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METHOD OF ANALYSES FOR TWO-DIMENSIONAL WATER WAVE PROBLEMS

by

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Abstract

One of the most powerful tools to analyze the boundary-value problems in water wave motion is the Green's function. However, to derive the Green's function which satisfies the imposed boundary conditions is sometimes difficult or impossible, especially in variable water depth. In this paper, a simple method of numerical analyses for two-dimensional boundary-value problems of small amplitude waves is proposed, and the wave transformation by fixed horizontal cylinders as an example of fixed boundaries, the wave transformation by and the motion of a cylinder floating on water surface as example of oscillating boundaries and the wave transformation by permeable seawall and breakwater as example of permeable boundaries are calculated and compared with experimental results.

I Introduction

The author (1971) has investigated the problem of wave transformation by permeable breakwater and seawall with vertical faces by the method of continuation of velocity potentials. Sollitt (1972) has also calculated the same problem by the similar method to the author's and recently Madsen and White (1976) have investigated the problem with long wave assumption. Such a problem can be analyzed theoreticall when the structure is of vertical faces, but as for the sloped-faces, it is possible only to estimate under several conventional assumptions.

The problem on wave transformation by and the motion of floating rectangular body in constant finite water depth area has been analyzed by one of the authors (1972) by the method of continuation of velocity potentials. Such a problem for floating cylinder with arbitrary cross-section shall be solved by means of Green's function, being derived by John (1950). However, the process is rather complicated and can not be applied to the case

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of variable water depth.

The proposed method in this paper is not to use Green's function but to use logarithmic function of the distance between the point on the boundary and the inner point of fluid region, according to Green's theorem. By means of our method, above-described problems concerning to the sloped-face permeable structures, the floating body in variable water depth area and so on are easily formulated and numerically analyzed. In the followings, the formulations and numerical evaluations for small amplitude waves are described and compared with experimental results.

II Green's Theorem and Identity Formula

We assume that a potential function $\phi(x,z)$ is defined in a closed domain enclosed by a curve D in (x,z) plane as shown in Fig.2-1. Indicating the point on the boundary curve D by (ξ,η), the outward normal by ν, the distance between the point (ξ,η) and a point (x,z) in the domain by r, that is, $r=\sqrt{(\xi-x)^2+(\eta-z)^2}$, and the constant reference length to the geometrical size of the domain by h_o, it follows by Green's theorem that the potential value at point (x,z) is provided by the potential values $\phi(\xi,\eta)$ and its normal derivatives $\partial\phi(\xi,\eta)/\partial(\nu/h_o)$ on the boundary curve as follows:

$$\phi(x,z)=\frac{1}{2\pi}\int_D\left[\phi(\xi,\eta)\frac{\partial \log(r/h_o)}{\partial(\nu/h_o)}-\frac{\partial\phi(\xi,\eta)}{\partial(\nu/h_o)}\log(r/h_o)\right]\frac{ds}{h_o} \qquad (2.1)$$

If the point (x,z) lies on the boundary at (ξ',η'), Eq.(2.1) leads to the Green's identity formula as follows:

$$\phi(\xi',\eta')=\frac{1}{\pi}\int_D\left[\phi(\xi,\eta)\frac{\partial \log(R/h_o)}{\partial(\nu/h_o)}-\frac{\partial\phi(\xi,\eta)}{\partial(\nu/h_o)}\log(R/h_o)\right]\frac{ds}{h_o} \qquad (2.2)$$

where $R=\sqrt{(\xi-\xi')^2+(\eta-\eta')^2}$

In Eq.(2.1) and (2.2), the integration denotes the line integral along the curve D. Then, dividing the boundary curve into N small elements by N points and indicating the length and the central point of the j-th element as ΔS_j and (ξ_j,η_j), as shown by Fig.2-2, Eq.(2.1) and (2.2) are approximated by the following summation equations, respectively.

$$\phi(x,z)=\frac{1}{2}\sum_{j=1}^{N}\left[\bar{E}_{xj}\phi(j)-E_{xj}\bar{\phi}(j)\right] \tag{2.3}$$

$$\phi(i)=\sum_{j=1}^{N}\left[\bar{E}_{ij}\phi(j)-E_{ij}\bar{\phi}(j)\right] \tag{2.4}$$

where

$$\phi(j)=\phi(\xi_j,\zeta_j), \qquad \bar{\phi}(j)=\partial\phi(\xi_j,\zeta_j)/\partial(\nu/h_0) \tag{2.5}$$

$$\begin{aligned} E_{xj}&=\frac{1}{\pi}\int_{\Delta S_j}\log\left(\frac{r_{xj}}{h_0}\right)\frac{dS}{h_0}, \quad \bar{E}_{xj}=\frac{1}{\pi}\int_{\Delta S_j}\frac{\partial}{\partial(\nu/h_0)}\log\left(\frac{r_{xj}}{h_0}\right)\frac{dS}{h_0}\\ E_{ij}&=\frac{1}{\pi}\int_{\Delta S_j}\log\left(\frac{R_{ij}}{h_0}\right)\frac{dS}{h_0}, \quad \bar{E}_{ij}=\frac{1}{\pi}\int_{\Delta S_j}\frac{\partial}{\partial(\nu/h_0)}\log\left(\frac{R_{ij}}{h_0}\right)\frac{dS}{h_0}\end{aligned} \tag{2.6}$$

E_{xj}, $\bar{E}_{xj}$, and E_{ij} , $\bar{E}_{ij}$ are integrated values over the j-th element refering to the point X = (x, z) and i = (ξ_i , ζ_i), respectively, and they are calculated numerically as follows:

$$\begin{aligned} E_{ij}&=\frac{1}{\pi}\log\left(\frac{R_{ij}}{h_0}\right)\cdot\frac{\Delta S_j}{h_0}, \quad & E_{ii}&=\frac{1}{\pi}\left(\log\frac{\Delta S_i}{2h_0}-1\right)\cdot\frac{\Delta S_i}{h_0}\\ \bar{E}_{ij}&=\theta_{ij}/2\pi, & \bar{E}_{ii}&=0\end{aligned} \tag{2.7}$$

where θ_{ij} is the subtending angle of the point i = (ξ_i, ζ_i) to the j-th element, and

$$R_{ij}=\sqrt{(\xi_j-\xi_i)^2+(\zeta_j-\zeta_i)^2}, \qquad \Delta S_j=\sqrt{(\Delta\xi_j)^2+(\Delta\zeta_j)^2}$$

$$\Delta\xi_j=\frac{1}{2}(\xi_{j+1}-\xi_{j-1}), \qquad \Delta\zeta_j=\frac{1}{2}(\zeta_{j+1}-\zeta_{j-1})$$

E_{xj} , $\bar{E}_{xj}$ are calculated, replacing the point i = (ξ_i , ζ_i) by X = (x,z) in Eq.(2.7).

