CHAPTER 153

WAVE TRANSMISSION THROUGH TRAPEZOIDAL BREAKWATERS

by

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Introduction

Previous publications resulting from this study dealt with certain aspects of the interaction of normally incident waves with a porous structure. Madsen and White (1976a), hereafter referred to as (I), developed a semi-empirical procedure for the prediction of reflection coefficients of rough impermeable slopes. Madsen (1974) with discussion by Kondo (1975) and Closure by Madsen (1976), collectively referred to as(II)hereafter, developed an explicit solution for the transmission and reflection coefficients of homogeneous rectangular crib-style breakwaters. When viewing the interaction of incident waves with a trapezoidal, multilayered breakwater as a problem of energy dissipation, the problem treated in (II) may be regarded as an idealized analysis accounting for the internal dissipation of energy within the structure, whereas (I) may be regarded as an idealized analysis of the energy dissipation on the seaward face of the breakwater, i.e., the external energy dissipation. The present paper presents a synthesis of the results obtained in (I) and (II) into an approximate procedure for the prediction of wave reflection from and transmission through trapezoidal, multilayered breakwaters.

The present paper presents only the major points of the approximate procedure. Thus, extensive references to (I) and (II) are made and only the determination of the hydraulically equivalent breakwater is presented in some detail. For a presentation of the entire development of this model for the reflection and transmission characteristics of trapezoidal, multilayered breakwaters the reader is referred to Madsen and White (1975) or Madsen and White (1976b) where detailed numerical examples of the use of the procedure may also be found.

Description of the Approximate Procedure

The basic assumptions of the approximate procedure are, of course, those inherent in the analyses and procedures developed in (I) and (II):

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(a) Relatively long normally incident waves which may be considered adequately described by linear long wave theory.

(b) The incident waves do not break on the seaward slope of the breakwater, so that the external energy dissipation may be considered mainly due to bottom frictional effects.

(c) The cover layer on the seaward slope of the breakwater consists of natural stones, so that the empirical relationships developed in (I) may be considered valid.

With these assumptions stated the following procedure is suggested as being physically realistic although approximate in nature.

For most multilayered trapezoidal breakwaters the stone size in the layer under the cover layer of the seaward slope is small relative to the stone size of the cover layer. As a first approximation the structure may therefore be regarded as resembling an impermeable rough slope. Thus, with the incident wave characteristics and the stone size of the cover layer as well as the seaward slope of the trapezoidal breakwater specified, the procedure developed in (I) may be used to account approximately for the energy dissipation on the seaward slope, i.e., the external energy dissipation may be estimated. This energy dissipation accounts approximately for the dissipation of energy associated with the top layer of stones in the cover layer. The remaining wave energy may be expressed as the energy associated with a progressive wave of amplitude

$$a_I = R_I a_i \tag{1}$$

in which a_i is the amplitude of the actual incident wave and R_I is the reflection coefficient determined by the procedure developed in (I).

With the energy dissipated on the seaward slope accounted for, the remaining energy is partitioned between reflected, transmitted and internally dissipated energy. This partition of energy is the problem dealt with in (II), and it is evaluated by regarding the remaining energy as an equivalent wave of amplitude a_I normally incident on an equivalent homogeneous rectangular breakwater. The role of this homogeneous rectangular breakwater is to reproduce the internal energy dissipation associated with the trapezoidal, multilayered breakwater, i.e., the two breakwaters should in this sense be hydraulically equivalent. A rational method for obtaining a homogeneous, rectangular breakwater which is hydraulically equivalent to a trapezoidal, multilayered breakwater is developed in the following section based on steady flow considerations. By employing the procedure developed in (II) the partition of the remaining wave energy among reflected, transmitted and internally dissipated energy is therefore approximately evaluated by determining the reflection coefficient, R_{II}, and the transmission coefficient, T_{II}, of the hydraulically equivalent homogeneous rectangular breakwater subject to an equivalent incident wave of amplitude a_I.

Having now accounted for the external as well as for the internal energy dissipation, the amplitude of the reflected wave is found to be

$$|a_r| = R_{II}a_I = R_I R_{II} a_i \tag{2}$$

and the transmitted wave amplitude is

$$|a_t| = T_{II}a_I = R_I T_{II} a_i \tag{3}$$

The approximate values of the reflection and transmission coefficient, R and T, of a trapezoidal, multilayered breakwater are therefore

$$R = \frac{|a_r|}{a_i} = R_I R_{II} \tag{4}$$

and

$$T = \frac{|a_t|}{a_i} = R_I T_{II} \tag{5}$$

It should be emphasized that the approximate procedure outlined above is predictive in the true sense of the word, i.e., no other information than what may be expected to be available is needed.

