CHAPTER 2

ON LONG-TERM STATISTICS FOR OCEAN AND COASTAL WAVES

Michel K. Ochi*

ABSTRACT

This paper discusses the statistical properties of long-term ocean and coastal waves derived from analysis of available data. It was found from the results of the analysis that the statistical properties of wave height and period obey the bi-variate log-normal probability law. The method to determine the confidence domain for a specified confidence coefficient is presented so that reliable information in severe seas where data are always sparse can be obtained from a contingency table. Estimation of the extreme significant wave height expected in the long-term is also discussed.

INTRODUCTION

For the design of ocean and coastal structures, it is very important to obtain information on wave characteristics over a period of time sufficiently long to cover the lifetime of the structures. Collection of data on wave height and period has been made by several researchers through either visual observations or measurements, and their results are usually presented in tabular form [1][2][3][4][5]. Although considerable attention has been given to statistical information of long-term wave height, we have as yet little information on the combined properties of wave height and period for the long-term.

A statistical contingency table on wave height and period provides valuable information for the design of ocean and coastal structures. However, data for severe seas, which are indeed necessary for design, are unreliable since such data are, without exception, sparse. One way to solve this problem is to represent wave statistical data by a certain probability law which governs the data, and then obtain necessary information for design from the probability function.

The purpose of this paper is to provide solutions to the problems cited in the foregoing discussion. For this, the joint probability

*David W. Taylor Naval Ship Research and Development Center, Washington, D.C., U.S.A.

distribution of wave height and period for data accumulated over many years is derived, and a method to establish the confidence domain for a specified confidence coefficient is presented. A method for predicting the extreme value of significant wave height from the long-term accumulation of data is also discussed.

LONG-TERM WAVE STATISTICS

Long-term wave statistics as defined in this study is the statistical information of wave characteristics such as wave height and period accumulated over a sufficiently long period of time. In accumulating data for long-term wave statistics, wave observations are usually made for 20-30 minute intervals. Wave height and period are expressed in terms of significant wave height (or average wave height) and zero-crossing period (or average wave period), respectively, and the accumulated data are usually presented in tabular form known as a contingency table.

Examples of a contingency table for long-term wave statistics are shown in Tables 1 through 3. Table 1, taken from Reference [1], shows the tabulation of significant wave height and zero-crossing period, both analyzed from records obtained at Weather Station I (59°N, 19°W) in the North Atlantic. The numbers given in the contingency table are those per 1,000 observations. Table 2 shows an example of data on significant wave height and period observed during 20 months (from November 1961 to March 1964) at Port Hueneme, California, presented in Reference [2]. Table 3, taken from Reference [3], shows an example of a contingency table on significant wave height and modal period where the wave spectrum has a peak value. The table provides information based on a total of 2,304 measurements in one year (from March 1972 to February 1973) at Tiner Point, New Brunswick, Canada. There are many other examples of contingency tables similar to those shown in these tables, Reference [4] and [5] for example.

Needless to say all of these long-term wave statistics are extremely valuable in providing information for the design of ocean and coastal structures. However, a problem always exists in the use of this information for design. That is, data in severe seas are unreliable, since they are, without exception, sparse. For instance, information above 10 ft (3.05 m) significant wave height in the example shown in Table 3 are few; only 15 cases are observed in a total of 2,304 observations which is equivalent to about 0.7 percent. This implies that it is not appropriate to use the numbers given in this sort of table in practice.

One way to solve this problem is to apply the statistical inference concept and establish the confidence domains from the data, taking into account the correlation between significant wave height and period. For this, it is necessary to find the joint probability distribution which is applicable to significant wave height and period.

PROBABILITY DISTRIBUTION FOR LONG-TERM WAVE STATISTICS

In order to derive the combined statistical characteristics of the two random variables, significant wave height, H_s, and zero-crossing period, T_o, we may first consider the marginal probability distribution of significant wave height. It is noted here that, although considerable attention has been given to statistical information of significant wave height, the probability function which is applicable to significant wave height is the focus of much criticism. Some claim that the log-normal distribution is appropriate [6][7][8], while others believe the data can be better fitted by the Weibull distribution [9][10]. The results of the present analysis, however, illustrate that significant wave height appears to follow the log-normal probability law over the range for the cumulative distribution up to 0.99. This conclusion appears to be valid for both ocean and coastal waves. To support this statement, the following discussion will be given:

Figure 1 shows the cumulative distribution function of significant wave height of ocean waves. The data are taken from the contingency table given in Table 1, and are plotted on log-normal probability paper. As can be seen in the figure, the data follow the log-normal distribution for the cumulative distribution up to 0.99. On the other hand, Figure 2 shows the same data plotted on Weibull probability paper. This figure shows that the data may also be represented by the Weibull probability distribution except for small significant wave height. Thus, one may have the impression that there is no significant difference in representing the statistical characteristics of significant wave height by the log-normal or Weibull probability law. However, if comparison is made between these two probability density functions and histograms, then the difference becomes pronounced.