Eq.(2.1) or (2.3), the Green's theorem, states that the potential function at any point in the domain is determined by its boundary-values and normal derivatives. In other words, to solve a boundary-value problem is equivalent to determine the boundary-values and its normal derivatives of the interested potential function.

Eq.(2.2) or (2.4), the Green's identity formula, states that the boundary-values $\phi(\xi,\zeta)$ and its normal derivatives $\bar{\phi}(\xi,\zeta)$ are in linear relationships which are defined by the geometrical shape of the domain.

This is the first set of relations between ϕ and $\bar{\phi}$ on the boundary.

Therefore, if another set of relations between ϕ and $\bar{\phi}$ is provided, it follows that they should be determined by solving the two set of relations, simultaneously. And, in our problems, this second relation is given by dynamical or kinematical boundary conditions on the boundaries of the interested domain.

III Wave Transformation by Fixed Cylinder

As an example of fixed boundaries, we consider the wave transmission through and wave forces to the semi-immersed cylinder with arbitrary cross-section in variable water depth area. In Fig.3-1, the origin O of the coordinate system is at still water surface, x- and z- axis are horizontal and vertically upwards, respectively. We assume that CDC' is a fixed cylinder at variable depth area, where the depth at sufficiently distant from the cylinder is constant h to the right and constant h' to the left and that the incident wave of frequency σ and amplitude ζ_i comes from the right. We take the geometrical boundaries AB and A'B' at x = ℓ and $-\ell'$, where the depths are h and h', respectively, and divide the fluid region into three parts (O), (I) and (O') as shown in the figure.

The fluid motion is assumed to have velocity potential with potential function $\phi(x,z)$ as shown by Eq.(3.1).

$$\Phi(x,z;t) = \frac{g\zeta_i}{\sigma}\phi(x,z)e^{i\sigma t} \tag{3.1}$$

where g is gravity acceleration and t is time. The potential functions in region (O),(I) and (O') are denoted by $\phi_o(x,z)$, $\phi(x,z)$ and $\phi_o'(x,z)$, respectively. Then, since region (O) and (O') are of constant depth and so far from the cylinder that the scattering waves are damped to be vanished, the potential functions for them are expressed simply by Eq.(3.2) and (3.3) without scattering terms.

$$\phi_o(x,z) = \left[e^{ik(x-\ell)} + \psi e^{-ik(x-\ell)}\right]\cdot A(kz) \tag{3.2}$$

$$\phi_o'(x,z) = \psi' e^{-ik'(x-\ell')}\cdot A(k'z) \tag{3.3}$$

In Eq.(3.2), the first term is for the incident wave and the second term is for reflected wave with complex reflection coefficient ψ . Eq.(3.3) is for transmitted wave with complex transmission coefficient ψ' .

The functions A(kz) and A(k'z) are given by Eq.(3.4) with wave numbers k and k' for region (O) and (O'), which are determined by Eq.(3.5). The reflection- and transmission coefficient K_r and K_t are provided by Eq.(3.6).

$$A(kz) = \frac{\cosh k(z+h)}{\cosh kh} \qquad A(k'z) = \frac{\cosh k'(z+h')}{\cosh k'h'} \tag{3.4}$$

$$kh \tanh kh = \frac{\sigma^2 h}{g} \qquad k'h' \tanh k'h' = \frac{\sigma^2 h'}{g} \tag{3.5}$$

$$K_r = |\psi| \qquad K_t = |\psi'| \tag{3.6}$$

Now, we consider the dynamical or kinematical conditions on the boundaries of fluid region (I).

On the free surface AC, C'A' at z = 0, we have Eq.(3.7).

$$\frac{\partial \phi}{\partial z} = \frac{\sigma^2}{g}\phi \quad \text{or} \quad \bar{\phi} = \frac{\partial \phi}{\partial(\nu/h_o)} = \Gamma\phi \quad \text{where } \Gamma = \frac{\sigma^2 h_o}{g} \tag{3.7}$$

and h_o is taken as the distance between point A and B'.

On the immersed surface of fixed cylinder CDC' and on bottom BB', we have Eq.(3.8) because of the impervious boundaries.

$$\frac{\partial \phi}{\partial \nu} = 0 \quad \text{or} \quad \bar{\phi} = \frac{\partial \phi}{\partial(\nu/h_o)} = 0 \tag{3.8}$$

Finally, on the geometrical boundaries AB ($x = \ell$) and A'B' ($x = -\ell'$), we have from Eq.(3.2) and (3.3)

$$\phi_o = (1+\psi)\cdot A(kz), \qquad \bar{\phi}_o = h_o \frac{\partial \phi_o}{\partial x} = i\lambda_o(1-\psi)\cdot A(kz) \tag{3.9}$$

$$\phi_o' = \psi'\cdot A(k'z), \qquad \bar{\phi}_o' = -h_o \frac{\partial \phi_o'}{\partial x} = -i\lambda_o'\psi' A(k'z) \tag{3.10}$$

$$\text{where} \quad \lambda_o = kh_o \qquad \lambda_o' = k'h_o \tag{3.11}$$

As shown in Fig.3-2, we divide the boundaries AC, CDC', C'A' and BB' into N_1, N_2, N_3 and N_4 elements, respectively and geometrical boundaries AB, A'B' into M and M' elements, and denote the potential functions on them by ϕ_1, ϕ_2, ϕ_3, ϕ_4 and ϕ_o, ϕ_o', respectively. Then, substituting the relations (3.7) ~ (3.10) into the Green's identity formula (2.4) for the fluid region (I), the following simultaneous linear equations with respect to the potential functions on the boundaries and coefficients ψ and ψ' are provided:

$$-\phi(i)+\sum_{j=1}^{N_1}(\bar{E}_{ij}-\Gamma E_{ij})\phi_1(j)+\sum_{j=1}^{N_2}\bar{E}_{ij}\phi_2(j)+\sum_{j=1}^{N_3}(\bar{E}_{ij}-\Gamma E_{ij})\phi_3(j)$$
$$+\sum_{j=1}^{N_4}\bar{E}_{ij}\phi_4(j)+\psi\sum_{r=1}^{M}G_{ir}A(kz_r)+\psi'\sum_{s=1}^{M'}G'_{is}A(k'z_s)$$
$$=-\sum_{r=1}^{M}G^*_{ir}A(kz_r) \tag{3.12}$$

where

$$G_{ir}=\bar{E}_{ir}+i\lambda_0E_{ir},\quad G'_{is}=\bar{E}_{is}+i\lambda'_0E_{is},\quad G^*_{ir}=\bar{E}_{ir}-i\lambda_0E_{ir} \tag{3.13}$$

