CHAPTER 181

DETERMINATION OF THE INTERFACIAL EDDY DIFFUSION COEFFICIENT OF A HIGHLY STRATIFIED ESTUARY

by

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INTRODUCTION

Estuaries may be sequentially classified into highly stratified, moderately mixed and vertically homogeneous. An important difference between moderately stratified or vertically homogeneous estuaries, and highly stratified estuaries (salt wedges) is that, in the former, tidal currents are sufficient to cause turbulent mixing of fresh water and sea water over the full depth of the estuary. In the latter, a distinct interface or interfacial layer exists which separates the two nearly homogeneous layers. The vertical advection of salt in this two-layer flow is the dominant process in maintaining the salt balance. This paper presents an analytical model describing this process. Experiments have been conducted in the laboratory to compare with the developed theory.

A large number of publications concerning estuarine dynamics are available for moderately mixed and vertically homogeneous estuaries. Relatively little information, however, is available for highly stratified estuaries. In an earlier work Keulegan (1949) defined the critical velocity of the upper layer at which the entrainment of the salt water layer starts. A summary of Keulegan's work may be found in Chappter 11 of the book, "Estuary and Coastline Hydrodynamics" edited by Ippen (1966). Recently Partheniades et. al. (1975) reviewed Schijf-Schonfeld's analytical work and Keulegan's experimental data with regard to the length, shape and shear stresses of a saline wedge. Wang (1975) compared his laboratory measurements of interfacial stresses with Lock's Theory.

THEORETICAL DEVELOPMENT

For a steady, incompressible, viscous, two-layer, stably stratified estuarial type flow, the governing equations are:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_1 \frac{\partial^2 u}{\partial y^2} \qquad \text{Bottom Layer (1)}$$

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$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_2 \frac{\partial^2 u}{\partial y^2} \qquad \text{Top Layer (2)}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{Continuity (3)}$$

The salt balance equation is:

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = E \frac{\partial^2 c}{\partial y^2} \qquad (4)$$

The symbols and coordinates are referred to in the following sketch. V is the velocity in the y-direction and E the eddy diffusion coefficient. Subscripts 1 and 2 refer to the bottom and top layers.

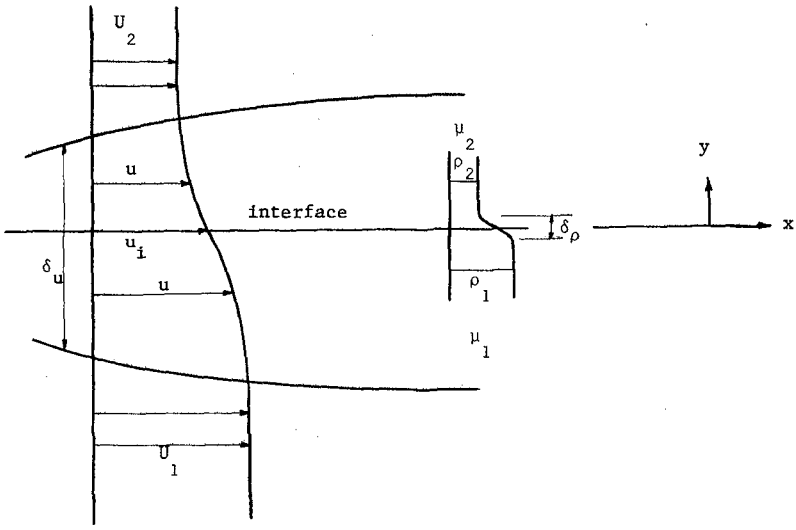

Figure 1. Definition Sketch

From laboratory observation, a highly stratified estuarial flow can be maintained if the flow of both layers are fairly uniform. However, this does not exclude turbulence from the vicinity of the interface due to the instability of the interface and the breaking of interfacial wavelets (Browand and Wang, 1972; Rumer, 1974; Wang 1975). It seems justified to assume self-similarity of velocity profiles such that:

$$\frac{u}{U_1} = \phi\left(\frac{y}{\delta}\right) = \phi(\eta) \tag{5}$$

where ϕ is a function to be determined, and δ is a vertical length scale such as the thickness of the viscous layer at the interface. This self-similarity assumption is analogous to the case of approximate self-similarity of the velocity profiles in turbulent wakes and boundary layers discussed by Townsend (1956). The observed self-similarity of concentration profiles facilitates the integration of the salt balance equation (4).