Figure 3 shows a comparison between the histogram, log-normal and Weibull probability density functions. As can be seen in the figure, the log-normal probability density function agrees reasonably well with the histogram over the entire range of significant wave heights. On the other hand, the Weibull probability density function agrees well with the histogram for large significant wave height, but the agreement is rather poor for small significant wave height. This is the general trend observed in the analysis of the significant wave height data.

Examples of statistical analysis of significant wave height for coastal waves are shown in Figures 4 through 7. The cumulative distribution function of the significant wave height measured at Tiner Point, Canada [3] is plotted on log-normal probability paper (Figure 4) as well as on Weibull probability paper (Figure 5). On the other hand, comparison between the two probability density functions and the histogram is shown in Figure 6. As can be seen in these figures, the same conclusion derived from analysis of the significant wave height for ocean waves may also be applicable to the significant wave height for coastal waves.

As another example for coastal waves, Figure 7 shows a comparison between the histogram and the log-normal distribution of the significant wave height obtained at Port Hueneme, California [2]. Thus, it may safely be concluded that the log-normal probability distribution represents reasonably well the histogram over the entire range of significant wave height for ocean as well as coastal waves.

In regard to the marginal probability distribution of zero-corssing wave period (or modal wave period), it was found through the present study that the wave period appears to follow the log-normal probability law for both ocean and coastal waves. Although an attempt was made to represent the wave period data by the Weibull probability distribution, the representation was extremely poor for all examples. Figures 8 through 10 show examples of a comparison between histograms and the log-normal probability density functions. The figures pertain to comparisons made for data observed at Station I in the North Atlantic (Figure 8), at Tiner Point, Canada (Figure 9), and at Port Hueneme, California (Figure 10), respectively. As can be seen in these comparisons, wave period appears to be represented by the log-normal probability law.

From the foregoing discussion, it has been concluded that significant wave height follows the probability distribution given by,

$$f(H_s) \sim \Lambda(\mu_{HS}, \sigma_{HS}) \tag{1}$$

where, $\Lambda(\mu_{HS}, \sigma_{HS})$ is the log-normal probability density function with parameters μ_{HS} and σ_{HS}. Similarly, the zero-crossing wave period follows the probability distribution given by,

$$f(T_o) \sim \Lambda(\mu_{T0}, \sigma_{T0}) \tag{2}$$

The modal period also obeys the same distribution law as given in Equation (2) but with different parameters.

Then, from the properties of the log-normal probability distribution, it can be derived that the combined statistical properties of significant wave height and period follow the bi-variate log-normal probability law which may be written as,

$$f(H_s, T_o) \sim \Lambda(\mu_{HS}, \sigma_{HS}, \mu_{T0}, \sigma_{T0}, \rho) \tag{3}$$

where, ρ is a correlation coefficient between two random variables, H_s and T_o, and its value can be determined from the data.

Various statistical properties of significant wave height and period can be derived based on Equation (3). First, it is possible from Equation (3) to evaluate the confidence domain for a specified confidence coefficient which will provide reliable information for severe seas where data are always sparse in the contingency tables. Also, the statistical properties of the zero-crossing period (or modal period) for a specified significant wave height can be obtained as the conditional log-normal probability distribution given by,

$$f(T_s|H_s) \sim \Lambda\left(\mu_{TO} + \rho \frac{\sigma_{TO}}{\sigma_{HS}} (\ell n\ H_s - \mu_{HS}),\ \sqrt{1-\rho^2}\ \sigma_{TO}\right) \qquad (4)$$

DERIVATION OF CONFIDENCE DOMAIN

Derivation of the confidence domain for a specified confidence coefficient may be made around the modal value which represents that combination of significant wave height and zero-crossing (or modal) period most likely to occur. For the log-normal distributions given in Equations (1) and (2), the modal value (H_*, T_*) is given by,

$$(H_*, T_*) \sim (e^{\mu_H - \sigma_H^2}, e^{\mu_T - \sigma_T^2}) \qquad (5)$$

Next, let us transform the joint probability density function given by Equation (3) into the following two new random variables, r and θ, shown in Figure 11.

$$\begin{cases} H_s = H_* + r \operatorname{Cos} \theta \\ T_o = T_* + r \operatorname{Sin} \theta \end{cases} \qquad (6)$$

By carrying out the transformation of random variables given in Equation (6), the joint probability density function of r and θ, $f(r,\theta)$ becomes,

$$f(r,\theta) = \frac{r}{\sigma_1 \sigma_2 (2\pi) \sqrt{1-\rho^2}\ (H_o + r\cos\theta)(T_o + r\sin\theta)}$$

$$\times\ \exp\left\{ -\frac{1}{2(1-\rho^2)} \left[\left(\frac{\ell n(H_o + r\cos\theta) - \mu_H}{\sigma_H} \right)^2 - 2\rho \left(\frac{\ell n(H_o + r\cos\theta) - \mu_H}{\sigma_H} \right) \right. \right.$$

$$\times\left(\frac{\ell n(T_o + r\sin\theta) - \mu_T}{\sigma_T}\right) + \left(\frac{\ell n(T_o + r\sin\theta) - \mu_T}{\sigma_T}\right)^2\Bigg]\Bigg\} \tag{7}$$