In above equations, the first term ϕ(i) should be written as follows, according to the position of point (i):

$$\text{For } i=1\sim N_1,\quad \phi(i)=\phi_1(j)\ ;\quad i=1\sim N_2,\quad \phi(i)=\phi_2(i);$$
$$i=1\sim N_3,\quad \phi(i)=\phi_3(j)\ ;\quad i=1\sim N_4,\quad \phi(i)=\phi_4(i); \tag{3.14}$$

For point (i) on AB and A'B', putting i= (ℓ, z_p) = (p), i= (-ℓ', z_q) = (q), we take

$$\phi(i)=(1+\psi)\cdot A(kz_p),\qquad \phi(i)=\psi'A(k'z_q) \tag{3.15}$$

Eq.(3.12) yields (N_1+ N_2+ N_3+ N_4+ 2) linear equations with respect to the same number of unknown quantities. Solving these equations, all of the unknowns are determined and by means of Eq.(2.3), the potential function at any point in fluid region is calculated, and at the same time those of regions (O) and (O') are obtained by Eq.(3.2) and (3.3).

The fluid pressure at point (j) = (ξ_j,ζ_j) on the immersed surface of the cylinder is given as

$$\frac{p(j)}{\rho g\zeta_0}=-i\phi_2(j)e^{i\sigma t} \tag{3.16}$$

Consequently, the horizontal and vertical resultant forces P_x and P_z and the resultant moment T around the point (x_0,z_0) are calculated as follows:

$$\frac{P_x}{\rho g \zeta_o h_o} = -i\, e^{i\sigma t} \sum_{j=1}^{N_2} \phi_2(j) \cdot \frac{\Delta \eta_j}{h_o} \tag{3.17}$$

$$\frac{P_z}{\rho g \zeta_o h_o} = i\, e^{i\sigma t} \sum_{j=1}^{N_2} \phi_2(j) \cdot \frac{\Delta \xi_j}{h_o} \tag{3.18}$$

$$\frac{T}{\rho g \zeta_o h_o^2} = i\, e^{i\sigma t} \sum_{j=1}^{N_2} \left(\frac{\xi_j - x_o}{h_o} \cdot \frac{\Delta \xi_j}{h_o} + \frac{\eta_j - z_o}{h_o} \cdot \frac{\Delta \eta_j}{h_o} \right) \phi_2(j) \tag{3.19}$$

The first calculated example is a semi-immersed circular cylinder whose center is fixed at still water surface on constant water depth area and whose diameter D is 0.8 times the water depth h. The geometrical surface AB and A'B' are taken at x = 3h and - 3h, respectively. The numbers of calculation points on the boundaries are taken as N_1= 20, N_2= 14, N_3= 20, N_4= 30 and M = M' = 20. The second example is double cylinders whose diameters are the same as above and whose centers are apart by three times the diameter D.

Fig.3-3 shows the calculated and measured transmission coefficients with respect to the non-dimensional frequency $\sigma^2 h/g$ or to the ratio of diameter to wave length D/L for the first and second examples, where the solid line and open circles are for single cylinder and the broken line and solid circles are for double cylinders. From the figure, it is seen that the transmission coefficient for single cylinder decreases gradually and the one for double cylinders decreases rapidly with increasing frequency and that the measured values are somewhat lower than the calculated values for higher frequencies but the tendencies of both are in good agreement. The discrepancies between measured and calculated values are thought to be due to the non-linear effect of measured waves. (The experiments were carried out in wave flume of length 22 m with water depth h = 40 cm and incident wave amplitude ζ_o = 3 ～ 4 cm.)

IV Wave Transformation by and the Motion of Floating Cylinder

In Fig.4-1, it is assumed that a cylinder of cross-section CDD'C' with gravity center at $(\overline{x}_o, \overline{z}_o)$ and center of bouyancy at (x_b, z_b) in equilibrium condition is moored by spring lines DE and D'E' with spring constant K on the variable sea bottom B'E'EB, and is subjected to the incident wave of frequency σ and small amplitude ζ_o from the right. Then, the position of the gravity center of the cylinder (x_0, z_0) and the rotation angle δ of the cylinder around gravity center at any time t in motion are expressed by the complex amplitude of horizontal and vertical displacements X, Z and of the rotation angle Θ as follows:

$$x_0 = \bar{x}_0 + X e^{i\sigma t}, \quad z_0 = \bar{z}_0 + Z e^{i\sigma t}, \quad \delta = \Theta e^{i\sigma t} \tag{4.1}$$

Similarly to the previous section III, the velocity potential is expressed by Eq.(3.1) and the potential function in region (O), (O') are by Eq.(3.2), (3.3) with reflection and transmission coefficients ψ and ψ'. And also, the potential function at free surface and at bottom in fluid region (I) are in the relation of Eq.(3.7) and (3.8), respectively. However, on the oscillating surface CDD'C', the normal derivatives of the potential function ϕ_2 is given by the following expression, due to the kinematical boundary condition:

$$\bar{\Phi} = i\Gamma\left[\frac{X}{\zeta_0}\frac{dz}{ds} - \frac{Z}{\zeta_0}\frac{dx}{ds} - \frac{\Theta a}{\zeta_0}\left\{\frac{x-\bar{x}_0}{a}\frac{dx}{ds} + \frac{z-\bar{z}_0}{a}\frac{dz}{ds}\right\}\right] \tag{4.2}$$

where a is a reference length to the horizontal size of the cross-section, for example, a is taken as half width for rectangular cylinder and as radius for circular cylinder. (x,z) is the coordinate of point on the surface CDD'C' and s is the length measured along CDD'C'.