Determination of the Equivalent Rectangular Breakwater

From the description given of the approximate method for obtaining the reflection and transmission coefficients of a trapezoidal, multilayered breakwater, the missing link for carrying out this analysis is the determination of the hydraulically equivalent homogeneous rectangular breakwater.

In Madsen and White (1976b) it was shown that a simple analysis, which essentially neglected unsteady effects, gave transmission and reflection coefficients equal to those obtained from the more complete analysis for structures of small width relative to the incident wavelength (II,eqs. 32 and 33). This observation suggests that a rational and reasonably simple determination of the hydraulically equivalent breakwater may be based on steady flow considerations. Therefore, a hydraulically equivalent breakwater is taken as the homogeneous rectangular breakwater which gives the same discharge, Q, as the discharge through the trapezoidal, multilayered breakwater. This definition will, according to the simple analysis presented in (II), preserve the equality of transmission coefficients for the two structures and hence essentially give the same internal dissipation. This definition of the equivalent breakwater is illustrated schematically in Figure 1.

Figure 1 shows schematically a trapezoidal, multilayered breakwater consisting of several different porous materials. These porous materials

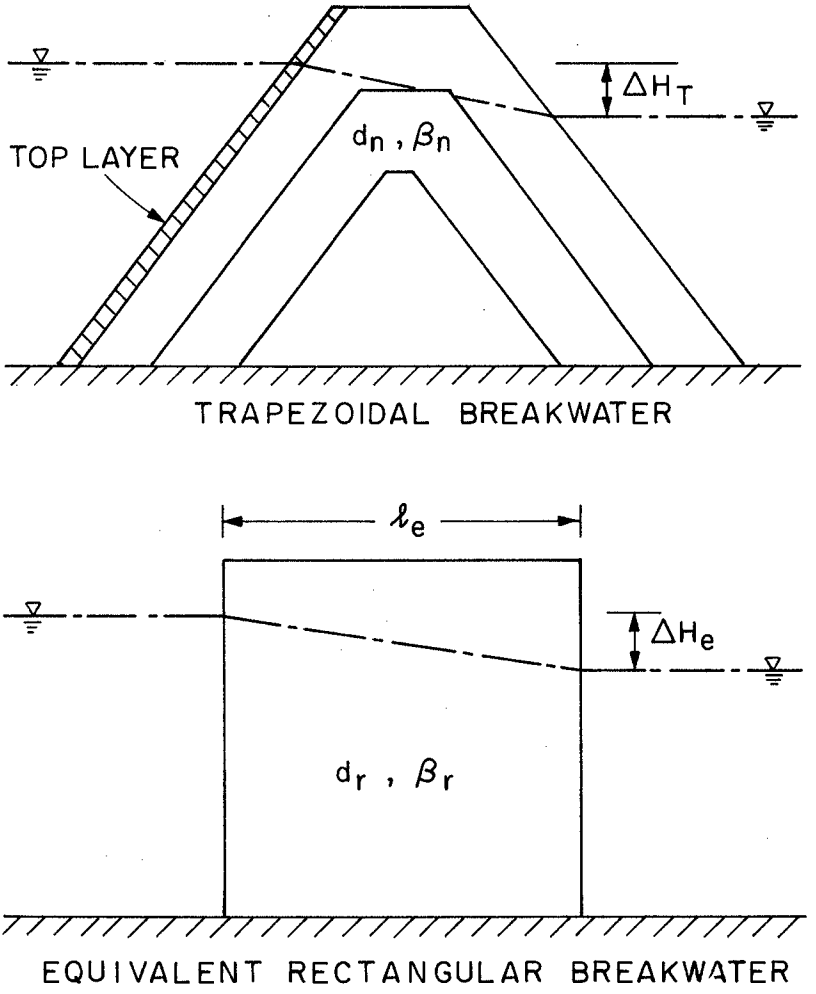

Figure 1. Definition Sketch of Trapezoidal Multilayered Breakwater and Its Hydraulic Equivalent Rectangular Breakwater.