In order to determine the confidence domain around the modal value (H_*, T_*), consider the conditional probability density function of r for a given θ. That is,

$$f(r|\theta) = \frac{f(r,\theta)}{\int_0^\infty f(r,\theta)\,dr} \tag{8}$$

Then, a point r_i which yields a contour curve whose enclosed area is equal to a specified confidence coefficient, γ, can be determined from the following relationship:

$$\int_0^{r_i} f(r|\theta)\,dr = \gamma \tag{9}$$

Figure 12 shows an example of a thus derived confidence domain using Draper's data obtained in the North Atlantic [1]. The black circle in the figure represents the combination of significant wave height and zero-crossing period which is most likely to occur. The closed curves given in the figure outline the statistical confidence domains inside of which the probabilities of occurrence of the wave conditions are the specified values. The numbers in the figure refer to the number of observations given in Table 1. These numbers, however, for convenience sake, are regrouped and are divided by 1,000 so that the total number is equal to 1.

Figures 13 and 14 show two examples of such confidence domains derived using data on coastal waves. Confidence domains using the data obtained at Tiner Point, Canada, are given in Figure 13, while those using the data obtained at Port Hueneme, California, are given in Figure 14. The numbers in these figures refer to the frequencies of observations given in Tables 2 and 3, respectively. As can be seen in Figures 12 through 14, the domain for a confidence coefficient of 0.99 sufficiently covers the measured data. The significant benefit of drawing the confidence domains is that information in severe seas where data are always sparse can be clearly estimated from the overall data given in the contingency tables.

ESTIMATION OF EXTREME SIGNIFICANT WAVE HEIGHT

One of the most important pieces of information concerning long-term wave statistics is the largest value (extreme value) of significant wave height. The information is of particular importance for the design of ocean and coastal structures.

Although the significant wave height follows the log-normal probability law, this holds only for the cumulative probability distribution up to 0.99 as was discussed in the previous section. Therefore, it is not appropriate to estimate the extreme significant wave height based on the log-normal distribution, since the value of the cumulative probability distribution will be greater than 0.99 for the extreme value. Instead, it may be evaluated by applying the concept of asymptotic distribution of extreme values which is applicable for any probability distribution if certain conditions are met. For this purpose, let us assume that the cumulative distribution function can be expressed asymptotically in the following form:

$$F(x) = 1 - e^{-q(x)} \tag{10}$$

Then, it can be derived that the probable extreme value in n number of observations, denoted by $\overline{Y}_n$, satisfies the following condition when n is large:

$$e^{-q(\overline{Y}_n)} = \frac{1}{n} \tag{11}$$

This in turn, implies that the probable extreme value can be evaluated from,

$$\frac{1}{1 - F(\overline{Y}_n)} = n \tag{12}$$

As an example, Figure 15 shows the left-hand side of Equation (12) (often called the return period) in logarithmic form using the significant wave height data obtained at Tiner Point, Canada. Since a total of 2,300 observations of significant wave height were made in one year, the probable extreme significant wave height expected to occur in 20 years, for example, can be estimated from the figure as the wave height for which the ordinate is $\ell n(2,300 \times 20) = 10.74$; namely, 18.0 ft (5.5 m).

Figure 16 shows the results of a similar analysis to that shown in Figure 15 but using data obtained at Port Hueneme, California. A total of 3,400 observations of significant wave height were made in 20 months for this example; hence the probable extreme significant wave height expected to occur in 20 years can be estimated from the figure as the wave height for which the ordinate is 10.62; namely, 9.7 ft (2.96 m).

CONCLUSIONS

This paper discusses statistical properties of the long-term ocean and coastal waves derived from analysis of available data. In the analysis, correlation between wave height and period is taken into consideration, and data on ocean waves accumulated in the North Atlantic as well as data on coastal waves obtained at Tiner Point, Canada, and Port Hueneme, California, are analyzed.

It is found from the results of analysis that statistical properties of both wave height and period are approximately represented by the log-normal probability law, and that the joint probability of wave height and period obeys the bi-variate log-normal probability law. This conclusion appears to be valid for both ocean and coastal waves.

The confidence domain of wave height and period for a confidence coefficient of 0.99 sufficiently covers the measured data. A significant benefit of drawing the confidence domains lies in obtaining information in severe seas where the data are always sparse in the contingency table.

The estimation of the largest significant wave height (extreme value) expected to occur in a specified long period of time is discussed by applying the concept of asymptotic statistical distribution of extreme values.