The complex amplitudes X, Z and Θ in Eq.(4.2) can be expressed by the potential function ϕ_2 on the immersed surface of cylinder, taking account of the following equations of motion of the cylinder:

$$M\frac{d^2 x_0}{dt^2} = P_x + F_x, \quad M\frac{d^2 z_0}{dt^2} = P_z + P_s + F_z$$
$$I_\theta \frac{d^2\delta}{dt^2} = T_\theta + T_s + M_\theta \tag{4.3}$$

where M is the mass of the cylinder; I_θ is the moment of inertia around the gravity center; P_x, P_z, T_θ are the resultant horizontal and vertical fluid forces and moment around gravity center due to the fluid pressure acting to the immersed surface; P_S, T_S are the restoring force and moment for vertical displacement and rotation of cylinder due to statical fluid pressure; F_x, F_z, M_θ are the mooring forces and moment by the mooring lines induced by the motion of the cylinder.

Indicating the fluid density by ρ, the draught in mooring condition by qh $(1 > q > 0)$, the mass M, the moment of inertia I_θ and the immersed volume of the cylinder V are expressed with positive constants ν_1, ν_2 and ν_3 as follows:

$$M = \nu_1 \rho a q h , \quad I_\theta = \nu_2 \rho a^2 (q h)^2 , \quad V = \nu_3 a q h \tag{4.4}$$

Since the fluid pressure on the immersed surface is expressed by Eq. (3.16), P_x, P_z and T_θ are given as follows:

$$\begin{aligned} P_x &= -i \rho g \zeta_0 e^{i\sigma t} \int_S \phi_2(x,z)\, dz \\ P_z &= i \rho g \zeta_0 e^{i\sigma t} \int_S \phi_2(x,z)\, dz \\ T_\theta &= i \rho g \zeta_0 e^{i\sigma t} \int_S \{(x-\bar{x}_0)dx + (z-\bar{z}_0)dz\}\, \phi_2(x,z) \end{aligned} \tag{4.5}$$

where integrations are taken along the surface CDD'C'.

Denoting the length of water line as $2\ell_0$, P_S and T_S are given as

$$P_S = -2\rho g \ell_0 Z e^{i\sigma t}, \quad T_S = -\rho g V\left\{\frac{3}{2}\frac{\ell_0^3}{V} - (\bar{z}_0 - Z)\right\} \Theta e^{i\sigma t} \tag{4.6}$$

For simplicity, we assume that the cross-section of the cylinder and the mooring condition are symmetrical with respect to the vertical line through the gravity center. Taking the angle of mooring line with horizontal as β and the mooring point on the cylinder as (a_0, b_0) and $(-a_0, b_0)$, the mooring forces and moment to the cylinder F_x, F_z and M_θ are expressed as follows:

$$\begin{aligned} F_x &= -2K(X - S\Theta)\cos^2\beta\, e^{i\sigma t}, \quad F_z = -2K\sin^2\beta\, e^{i\sigma t} \\ M_\theta &= 2KS(X - S\Theta)\cos^2\beta\, e^{i\sigma t} \end{aligned} \tag{4.7}$$

where

$$S = b_0 - \bar{z}_0 - (a_0 - \bar{x}_0)\tan\beta$$

Substituting Eq. (4.1) (4.4) (4.5) (4.6) and (4.7) into Eq. (4.3), it follows that X, Z and Θ are expressed by $\phi_2(x,z)$.

$$\frac{X}{\zeta_0} = \frac{i}{\gamma}\int_S \phi_2(x,z)\cdot\left\{k_{x\theta}\frac{x-\bar{x}_0}{a}\frac{dx}{a} + \left(k_{x\theta}\frac{z-\bar{z}_0}{a} - \alpha_3\right)\frac{dz}{a}\right\} \tag{4.8}$$

$$\frac{Z}{\zeta_0} = \frac{i}{\alpha_2}\int_S \phi_2(x,z)\frac{dx}{a} \tag{4.9}$$

$$\frac{\Theta a}{\zeta_0} = \frac{i}{\gamma}\int_S \phi_2(x,z)\cdot\left\{\alpha_1\frac{x-\bar{x}_0}{a}\frac{dx}{a} + \left(\alpha_1\frac{z-\bar{z}_0}{a} - k_{x\theta}\right)\frac{dz}{a}\right\} \tag{4.10}$$

where

$$\gamma = \alpha_1\alpha_3 - k_{x\theta}^2, \qquad \alpha_1 = k_{xx} - \nu_1\frac{qh}{h_0}\Gamma,$$

$$\alpha_2 = k_{zz} - \nu_1\frac{qh}{h_0}\Gamma + \frac{2l_0}{a}, \quad \alpha_3 = k_{\theta\theta} - \nu_2\frac{(qh)^2}{a h_0}\Gamma$$

$$+\frac{2}{3}\left(\frac{l_0}{a}\right)^3 - \nu_3\frac{qh}{a}\frac{\bar{z}_0 - z_b}{a}, \quad k_{xx} = \frac{2K}{\rho g a}\cos^2\beta \tag{4.11}$$

$$k_{zz} = \frac{2K}{\rho g a}\sin^2\beta, \quad k_{\theta\theta} = \frac{2KS^2}{\rho g a^3}\cos^2\beta, \quad k_{x\theta} = \frac{2KS}{\rho g a^2}\cos^2\beta$$