are identified by their stone size, d_n, and their hydraulic characteristics, β_n, in the flow resistance formula (II, eq. 18). To keep the following determination of the equivalent breakwater reasonably simple, the flow resistance is assumed to be purely turbulent although in principle it is possible to perform the determination of the equivalent breakwater based on the more general form of the Dupuit-Forchheimer resistance formula. Since the energy dissipation associated with the top layer of stones on the seaward slope has been accounted for in the determination of the external energy dissipation, the rectangular homogeneous breakwater which accounts approximately for the internal dissipation should be hydraulically equivalent to the trapezoidal, multilayered breakwater with the top layer of cover stones removed.

The homogeneous rectangular breakwater consists of a reference material of stone size, d_r, and hydraulic characteristics, β_r. The reference material should be taken to be representative of the porous materials of the multilayered breakwater. In keeping with the assumption of long waves the flow is assumed to be essentially horizontal and the horizontal discharge velocity is found from

$$\rho g \frac{\Delta H_e}{\ell_e} = \rho \beta_r U^2 \quad , \tag{6}$$

in which ℓ_e is the width of the equivalent breakwater and ΔH_e is the head difference defined in Figure 1, ρ is the fluid density and g is the gravitational acceleration. The discharge per unit length is therefore obtained from equation (6) to be

$$Q = Uh_o = \left\{\frac{g\Delta H_e}{\beta_r}\right\}^{1/2} \frac{h_o}{\sqrt{\ell_e}} \tag{7}$$

in which h_o is the depth of water.

To evaluate the discharge per unit length of the trapezoidal, multilayered breakwater a horizontal slice of height, Δh_j, is shown schematically in Figure 2. This horizontal slice consists of segments of the different porous materials of lengths ℓ_n. From the assumption of purely horizontal flow it follows that the discharge velocity of the slice considered, U_j, must be the same in all segments and the total head loss across the breakwater must be equal to ΔH_T, the head difference shown in Figure 1. From this it is seen

$$\Delta H_T = \sum_n (\Delta H_n) \quad . \tag{8}$$

in which ΔH_n is the head loss associated with the segment of length, ℓ_n,

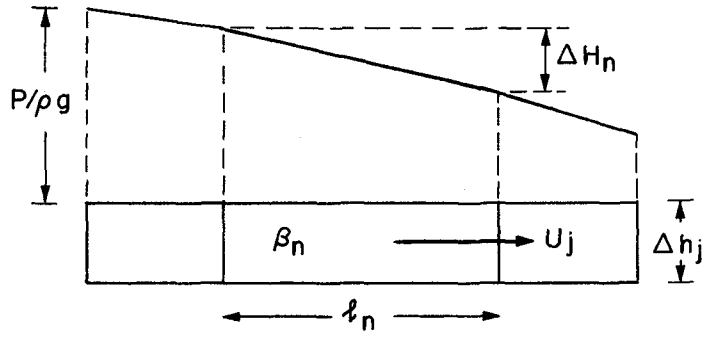

Figure 2. Horizontal slice of thickness, Δh_j, of multilayered breakwater.

and hydraulic characteristics, β_n. From equation (6) it is seen that

$$\Delta H_n = \beta_n \ell_n \frac{U_j^2}{g} = \beta_r \frac{U_j^2}{g} \left(\frac{\beta_n}{\beta_r} \ell_n\right) \quad , \tag{9}$$

in which β_r is the hydraulic characteristic of the reference material. Equation (8) may therefore be written:

$$\Delta H_T = \beta_r \frac{U_j^2}{g} \sum_n \left(\frac{\beta_n}{\beta_r} \ell_n\right) \quad , \tag{10}$$

in which the summation is carried out over the n different porous materials of the horizontal slice of thickness, Δh_j.