REFERENCES

1. Draper, L. and Squire, E.M., "Waves at Ocean Weather Ship Station INDIA," Trans. Roy. Inst. Nav. Arch., Vol 109, 1967.

2. Thompson, E.F., "Wave Climate at Selected Locations Along U.S. Coasts," Coastal Eng. Res. Center, Tech. Rep. 77-1, 1977.

3. Khanna, J. and Andru, P., "Lifetime Wave Height Curve for Saint John Deep Canada," Proc. Symp. on Ocean Wave Measure. and Analysis, Vol 1, 1974.

4. Hogben, N. and Lumb, F.E., Ocean Wave Statistics, National Physical Laboratory, H.M. Stationery Office, London, 1967.

5. Walden, H., "Die Eigenschaften der Meerswellen in Nordatlantischen Ozean," Deutscher Wetterdienst, Einzelveröffentlichungen, Nr 41, 1964 (in German).

6. Jasper, N.H., "Statistical Distribution Patterns of Ocean Waves and of Wave-Induced Ship Stresses and Motions, with Engineering Applications," Trans. Soc. Nav. Arch. & Mar. Eng., Vol 64, 1956.

7. Davidan, I.N. et al., "The Results of Experimental Studies on the Probabilistic Characteristics of Wind Waves," Proc. Dynamics of Marine Vehicles and Structures in Waves, 1974.

8. Ward, E.G., Evans, D.J., and Pompa, J.A., "Extreme Wave Heights Along the Atlantic Coast of the United States," Proc. Offshore Tech. Conference, OTC 2846, 1977.

9. Houmb, O.G., and Overvik, T., "Parameterization of Wave Spectra and Long Term Joint Distribution of Wave Height and Period," Proc. Behavior of Offshore Structures, 1976.

10. Nordenstrom, N., "Method for Predicting Long Term Distribution of Wave Loads and Probability of Failure for Ships," Appendix I, Long-Term Distribution of Wave Height and Period, Det Norske Veritas Report No. 69-21-S, 1969.

Table 1: Statistical data on wave height and period obtained at Weather Station I in the North Atlantic [1]

ZERO-CROSSING WAVE PERIOD (SEC.) / SIGNIFICANT WAVE HEIGHT

Significant wave height	6.0–6.5	6.5–7.0	7.0–7.5	7.5–8.0	8.0–8.5	8.5–9.0	9.0–9.5	9.5–10.0	10.0–10.5	10.5–11.0	11.0–11.5	11.5–12.0	12.0–12.5	12.5–13.0	13.0–13.5	13.5–14.0	Total
46–48 FT (14.0–14.6 M)																1	1
44–46 (13.4–14.0)																	0
42–44 (12.8–13.4)																	0
40–42 (12.2–12.8)															1		1
38–40 (11.6–12.2)												1	1				2
36–38 (11.0–11.6)										1			1				2
34–36 (10.4–11.0)										1	1	1	1	1		1	6
32–34 (9.8–10.4)											2	1	1				4
30–32 (9.2–9.8)									1	2		1			1		5
28–30 (8.5–9.2)								1	1	1	2	2	2	1		1	11
26–28 (7.9–8.5)									1		4	1	1	1			8
24–26 (7.3–7.9)								1	2	4	7	1	1			1	17
22–24 (6.7–7.3)				1			1	3	4	3	3	1		2			18
20–22 (6.1–6.7)					1		1	1	5	5	4	1	1				19
18–20 (5.5–6.1)					1	2	6	5	9	3	6	1	2	1			36
16–18 (4.9–5.5)			1		2	4	14	7	10	6	4	1	3	1	1	1	55
14–16 (4.3–4.9)				1	4	4	13	9	10	10	8	4	3	1		1	68
12–14 (3.7–4.3)				2	8	8	19	14	15	9	5	4	2	1			87
10–12 (3.1–3.7)		2	2	4	13	18	24	22	16	8	5	3	1				118
8–10 (2.4–3.1)	1	1	2	12	21	26	39	24	12	8	3	1	1				151
6–8 (1.8–2.4)		3	5	15	39	23	42	20	13	5	2						167
4–6 (1.2–1.8)		4	7	24	39	24	24	12	6	1	1						142
2–4 (0.6–1.2)	3	4	3	12	17	16	12	5	1								73
0–2 (0–0.6)		1			2		1	2		1		1	1				9
Total	4	5	20	71	147	125	196	126	106	68	57	25	22	9	3	6	1000

Table 2: Statistical data on wave height and period obtained at Port Hueneme, California [2]