Introducing Eq.(4.8)(4.9)(4.10) into Eq.(4.2), $\bar{\phi}_2$ on the immersed surface of cylinder is written by ϕ_2 as follows:

$$\bar{\phi}_2(x,z) = \Gamma\int_S \phi_2(u,v)\cdot F(x,z;u,v) \tag{4.12}$$

where

$$F(x,z;u,v) = \left[\left(\frac{1}{\alpha_2} + \frac{\alpha_1}{\gamma}\cdot\frac{u-\bar{x}_0}{a}\cdot\frac{x-\bar{x}_0}{a}\right)\frac{dx}{ds} - \frac{1}{\gamma}\left(k_{x\theta} - \alpha_1\frac{z-\bar{z}_0}{a}\right)\cdot\right.$$

$$\left.\cdot\frac{u-\bar{x}_0}{a}\frac{dz}{ds}\right]\frac{du}{a} + \frac{1}{\gamma}\left[\left(\alpha_1\frac{v-\bar{z}_0}{a} - k_{x\theta}\right)\frac{x-\bar{x}_0}{a}\cdot\frac{dx}{ds} + \left\{\left(\alpha_3 - \right.\right.\right.$$

$$\left.\left.\left. k_{x\theta}\frac{z-\bar{z}_0}{a}\right) - \left(k_{x\theta} - \alpha_1\frac{z-\bar{z}_0}{a}\right)\frac{v-\bar{z}_0}{a}\right\}\frac{dz}{ds}\right]\frac{dv}{a} \tag{4.13}$$

where (x,z) and (u,v) are the coordinates of the points on the immersed surface. Indicating the calculation points on the surface as (ξ_j, ζ_j) and (ξ_m, ζ_m), corresponding to (x,z) and (u,v), Eq.(4.12) is written as follows:

$$\bar{\phi}_2(j) = \Gamma\sum_{m=1}^{N_2} F(j,m)\cdot\phi_2(m) \tag{4.14}$$

Similarly to the preceding section III, applying Eq.(3.7)(3.8)(3.9)(3.10) and (4.14) to the Green's identity formula (2.4) for the fluid region (I), we have linear simultaneous equations with respect to the potential functions Φ on the boundaries and ψ , ψ'. They are witten by replacing the term $\sum_{j=1}^{N_2} \bar{\bar{E}}_{ij} \Phi_2(j)$ in Eq.(3.12) by $\sum_{m=1}^{N_2} \sum_{j=1}^{N_2}{}' [\delta_{jm} \bar{\bar{E}}_{ij} - \Gamma E_{ij} F(j,m)] \Phi_2(m)$,where δ is Kronecker's delta and $\delta_{jm} = 0$ ($j \neq m$) : = 1 ($j = m$).

Solving the equations, we can obtain all of the boundary-values of potential function of region (I) and the transmission-, reflection coefficient, similarly to the section III. Then, the amplitudes of motion of cylinder are calculated by Eq.(4.8)(4.9)(4.10) and also the mooring force F to the wave-side mooring line DE is calculated as follows:

$$\frac{F}{\zeta_i K} = \left[\frac{X}{\zeta_i} + \frac{Z}{\zeta_i} \tan\beta - \frac{\Theta a}{\zeta_i} \frac{S}{a} \right] \cos\beta \cdot e^{i\sigma t}. \tag{4.15}$$

The mooring force F' to the lee-side line D'E' is given by replacing β by $-\beta$ in above expression.

As an example, we consider the case when a circular cylinder is moored on constant water depth h. The diameter D = 2a is 0.914h, the draught is 0.67h (q=0.67), the mooring points on the cylinder are ($\pm$ 0.486h, - 0.114h) and $\nu_1 = 1.467$, $\nu_2 = 0.670$, $\nu_3 = 1.646$. The cylinder is of uniform density 0.584 and the center is at 0.114h below still water surface. The spring constant $K/\rho ga$ is 0.227 and mooring angle β is 33°. Fig.4-2 is the calculated (solid line) and measured (open circles) transmission coefficients with respect to the non-dimensional frequency or to the ratio of diameter to the wave length D/L. Experiments were carried out in wave flume with water depth h = 35 cm and a circular cylinder of diameter D = 32 cm, whose center was at depth 4.0 cm below still water level in equilibrium condition. The figure shows that the calculated and measured values are in good agreement. Moreover, it shows an interesting fact that the incident wave is perfectly intercepted even by floating cylinder, if the frequency $\sigma^2 h/g$ is 0.42 and 1.74, that is, D/L is 0.10 and 0.26. Fig.4-3 is the calculated reflection coefficient and amplitudes of motion of cylinder.

V Wave Transformation by Permeable Seawall and Breakwater

In Fig.5-1, suppose that ABC is a permeable seawall placed on impervious bottom BCO'. The geometrical boundary is taken at OO', which is sufficiently distant from the seawall and of constant water depth h. Dividing the fluid region into three regions (O), (I) and (II), the velocity potentials in region (O) and (I) are assumed to be expressed in the form of Eq.(3.1) with potential functions $\phi_0(x,z)$ and $\phi(x,z)$, respectively. In permeable region (II), indicating the quantities by superscript * , the mass and momentum equations are written with horizontal and vertical fluid velocities u*, w* and fluid pressure p* as follows:

$$\begin{aligned} &\frac{\partial u^*}{\partial x}+\frac{\partial w^*}{\partial z}=0 \\ &\frac{1}{V}\frac{\partial u^*}{\partial t}=-\frac{1}{\rho}\frac{\partial p^*}{\partial x}-\frac{\mu}{V}u^*-\frac{\varepsilon(1-V)}{V}\frac{\partial u^*}{\partial t} \\ &\frac{1}{V}\frac{\partial w^*}{\partial t}=-\frac{1}{\rho}\frac{\partial p^*}{\partial z}-g-\frac{\mu}{V}w^*-\frac{\varepsilon(1-V)}{V}\frac{\partial w^*}{\partial t} \end{aligned} \tag{5.1}$$

where ρ is the fluid density, V is porosity of the seawall, μ is the coefficient of drag force to the porous material which is linearized to be proportional to the fluid velocity and ε is the added mass force coefficient to the material. The fluid motion represented by Eq.(5.1) has velocity potential, which is expressed by Eq.(5.2) with potential function ϕ^* , and fluid velocities, pressure and surface profile are provided by Eq.(5.3).

$$\Phi^*(x,z;t)=\frac{g\zeta_0}{\sigma}\phi^*(x,z)e^{i\sigma t} \tag{5.2}$$

$$\begin{aligned} &u^*=\partial\Phi^*/\partial x,\quad w^*=\partial\Phi^*/\partial z,\quad p^*/\rho g\zeta_0=-i\beta\phi^*(x,z)e^{i\sigma t} \\ &\zeta^*/\zeta_0=-i\beta\phi^*(x,0)e^{i\sigma t},\quad \beta=\frac{\alpha}{V},\ \alpha=1+\varepsilon(1-V)+i\mu/\sigma \end{aligned} \tag{5.3}$$