With equation (5), the interfacial shear stress is given by:

$$\tau_i = \mu\left(\frac{\partial u}{\partial y}\right)_{y\to 0} = \frac{\mu}{\delta_1} U_1 \phi_1'(0) \tag{6}$$

the interfacial friction factor is defined as (Lock 1951)

$$f_i = \frac{\tau_i}{\rho_1 U_1^2}\sqrt{\frac{U_1 x}{\nu_1}} \tag{7}$$

Eliminating τ_i between equation (6) and (7), we have

$$\left.\frac{\partial u}{\partial y}\right|_{y\to 0-} = \frac{1}{\mu_1}\left(f_i \rho_1 U_1^2 \sqrt{\frac{\nu_1}{U_1 x}}\right) \tag{8}$$

Let

$$\eta_1 = \phi_1\left(\frac{y}{\delta_1}\right) = y\sqrt{\frac{U_1}{\nu_1 x}}$$

integrating equation (8) and applying the condition

$$u \to u_i \quad \text{when } y \to 0-$$

We obtain

$$u = f_i U_1 \eta_1 + u_i \tag{9}$$

The continuity equation (3) implies

$$v = \int \left(- \frac{\partial u}{\partial x} \right) \partial y = \frac{1}{4} f_i U_1 \eta_1^2 \sqrt{\frac{\nu_1}{U_1 x}} \tag{10}$$

substituting (9) and (10) into the salt balance equation (4) and applying the chain rule we have

$$\frac{\partial^2 c}{\partial \eta^2} + \frac{\nu}{E} \left(f_i \frac{\eta^2}{4} + \frac{u_i}{U_1} \frac{\eta}{2} \right) \frac{\partial c}{\partial \eta} = 0 \tag{11}$$

Let

$$P = \frac{\partial c}{\partial \eta}, \quad P = \frac{\partial^2 c}{\partial \eta^2}$$

the equation (11) may be solved,

$$P = A_0 \operatorname{Exp} \left[-f_i \frac{1}{4} \frac{\nu}{E} \frac{\eta^3}{3} - \frac{u_i}{U_1} \frac{\nu}{E} \frac{\eta^2}{4} \right]$$

Integrating to get

$$C = A_0 \int^{\eta} e^{-\left(f_i \frac{1}{4} \frac{\nu}{E} \frac{\xi^3}{3} + \frac{u_i}{U_1} \frac{\nu}{E} \frac{\xi^2}{4} \right)} \partial \xi + A_1 \tag{12}$$

Near the interface

$$\xi < 1 \text{ and } f_i << 1.$$

Therefore,

$$\left| f_i \frac{1}{4} \frac{\nu}{E} \frac{\xi^3}{3} \right| << \left| \frac{u_i}{U_1} \frac{\nu}{E} \frac{\xi^2}{4} \right|$$

Neglecting the cubic term, we have

$$C = A_0 \int^{\eta} e^{-\left(\frac{u_i}{U_1} \frac{\nu}{E} \frac{\xi^2}{4} \right)} \partial\xi + A_1 \ . \tag{13}$$

The conditions are:

$$C\ (\ \eta = -\infty\ ) = C_1 \text{ implies } A_1 = C_1$$

$$C\ (\ \eta = +\infty\ ) = A_0 \int^{\eta} e^{-\beta\xi^2}\ \partial\xi + C_1 = A_0 \sqrt{\frac{\pi}{\beta}} + C_1$$

Where

$$\beta = \frac{1}{4} \frac{u_i}{U_1} \frac{\nu}{E}$$

Then,

$$A_0 = -(\ C_1 - C_2\ ) \sqrt{\frac{\beta}{\pi}}$$

Substituting β , A_0 and A_1 into equation (13), we obtain the concentration distribution

$$C = -(C_1 - C_2)\sqrt{\frac{\beta}{\pi}}\int^{\eta} e^{-\beta\xi^2}\,\partial\xi + C_1 \tag{14}$$

The concentration gradient at the interface is obtained by differentiating equation (14) with respect to y.

$$\left.\frac{\partial c}{\partial y}\right|_{y\to 0} = -(C_1 - C_2)\sqrt{\frac{u_i}{4\pi E x}} \tag{15}$$

For a given station from the river mouth, E is ready to be determined if the concentration gradient and velocity at the interface are known.

EXPERIMENTAL RESULTS AND DISCUSSION

Two streams of salt water of different concentrations $C_1 > C_2$, are brought to the open channel entrance to form a two-layer type flow field. The velocities of both layers are adjusted by control valves. For a given concentration difference $\Delta C = C_1 - C_2$, the degree of mixing at the interface is proportional to the velocity differential $\Delta U = U_1 - U_2$. For this experiment, the ΔU is maintained in such a way that the interface is sharp. The bottom layer is dyed red for visual observations. A standard TSI hot-film probe is used for velocity measurement. Variations in electrical conductivity is used to detect the variations in concentration. Both hot-film sensor and conductivity probes are mounted together in the same probe holder. The velocity profile and concentration profile at various x-stations are obtained simultaneously by traveling these two probes slowly across the interface. The interfacial velocity u_i and the concentration gradients are obtained from the velocity and density profiles.