3454 OBSERVATIONS SUMMARY FOR 28 MONTHS NOV 61 THROUGH MAR 64

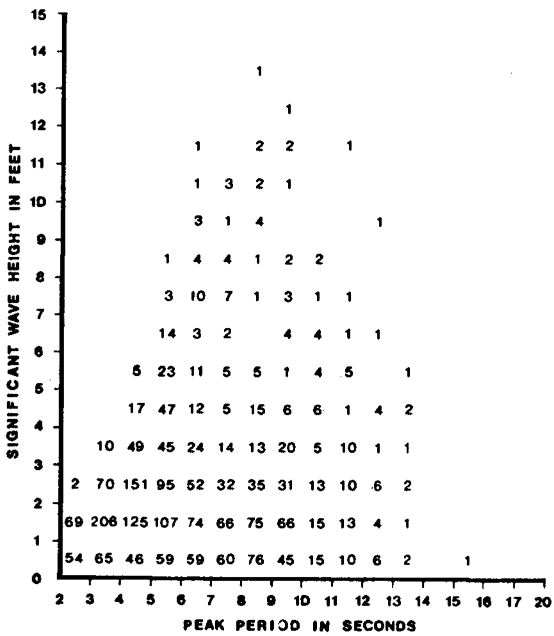
PERIOD (SECS) HEIGHT (FT)

PERIOD (SECS)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	TOT.	CUM. TOT.	ROW AVG.
.0 - 1.9	05														1000	.00
2.0 - 2.4															1000	.00
2.5 - 2.9															1000	.00
3.0 - 3.4	5	3												8	1000	1.11
3.5 - 3.9	3	7												10	992	1.30
4.0 - 4.9	3	12	8	2										24	982	1.69
5.0 - 5.9	8	20	17	10	2	2			1					69	930	1.97
6.0 - 6.9	7	12	12	10	2	2	1	1						78	892	2.00
7.0 - 7.9	8	10	10	7	3	1	1	1	1					88	822	1.97
8.0 - 8.9	8	29	21	20	7	5	1	1	1					95	754	2.29
9.0 - 9.9	8	37	31	23	8	4	2	2						121	654	2.25
10.0 -10.9	9	48	32	21	13	8	5	1	1		1			139	318	2.39
11.0 -11.9	12	39	20	22	9	7	3	1	1		1			124	394	2.29
12.0 -12.9	9	37	26	20	8	9	5	1	1	1				115	275	2.20
13.0 -13.9	7	22	17	12	5	3	3	1						70	180	2.30
14.0 -14.9	4	21	13	12	5	5	2	1						67	65	2.31
15.0 -15.9		3	1	1										5	18	1.95
16.0 -16.9	1	8	8	1	2	1	1							18	13	0.39
17.0 -17.9		1												1	2	1.50
18.0 -18.9	1													1	1	1.30
TOTAL	133	330	221	182	83	44	28	18	8	2	2		1			2.10
CUM. TOTAL	1000	867	529	307	125	62	18	10	8	8	3	1	1			
COL. AVG.	9.504	9.70	10.08	10.33	10.87	10.83	11.39	10.90	9.21	12.00	12.00	10.50	18.90	15.00		

AVERAGE SIG. HEIGHT = 2.13 FT
VARIANCE OF SIG. HEIGHT = 1.06 FT SQ
STANDARD DEVIATION OF HEIGHT = 1.03 FT

AVERAGE WAVE PERIOD = 10.08 SECS
VARIANCE OF WAVE PERIOD = 8.40 SEC SQ
STANDARD DEVIATION OF PERIOD = 2.90 SECS

Table 3: Statistical data on wave height and period obtained at Tiner Point, Canada [3]



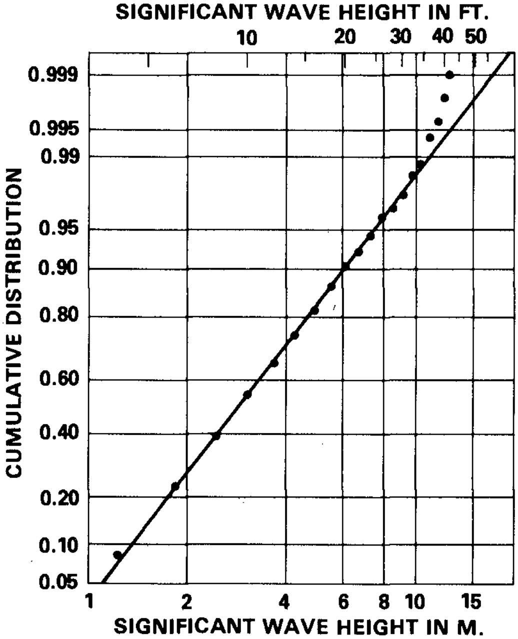


Figure 1: Cumulative distribution function of significant wave height plotted on log-normal probability paper (Data from Reference 1)