The potential function ϕ_0 in region (O) is given by Eq.(3.2), so that the boundary conditions of fluid region (I) are provided by Eq.(3.7) on $\overrightarrow{OA}$, by Eq.(3.8) on $\overrightarrow{CO'}$ and by Eq.(3.9) on $\overrightarrow{O'O}$. As for the conditions on $\overrightarrow{AC}$, since the mass flux and energy flux through the boundary AC should be continuous, it follows from Eq.(3.16) and (5.3) that

$$\bar{\Phi}^* = \bar{\Phi} , \qquad \Phi^* = \frac{1}{\beta} \Phi \tag{5.4}$$

As for the porous region (II), we have Eq.(5.5) on free surface $\overrightarrow{AB}$ from the kinematical condition, and Eq.(5.6) on impervious boundary BC.

$$\frac{\partial \Phi^*}{\partial Z} = \alpha \frac{\sigma^2}{g} \Phi^* \quad \text{or} \quad \bar{\Phi}^* = \alpha \Gamma \Phi^* , \quad \Gamma = \frac{\sigma^2 h_0}{g} \tag{5.5}$$

$$\frac{\partial \Phi^*}{\partial Z} = 0 \quad \text{or} \quad \bar{\Phi}^* = 0 \tag{5.6}$$

As shown in Fig.5-2, denoting the potential functions on the boundaries $\overrightarrow{OA}$, $\overrightarrow{AC}$, $\overrightarrow{CO'}$ and $\overrightarrow{O'O}$ by Φ_1 , Φ_2 , Φ_3 , Φ_0 and on the boundaries $\overrightarrow{BA}$, $\overrightarrow{AC}$ and $\overrightarrow{CB}$ by Φ_1^*, Φ_2^* and Φ_3^*, dividing these boundaries into N_1, N_2, N_3, M and N_1^* , N_2, N_3^* and taking the outward normal for region (I) and inward normal for region (II), and applying the boundary conditions (3.7) (3.8) (3.9) to the Green's identity formula (2.4) for region (I) and conditions (5.4) (5.5) and (5.6) to Eq.(2.4) for region (II), we have the following equations:

(i) For fluid region (I):

$$-\Phi(i) + \sum_{j=1}^{N_1} (\bar{E}_{ij} - \Gamma E_{ij}) \Phi_1(j) + \sum_{j=1}^{N_2} [\bar{E}_{ij} \Phi_2(j) - E_{ij} \bar{\Phi}_2(j)]$$
$$+ \sum_{j=1}^{N_3} \bar{E}_{ij} \Phi_3(j) + \psi \sum_{r=1}^{M} G_{ir} A(kZ_r) = - \sum_{r=1}^{M} G_{ir}^* A(kZ_r) \tag{5.7}$$

(i = 1 $\sim N_1$, 1 $\sim N_2$, 1 $\sim N_3$ and (0, Z_p) on O'O)

(ii) For porous region (II):

$$\Phi^*(i) + \sum_{j=1}^{N_1^*} (\bar{E}_{ij}^* + \alpha \Gamma E_{ij}^*) \Phi_1^*(j) + \sum_{j=1}^{N_2} [\frac{1}{\beta} \bar{E}_{ij}^* \Phi_2(j)$$
$$- E_{ij}^* \bar{\Phi}_2(j)] + \sum_{j=1}^{N_3^*} \bar{E}_{ij}^* \Phi_3^*(j) = 0 \tag{5.8}$$

(i = 1 $\sim N_1^*$, 1 $\sim N_2$, 1 $\sim N_3^*$)

Eq.(5.7),(5.8) are ($N_1 + 2 N_2 + N_3 + N_1^* + N_3^* + 1$) linear equations with respect to the same number of unknowns Φ_1 , Φ_2 , $\bar{\Phi}_2$, Φ_3 , ψ , Φ_1^* and Φ_3^* . Consequently, solving these equations simultaneously, we can determine all of the unknowns, from which the potential values at points in fluid region are calculated by Eq.(2.3).

The surface wave profiles are calculated as follows:

$$\text{From B to A:} \quad \zeta^*(j) = -i\beta\,\phi_i^*(j)\,e^{i\sigma t} \qquad j = 1 \sim N_i^*$$
$$\text{From A to O:} \quad \zeta(j) = -i\,\phi_i(j)\,e^{i\sigma t} \qquad j = N_i \sim 1 \tag{5.9}$$

Fig.5-3 and 5-4 are the calculated and measured reflection coefficients with respect to non-dimensional frequency $\sigma^2 h/g$ for model seawall of 1:1 slope and of vertical face, respectively, made by quarry stones of mean diameter 6 cm with porosity V = 0.43 in constant water depth h = 40 cm. The widths of both seawalls at still water level are equal to twice the water depth h. The solid lines in figures are calculated values, taking V = 0.5, μ/σ = 1.0 and ε = 0 for all frequencies. The measured and calculated values are almost in good agreement.

Wave transformation by permeable breakwater is analyzed in the similar manner. In Fig.5-5, solid line, solid circles and broken line, solid triangles are the calculated and measured reflection and transmission coefficients, respectively, for model permeable breakwater with 1:1.5 sloped faces and the width at still water level h. Other conditions are the same as the seawall. The calculated values are somewhat different from measured values but the tendencies are nearly in agreement. Fig.5-6 is for permeable breakwater model with rectangular cross-section of width 2h. The measured and calculated values are in good agreement.

Fig.5-7 is the calculated distribution of equi-potential lines (solid lines) and its orthogonals (broken lines) for permeable breakwater in Fig. 5-5 at $\sigma t = 0^\circ$, 30°, 60° and 90°, when the incident wave crest approaches to the breakwater.

VI Conclusions

It is clear that the proposed method provides a convenient and simple analysis for two-dimensional boundary-value problems of small amplitude waves. And, if the difficulties arising in solving simultaneous equations of so many unknown quantities were overcome, this method is extented directly to the problem of three-dimensional waves and also to the finite amplitude wave problems by means of perturbation method.