From equation (10) the discharge associated with the slice of thickness Δh_j is found to be

$$\Delta Q_j = U_j \Delta h_j = \left\{\frac{g \Delta H_T}{\beta_r}\right\}^{1/2} \frac{\Delta h_j}{\left\{\sum \left(\frac{\beta_n}{\beta_r} \ell_n\right)\right\}^{1/2}} \quad , \tag{11}$$

and by adding the contributions from all horizontal slices of the trapezoidal breakwater one obtains

$$Q = \sum_j \Delta Q_j = \left\{\frac{g\Delta H_T}{\beta_r}\right\}^{1/2} h_o \sum_j \left(\frac{1}{\left\{\sum_n \left(\frac{\beta_n}{\beta_r}\right)^{1/2} \ell_n\right\}} \frac{\Delta h_j}{h_o}\right) \quad . \tag{12}$$

Thus, requiring that the discharges per unit length given by equations (7) and (12) be identical, the width, ℓ_e, of the equivalent rectangular breakwater is

$$\ell_e = \left\{\sum_j \left(\frac{1}{\left\{\sum_n \left(\frac{\beta_n}{\beta_r}\right)^{1/2} \ell_n\right\}} \frac{\Delta h_j}{h_o}\right)\right\}^{-2} \left(\frac{\Delta H_e}{\Delta H_T}\right) \quad . \tag{13}$$

This equation shows that the width of the equivalent breakwater may be determined from knowledge of the configuration of the trapezoidal, multilayered breakwater and the corresponding head differences, ΔH_e and ΔH_T.

As described in the previous section the equivalent breakwater is subject to an equivalent incident wave of amplitude a_I given by equation (1). A simplified analysis of the interaction of incident waves and a rectangular homogeneous breakwater of small width relative to the length of the incident waves was presented in (II). This simplified analysis essentially neglected unsteady effects and any phase difference between the incident, reflected, and transmitted waves. The runup on the seaward slope of the hydraulically equivalent rectangular breakwater is taken as a representative value of the head difference, ΔH_e, across the equivalent breakwater. With this assumption, which neglects the influence of a transmitted wave of small amplitude, one obtains

$$\Delta H_e = (1 + R_{II})\, a_I = (1 + R_{II})\, R_I\, a_i \quad , \tag{14}$$

in which R_{II} is the reflection coefficient of the equivalent breakwater, determined by the procedures developed in (II).

The value of the head difference across the trapezoidal breakwater, ΔH_T, is in accordance with the argument presented for the equivalent rectangular breakwater taken as the runup on the seaward slope of the trapezoidal breakwater. This runup may in principle be determined by the procedure developed in (I). However, there is reason to believe that such an estimate, which would correspond to an impermeable slope, would be somewhat on the high side. In general one may, however, take

$$\Delta H_T = 2R_u\, a_i \quad , \tag{15}$$

where R_u is the best estimate available for the ratio of the runup to the

incident wave height $H_i = 2a_i$ for given slope characteristics. If R_u is taken as determined from Figure 3 in (I) the estimate of ΔH_T is expected to be conservative.

Equations (14) and (15) show that the ratio

$$\frac{\Delta H_e}{\Delta H_T} = \frac{(1 + R_{II})R_I}{2R_u} \quad , \tag{16}$$

is a function of the reflection coefficient, R_{II}, of the equivalent breakwater. Since this reflection coefficient cannot be determined until the width of the equivalent breakwater, ℓ_e, is known one is faced with a tedious iterative procedure. However, in most cases a sufficiently accurate estimate of R_{II} may be obtained by assuming initially that $\Delta H_e/\Delta H_T$ is unity and use this estimate along with the best estimate of R_u to obtain a new value of $\Delta H_e/\Delta H_T$ from equation (16).

To evaluate Equation (13) the hydraulic characteristics, β_n, of the various layers must be known. Since β_n in general is unknown the use of empirical relationship suggested in (II, eq. 40) is necessary. This relationship

$$\beta = \beta_o \frac{1-n}{n^3} \frac{1}{d} \tag{17}$$

is strongly dependent on the value of the porosity, n, which must be assumed. Since this empirical relationship is valid only for natural stones, in fact it is established for sands, its use is limited to rubble-mound breakwaters. The empirical, dimensionless constant β_o is a function of stone shape, etc., and $\beta_o = 2.7$, as suggested in (II) seems to be a reasonable value. Since equation (13) depends on the relative value of β_n/β_r the value of β_o is immaterial so long as it may be assumed the same for all layers.