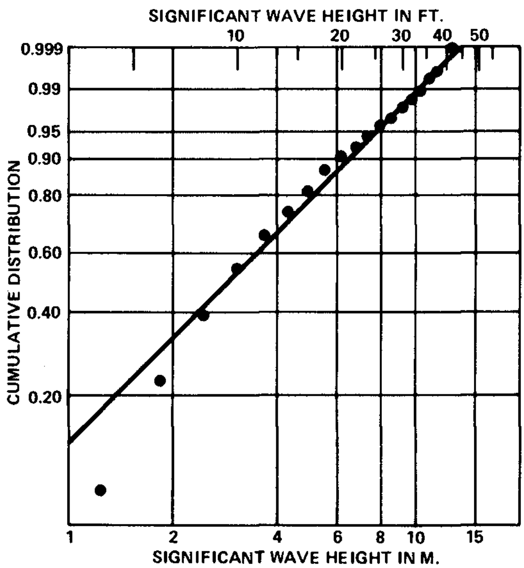


Figure 2: Cumulative distribution function of significant wave height plotted on Weibull probability paper (Data from Reference 1)

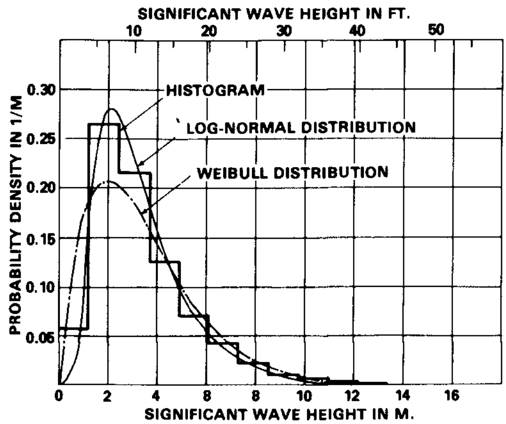


Figure 3: Comparison between histograms of significant wave height and log-normal and Weibull probability density functions (Data from Reference 1)

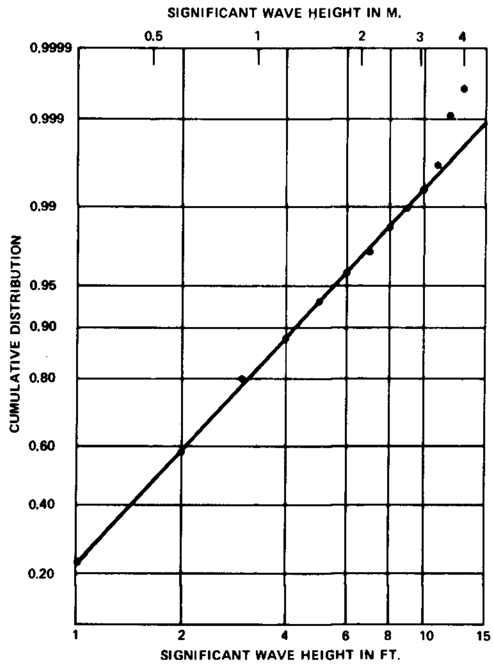


Figure 4: Cumulative distribution function of significant wave height plotted on log-normal probability paper (Data from Reference 3)

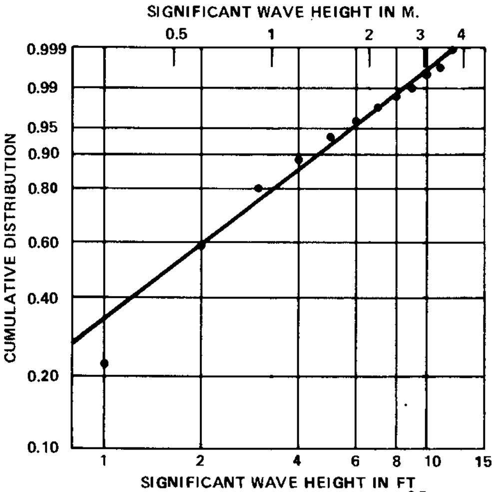


Figure 5: Cumulative distribution function of significant wave height plotted on Weibull probability paper (Data from Reference 3)

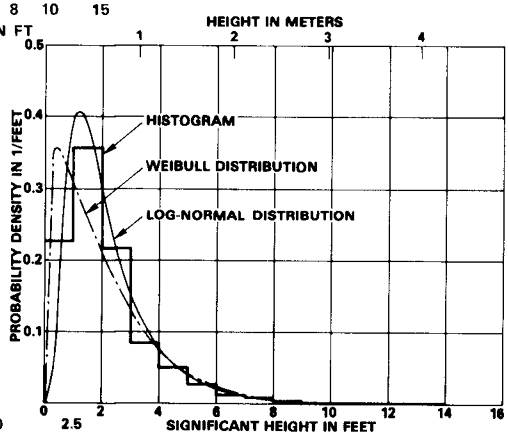


Figure 6:
Comparison between histogram of significant wave height and log-normal and Weibull probability density functions (Data from Reference 3)