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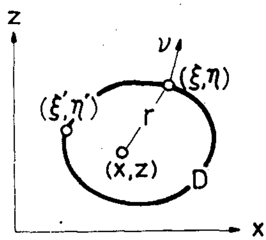


Fig. 2-1 Definition Sketch

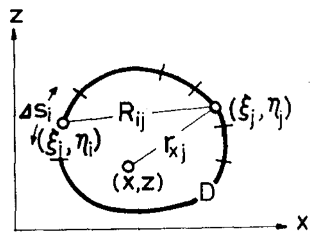


Fig.2-2 Definition Sketch

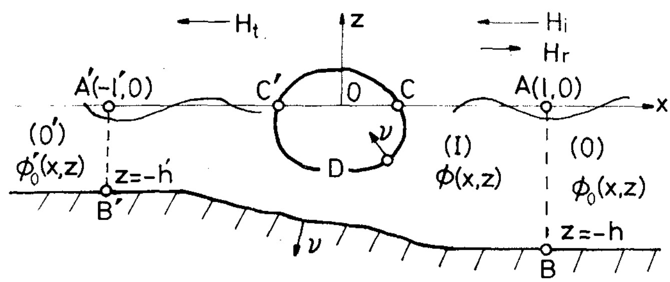


Fig. 3-1 Definition Sketch for Fixed Cylinder

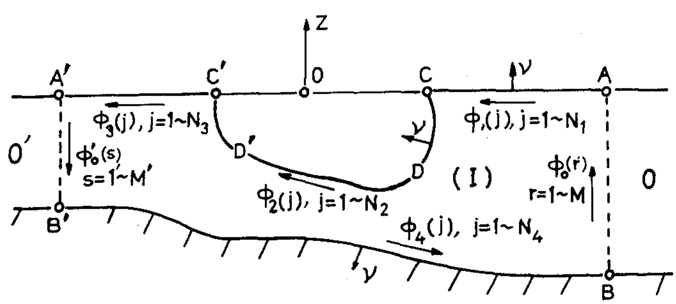


Fig.3-2 Definition of Potential Functions

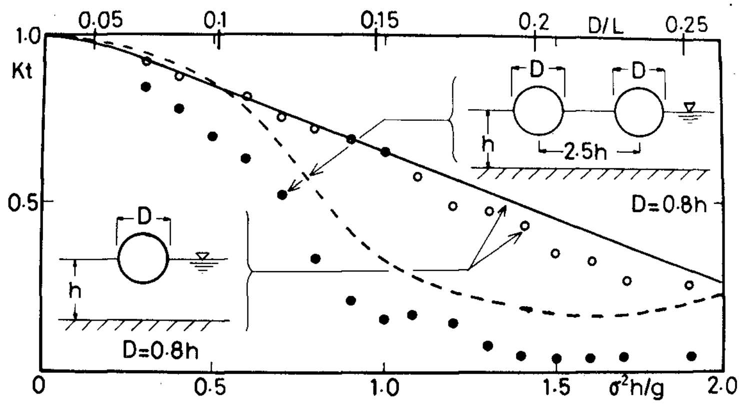


Fig.3-3 Transmission Coefficient of Single and Double Cylinders

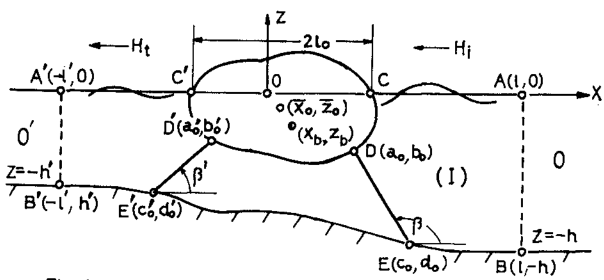


Fig.4-1 Definition Sketch

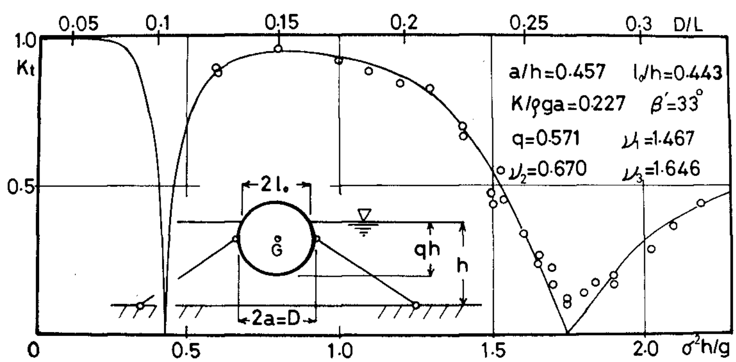


Fig.4-2 Transmission coefficient of moored floating cylinder

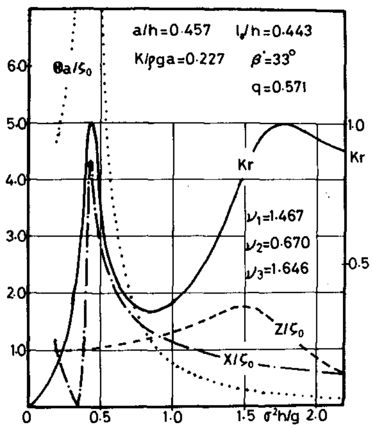


Fig. 4-3 Amplitude of motions and Reflection coefficient

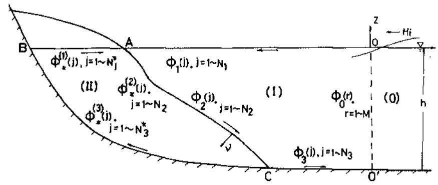


Fig. 5-1 Definition Sketch for Permeable Seawall

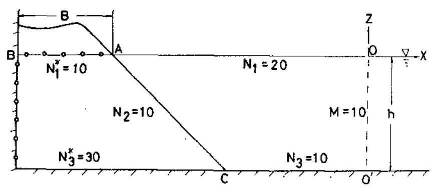


Fig. 5-2 Calculated Cross-Section of Permeable Seawall