Computation of Transmission and Reflection Coeffecients for Trapezoidal, Multilayered Breakwaters

To apply the approximate method outlined in the preceeding sections it is assumed that the incident wave characteristics are known, i.e., $H_i = 2a_i$, h_o, T or L are given. The procedure is therefore

a. Determination of the External Dissipation. The first step is to estimate the external energy dissipation on the seaward slope using the procedure developed in (I). To perform this analysis the necessary information, in addition to the incident wave characteristics, is

$$\text{Surface Roughness} = d_I \; ; \; \text{Seaward Slope} = \tan\beta_s \tag{18}$$

This information is sufficient for carrying out the procedure developed in (I) for the determination of the reflection coefficient of impermeable rough slopes, R_I. Since the rate of energy dissipation on a rough impermeable slope is related to its reflection coefficient this step in the analysis may be viewed as an approximate evaluation of the energy dissipation on the seaward slope of a trapezoidal, multilayered breakwater.

The empirical relationships for the frictional effects established in (I) are valid only for slopes whose roughness is adequately modeled by gravel, i.e., natural stones, which is reflected in the previously stated assumption, (c).

b. Determination of the Internal Dissipation. Having determined the energy dissipation on the seaward slope, the remaining wave energy is equivalent to an equivalent incident wave of amplitude a_I given by equation (1). The partition of the remaining energy among reflected, transmitted and internally dissipated energy is estimated from the procedures developed in (II). To perform this analysis the hydraulically equivalent breakwater is determined as outlined in the previous section. The information necessary for the determination of the hydraulically equivalent rectangular, homogeneous breakwater is

Breakwater Geometry;

Material Properties (including porosity); (19)

Runup on trapezoidal breakwater.

The use of equation (17) to estimate the hydraulic characteristics of a porous material reduces the necessary information to generally available quantities except for the porosity and the runup on the seaward slope in order for equation (13) to be evaluated. Use of $n \approx 0.4$ seems reasonable in conjunction with $\Delta H_e/\Delta H_T = 1$ for preliminary calculations.

With these assumptions the equivalent breakwater is readily determined and the explicit procedure developed in (II) may be used to obtain an estimate of the partition of the remaining energy among reflected, transmitted and internally dissipated energy, i.e., the reflection and transmission coefficients R_{II} and T_{II}, respectively, of the equivalent breakwater may be obtained.

c. Determination of the Transmission and Reflection Coefficient of Trapezoidal, Multilayered Breakwaters. From the results obtained in steps a and b it is now possible to estimate the transmission and reflection coefficient of a trapezoidal, multilayered breakwater since the external as well as the internal energy dissipation has been accounted for. From the description of the procedure given previously it follows that the transmitted and reflected wave amplitudes, $|a_t|$ and $|a_r|$, are given by

$$|a_t| = T_{II}a_I = T_{II}R_I a_i \tag{2}$$

and

$$|a_r| = R_{II}a_I = R_I R_{II} a_i \quad . \tag{3}$$

The transmission coefficient, T, and reflection coefficient, R, are therefore obtained from

$$T = \frac{|a_t|}{a_i} = T_{II} R_I \tag{4}$$

and

$$R = \frac{|a_r|}{a_i} = R_I R_{II} \quad . \tag{5}$$

Comparison Between Predicted and Observed Transmission and Reflection Coefficients of a Trapezoidal, Multilayered Breakwater

The preceding section has illustrated the use of the approximate method for the determination of transmission and reflection coefficients of trapezoidal, multilayered breakwaters. This procedure was followed choosing values of the incident wave characteristics and breakwater characteristics corresponding to the laboratory experiments performed by Sollitt and Cross (1972), and the predictions may therefore be compared directly with the experimentally observed values of the transmission and reflection coefficients given by Sollitt and Cross (1972, App. G). This comparison between predicted and observed transmission and reflection coefficients is shown in Figure 3, where the values of T and R are plotted against the incident wave steepness, H_i/L, corresponding to a value of $h_o/L = 1/12$, i.e., relatively long waves as assumed throughout this paper.

From the comparison presented in Figure 3 the predicted reflection coefficients are in excellent agreement with the observed reflection coefficients for lower values of the incident wave steepness. For larger values of the incident wave steepness the predicted reflection coefficient is seen to increase slightly whereas the observed reflection coefficients exhibit a decreasing trend with increasing wave steepness. As discussed by Madsen and White (1975 and 1976b) this trend of the experimental reflection coefficients is generally observed and may be partly due to experimental errors in the determination of the reflection coefficient. This was discussed briefly in (I) and in detail in Madsen and White (1975 and 1976b).