The theoretical concentration distribution is compared to the measured profile in Figure (2). The theory gives a less steep slope at the interface than was actually measured. The discrepancy may be explained as the following:

In the process of integration a common vertical scale length is used. This means the theory is developed under the consideration that viscous layer thickness δ_u and diffusion layer thickness δ_ρ is of the same order of magnitude

$$\frac{\delta_u}{\delta_\rho} \simeq 1$$

A close examination of the measured data reveals that in the experiment

$$\frac{\delta_u}{\delta_\rho} >> 1$$

The viscous layer thickness δ_u is plotted against longitudinal x-axis in Figure (3), and the diffusion layer thickness δ_ρ versus x is shown in Figure (4).

For a given x-station, equation (15) indicates that the eddy diffusion coefficient E is inversely proportional to the square of the concentration gradient at the interface. For this experiment run,

$$\frac{E_{theory}}{E_{measured}} \simeq 6.2$$

It may be concluded that the relative size of the viscous layer thickness and diffusion layer thickness is important in determining the eddy diffusion coefficient. The developed theory herein predicts concentration distribution and eddy diffusion coefficient E for $\delta_u = \delta_\rho$. The theory gives larger E values and less steep slopes of the concentrations profile at the interface when $\delta_u > \delta_\rho$, smaller E values and steeper concentration profile slopes at the interface when $\delta_u < \delta_\rho$.

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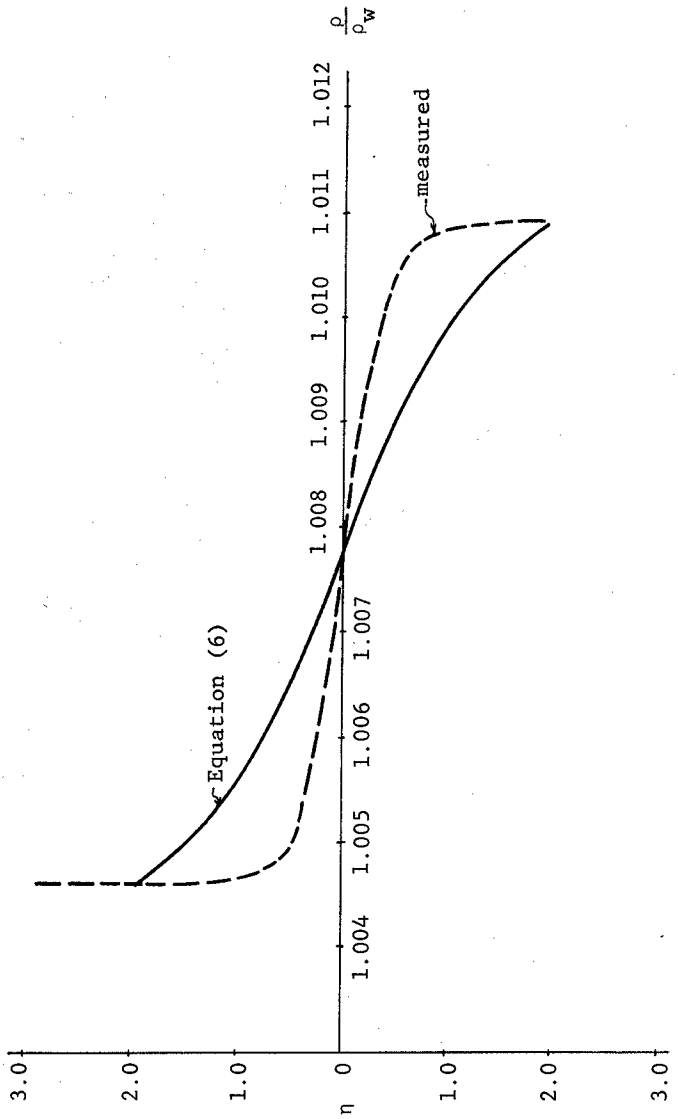
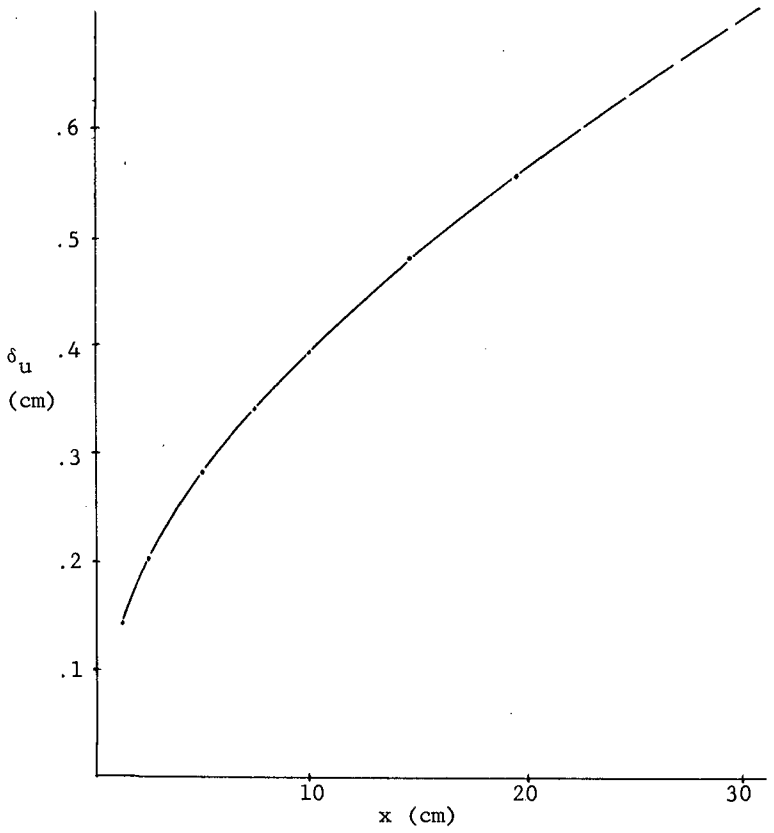