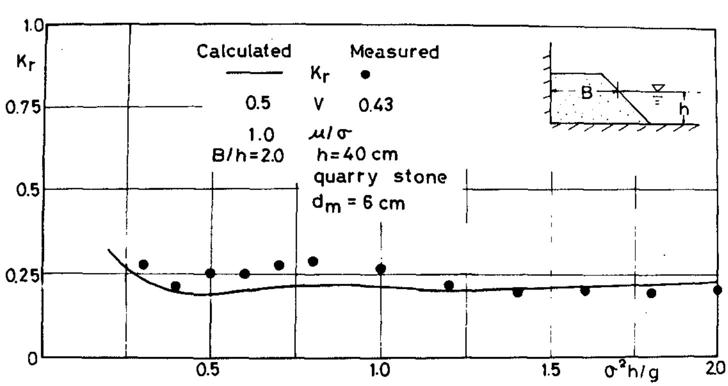


Fig. 5-3 K_r for Sloped-Face Seawall

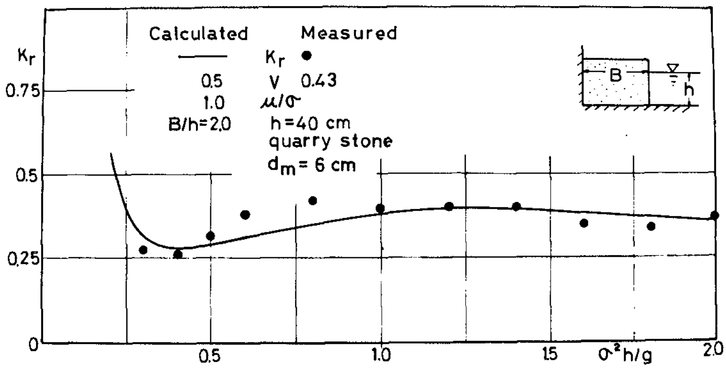


Fig. 5-4 K_r for Vertical-Face Seawall

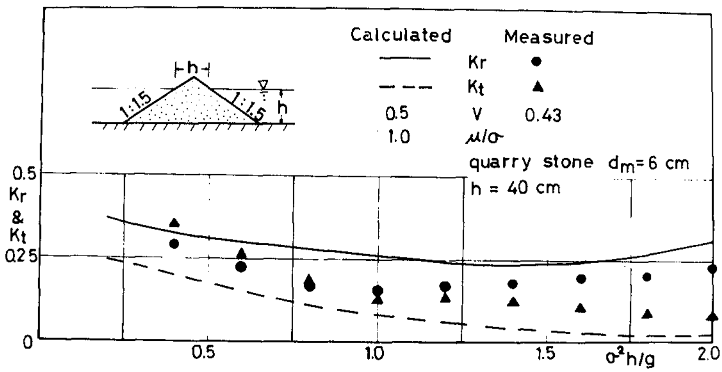


Fig. 5-5 K_r and K_t for Breakwater with Sloped Faces

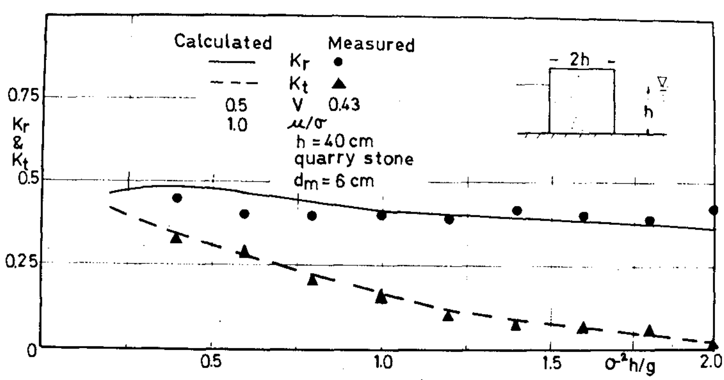
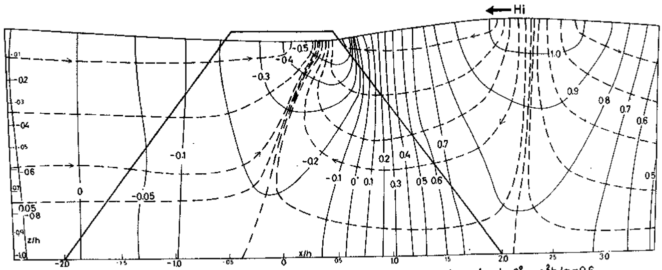
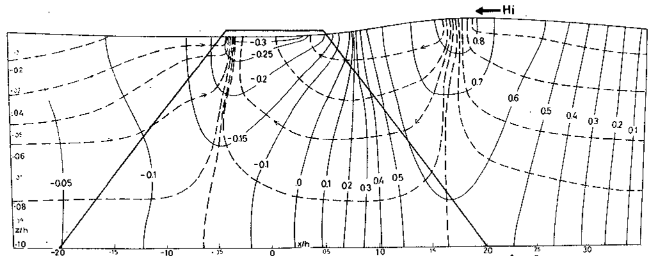


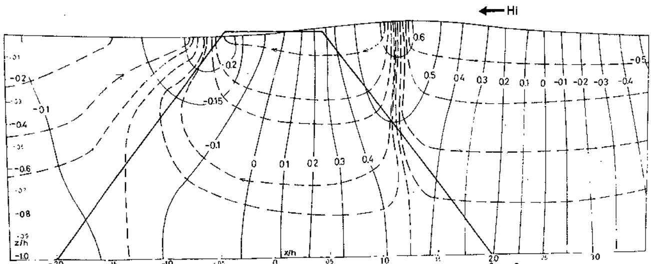
Fig. 5-6 K_r and K_t for Vertical-Face Breakwater



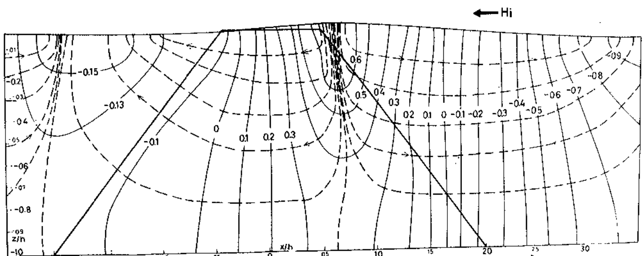
a Distribution of Velocity Potentials for Type A Breakwater at $\sigma t = 0°$, $\sigma^2 h/g = 0.6$



b Distribution of Velocity Potentials for Type A Breakwater at $\sigma t = 30°$, $\sigma^2 h/g = 0.6$



c Distribution of Velocity Potentials for Type A Breakwater at $\sigma t = 60°$, $\sigma^2 h/g = 0.6$



d Distribution of Velocity Potentials for Type A Breakwater at $\sigma t = 90°$, $\sigma^2 h/g = 0.6$

Fig. 5-7