The transmission coefficients predicted based on the assumption $\Delta H_e/\Delta H_T = 1$ are seen to be lower than the experimentally obtained values. This, of course, is the expected type of discrepancy since the runup on the seaward slope of the trapezoidal breakwater is almost certain to exceed the runup on the equivalent rectangular breakwater. Adopting the theoretical value of the runup, R_u, on the trapezoidal breakwater predicted by the procedure developed in (I) is expected to give transmission coefficients

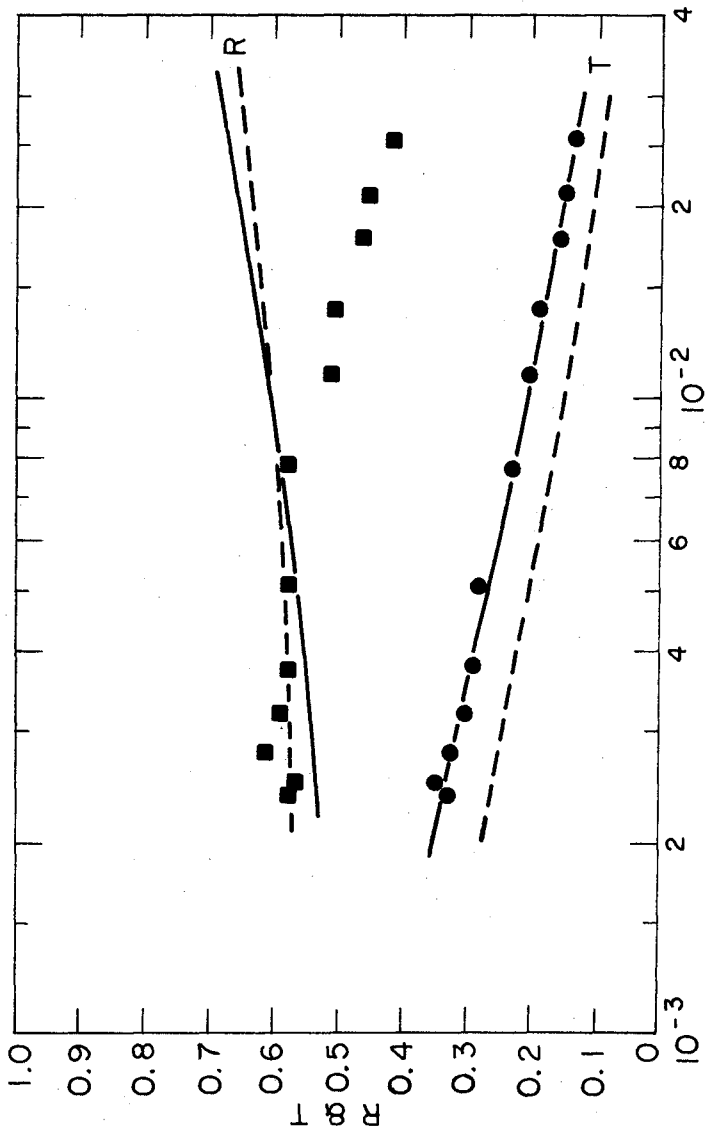

Figure 3. Comparison of predicted and observed reflection and transmission coefficient of trapezoidal, multilayered breakwater tested by Sollitt and Cross (1972, App. G, kh = 0.5). ■ Experimental reflection coefficient, ● experimental transmission coefficient, — — prediction based on $\Delta H_e/\Delta H_T = 1$, ——— prediction based on $\Delta H_e/\Delta H_T$ given by equation (16)

slightly on the high side. This anticipated behavior is not exhibited by the predicted transmission coefficients plotted in Figure 3. In fact, the agreement between observed and predicted transmission coefficients is excellent.

A slightly different estimate of the runup on the seaward slope of a trapezoidal breakwater may be obtained by adopting, for example, the results obtained by Jackson (1968), who reported values of R_u approximately equal to unity for test conditions similar to those of Sollitt and Cross (1972). In the present case this value of R_u would result in a slightly lower prediction of the transmission coefficient than the prediction indicated by the full line in Figure 3.

The procedure developed here for the prediction of transmission and reflection coefficients of a trapezoidal, multilayered breakwater did not rely on the experimental data shown in Figure 3 to obtain a "good fit." The overall comparison between predicted and observed transmission and reflection coefficients, which is analogous to the comparison given by Sollitt and Cross (1972, Fig. 4-14), must therefore be considered very good.