VERTICAL DENSITY DISTRIBUTIONS - A COMPARISON OF THEORY AND MEASURED PROFILE.

Figure 2



THE VISCOUS LAYER THICKNESS δ_u VS. X-AXIS

Figure 3

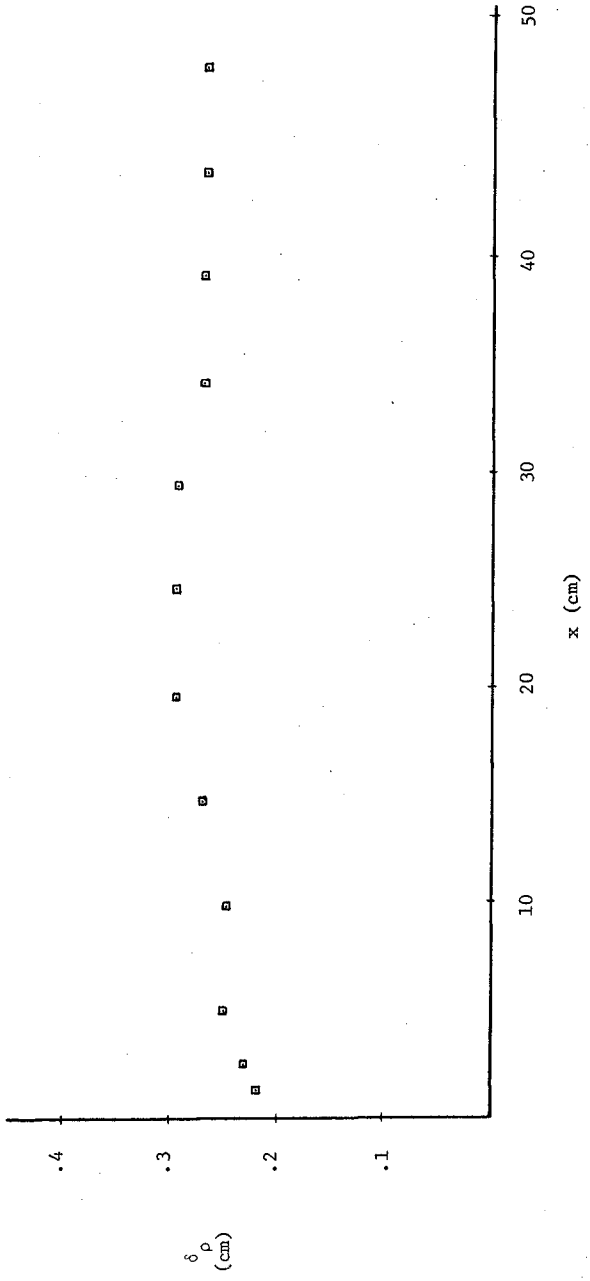


Figure 4. The Diffusion Layer Thickness δ_ρ Vs. X-axis