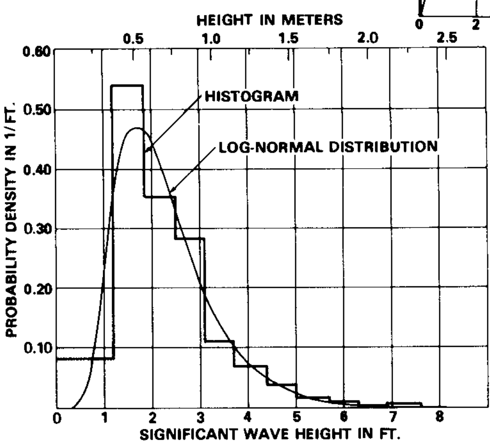


Figure 7:
Comparison between histogram of significant wave height and log-normal probability density function (Data from Reference 2)

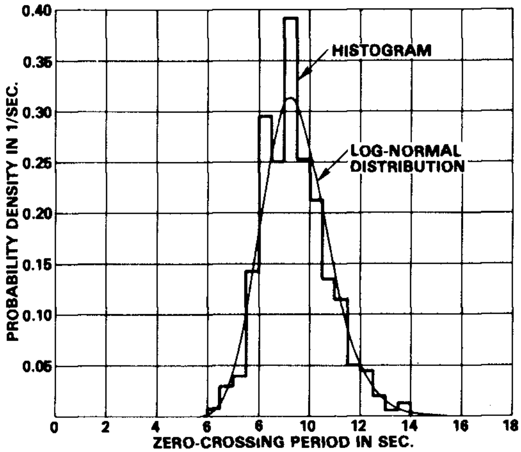
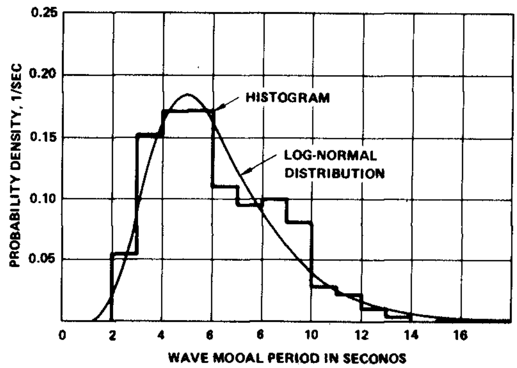


Figure 8: Comparison between histogram of zero-crossing period and log-normal probability density function (Data from Reference 1)

Figure 9: Comparison between histogram of modal period and log-normal probability density function (Data from Reference 3)



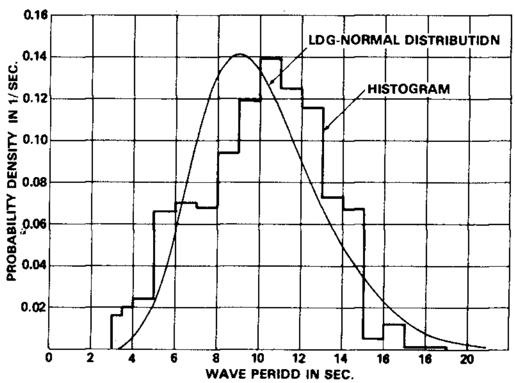


Figure 10: Comparison between histogram of wave period and log-normal probability density function (Data from Reference 2)

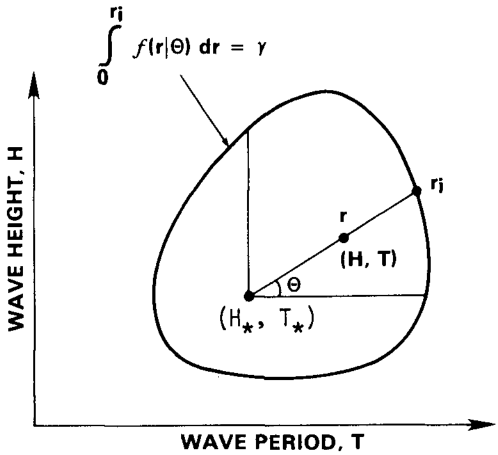


Figure 11: Pictorial sketch illustrating transformation of random variables

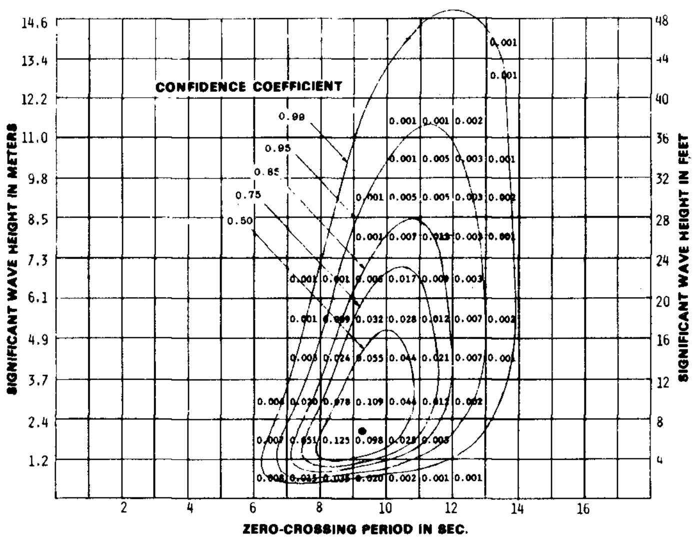


Figure 12: Comparison of domains of significant wave height and zero-crossing period for various confidence coefficients (Data from Reference 1)