Summary and Conclusions

This paper presents the synthesis of the results of an analytical study of the reflection and transmission characteristics of porous rubble-mound breakwaters. An attempt was made at making the procedures entirely self-contained by introducing empirical relationships for the hydraulic characteristics of the porous material (II) and by establishing experimentally an empirical relationship for the friction factor that expresses energy dissipation on the seaward slope of a breakwater, (I).

The results are presented in graphical form and require no use of computers, although the entire approach could be programmed. The procedures were developed in such a manner that the information required to carry out the computations can be expected to be available. Thus, for a trapezoidal, multilayered breakwater subject to normally incident, relatively long waves the information required is:

(a) Breakwater configuration: breakwater geometry and stone size and porosity of the breakwater materials

(b) Incident wave characteristics: wave amplitude, period, and water depth.

Only the porosity of the breakwater materials may be hard to come by. It is recommended that the sensitivity of the results to the estimate of the porosity, n, be investigated.

The hydraulic flow resistance in the porous medium is expressed by a Dupuit-Forchheimer relationship and empirical formulas are adopted. The investigation shows that reasonably accurate results are obtained by taking

$$\beta_o \simeq 2.7$$
$$\alpha_o \simeq 1150 \qquad (20)$$

in equation (40) of (II). To estimate reflection and transmission characteristics of a prototype structure only the value of β_o needs to be known. For laboratory experiments the value of the ratio, α_o/β_o is important in assessing the influence of scale effects. In a laboratory setup it is possible to determine the best values of α_o and β_o from the simple experimental procedure used by Keulegan (1973). Thus, it was found that the porous materials tested by Sollitt and Cross (1972) showed a value of $\alpha_o = 2{,}700$, a better value than that given by equation (20). However, the important thing to note is that the analysis carried out in (II) presents a method for assessing the severity of scale effects in hydraulic models of porous structures. The empirical relationships for the flow resistance of porous materials have been demonstrated to be fairly good for porous materials consisting of gravel-size stones, diameter less than 2 inches (5 centimeters), which is a considerable extension of the conditions from which they were derived (sand-size). The use of the formulas for rubble-mound breakwaters is, however, a further extension and caution is recommended.

The energy dissipation on a rough, impermeable slope was investigated in (I). The experimental investigation revealed the need for an accurate method for the determination of reflection coefficients from experimental data. The simple procedure of seeking out the locations where the wave amplitudes are maximum and minimum, respectively, may lead to reflection coefficients which are much too low, unless the recorded surface elevation is analyzed and only the amplitude of the first harmonic motion is used to determine the reflection coefficient. Accurately determined reflection coefficients for slopes with roughness elements consisting of gravel led to an empirical determination of the friction factor (I, eqs. 32 and 34) expressing the energy dissipation on a rough slope due to bottom friction. Adopting this empirical relationship a procedure for estimating the reflection coefficient of rough impermeable slopes was developed. This procedure was quite accurate in reproducing the experimentally obtained reflection coefficients in a separate set of experiments. The procedure for the determination of the reflection coefficient of rough impermeable slopes is limited to slopes having roughness elements consisting of natural stones. To make the procedure generally applicable, empirical relationships for the friction factor should be determined for slopes whose roughness elements consist of models of concrete armor units.

The synthesis of the investigation presented in this paper, is the development of an approximate procedure for the prediction of the reflection and transmission characterisitcs of trapezoidal, multilayered breakwaters. This procedure is entirely self-contained and yields excellent results when compared with the model scale experimental results obtained by Sollitt and Cross (1972).

It is emphasized that the analytical model for the reflection and transmission characteristics of trapezoidal, multilayered breakwaters developed here needs further verification before it can be used with complete confidence. However, the good agreement between predictions and observations exhibited in Figure 3 is encouraging and does indicate that a simple analytical model which may be used for preliminary design of rubble-mound breakwaters has been developed.

To improve the physical realism of the approximate model for the interaction of waves and porous structures a more accurate description of the flow over the sloping seaward face of the porous breakwater should be developed. Development of such an analytical model is presently being pursued and should lead to improved results, in particular, in the prediction of runup on the permeable seaward slope. In addition an experimental investigation is called for to resolve the problem of whether or not the discrepancy between predicted and observed reflection coefficients, exhibited in Figure 3, is due mainly to experimental errors.

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