order to estimate the upper tail of $F(H_s)$. In Figs. 66, 67, 69, estimates of $F(H_s)$ and $n(H_s)$ in the "winter season" (October–March) and "summer season" (April–September) during one year in Owers Lightvessel and 2 years in Bilbao, are compared with the estimates obtained with complete years. Both stations were selected for the completeness of their observations. The resulting extremal predictions are compared in Fig. 70: The "winterly" predictions are higher (for $T=500=1$ m. at Bilbao and 0.5 m. at Owers L.). This is a natural consequence of having used the same distribution (Exponential) for the "winterly" and "yearly" estimates: since not all the H_s values of the "winter season" are higher than the values of the "summer season", the slope of the exponential line for the winterly estimate is steeper than for the whole year estimate. "Winterly" predictions are bound to stay systematically higher than yearly ones, unless distribution functions are used which can be made to converge in their upper tails (different functions, or maybe variable-parameters functions). This possibility is above our present knowledge of the behaviour of geophysical variables. It must be indicated that the use of more flexible functions may worsen the situation: when the Weibull distribution was used instead of the Exponential, the difference between predictions was 2.0 m. for Owers Lightvessel.

The different $n(H_s)$ estimates did not appreciably account for the variability in extremal predictions. However, the "winterly" points form a peculiar double arch above (higher durations) the complete-year line. The same pattern can be seen in Fig. 68 corresponding to 6 "winterly seasons" recorded by rescue ship "Panita" (North Sea). These points do not show the linear trend which was clear in the complete-year estimates.

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