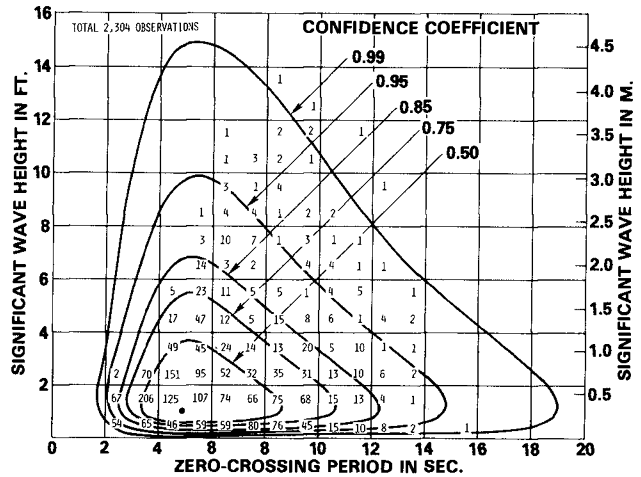


Figure 13: Comparison of domains of significant wave height and modal period for various confidence coefficients (Data from Reference 3)

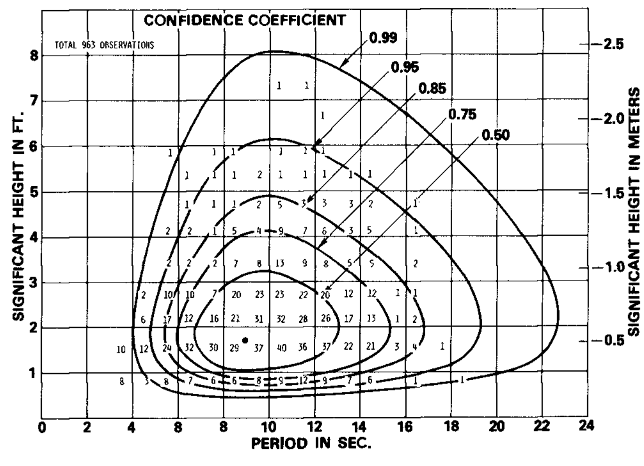


Figure 14: Comparison of domains of significant wave height and period for various confidence coefficients (Data from Reference 2)

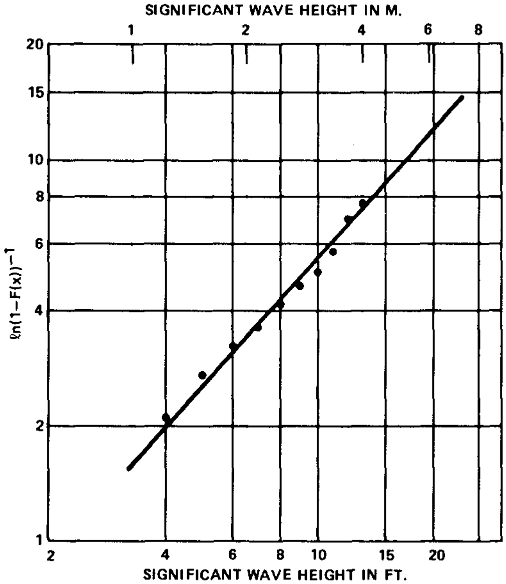


Figure 15: Probable extreme significant wave height at Tiner Point, Canada (Data from Reference 3)

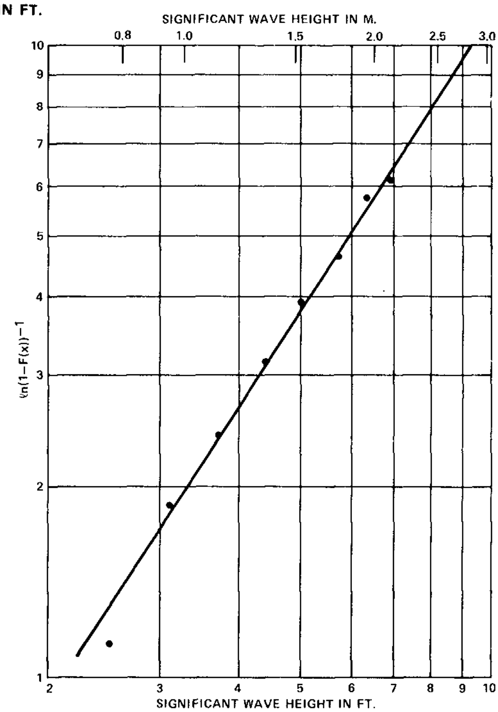


Figure 16: Probable extreme significant wave height at Port Hueneme, California (Data from Reference 2)