CHAPTER 139

NEAR-BOTTOM WATER MOTION UNDER OCEAN WAVES

by

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SUMMARY

A two-year ocean experiment involving wave-induced forces on a test pipe mounted on the sea floor [Grace and Nicinski (1976)] involved the measurement of various quantities other than the pipe forces per se. A pair of these involved surface wave characteristics and wave-induced water motion at the level of the pipe centerline but off to one end of the pipe. These wave-kinematics data have been combined, and the results of this work make up this paper in which the emphasis is on the deterministic approach to data interpretation. Presented are comparisons of the velocity and acceleration data with the predictions of Airy and stream function theories plus discussion of the dispersion of the field data. The primary intent of the paper is to suggest to designers of bottom-laid structures, such as pipes, how values of the peak velocity and maximum acceleration of the water motion associated with a non-breaking design wave of specified characteristics can be chosen.

TEST SITE AND WAVE CONDITIONS

A site was chosen on a moderately level area of coral rock bottom 1400 feet from the reclaimed shoreline near Kewalo Basin, the fishing and tour boat harbor for Honolulu, where the water depth was 37 feet. Peak-to-trough tidal variations in Hawaiian waters are in the 1-to-2-foot range, so that the depth can be considered constant for all practical purposes.

We installed at this test site various structures. The major one related to the topic of this paper consisted of a heavy base composed of steel I beams and a wave mast bolted to it. This mast consisted of two parts; the lower one remained vertical and occupied approximately half the water column, whereas the upper one tilted down when not deployed and vertically upwards when a buoyancy chamber mounted permanently on it was blown. Both 3-inch and 2-inch-diameter steel pipe were used in the mast. The graduated upper part of the mast in part resembled the mast of a sailboat. A line over a pulley enabled us to pull the top of a 15-foot-long, wire-wound electrical wave staff to the summit; an acme thread bracket well down the tilting mast portion permitted us to then tighten the staff parallel to the mast. A cable ran from an oscillator at the top of the staff over the water to the project boat where suitable power supply and recording instrumentation were available. The boat, incidentally, was a 31-foot-long catamaran with 12-foot beam, an excellent work platform for our purposes.

A ducted-impeller velocity sensor was attached, by means of U bolts, to a small pipe cantilevered out from the end of the base for the 16-inch test pipe. The sensor was always located 15 inches from the bottom and 38 1/2 inches from

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the end of the test pipe. The latter dimension was a compromise (for the pipe force work) between having the sensor close enough to represent the water motion incident upon the test pipe and yet far enough away not to be influenced by water motion around the end of it. A cable extended from the current meter up to associated d.c. power supply and recorder in the instrument shack on the boat.

We gathered concurrent data for waves and associated water motion on twelve different occasions, and the peak wave height measured overall was about 12 1/2 feet. However, we did encounter waves too big to measure accurately; one of these, a fluke, overtopped everything by quite a margin, severely bent the wave mast even though stiffened by a stout line from its top, and gave us all a fright; another snapped off the whole (repaired) mast at the base, along with all five stays set seaward, and wrote an end to wave-measuring for the remainder of the project.

Our test waves arrived between March and September of 1975 and 1976. The origin of these swells was in the Southern Hemisphere chiefly in the Tasman Sea as well as in the Pacific and Great Southern Ocean east of New Zealand. Swell from the latter source is generally the larger in Hawaii. Measured periods ran from 7 to 19 seconds, but by far the bulk of the observations lay in the range between 12 and 17 seconds.

PAST RELATED WORK

We were by no means the first to carry out at-sea measurements of waves and wave-induced water motion in the sea. Various types of field investigations have been run, and these are typified by Inman and Nasu (1956), Shonting (1967a, b) and Thornton and Krapohl (1974). A considerable amount of laboratory research has also been carried out on wave-induced water motion. Perhaps the most significant of such studies was the work of Goda (1964), and a particularly relevant part of his data, as far as this project is concerned, is shown in Tables 1 and 2. These data were kindly supplied to the writer by Goda for reworking.

The variables shown in Tables 1 and 2 are as follows: $\bar{H}$ is the average and s_H the unbiased standard deviation of a sample of ten ostensibly identical waves of period T; d is the water depth; g is the acceleration due to gravity; c_{Airy} and L_{Airy} are the wave celerity and wave length predicted by Airy wave theory; $u_{c_{Airy}}$ and $u_{c_{s.f.}}$ are the peak horizontal flow speeds under the waves, for a measurement location off the bottom of 0.13 times the depth, predicted by Airy and stream function theories; $\bar{U}_c$ and s_u are the mean and standard deviation of the sample of ten measured peak flow velocities for each wave, and r_{HU} is the product-moment correlation coefficient between measured wave heights and velocities.

Table 1: Characteristics of Selected Waves from Goda (1964)

Test No.	$\bar{H}$ (cm)	$\bar{H}/d$	s_H (cm)	T (sec)	$T\sqrt{g/d}$	c_{Airy} (cm/sec)
44	36.1	0.24	0.6	2.38	6.09	316
45	52.7	0.35	1.0	2.36	6.04	315
46	58.5	0.39	1.1	2.43	6.21	318
47	34.6	0.23	0.7	2.98	7.62	340
48	55.9	0.37	0.8	3.14	8.03	344
49	28.6	0.19	0.2	4.15	10.61	361
50	47.0	0.31	0.3	4.16	10.64	361
51	64.5	0.43	0.6	4.17	10.66	361
52	57.3	0.38	0.8	5.77	14.76	372
53	64.3	0.43	0.6	7.89	20.18	377
54	77.2	0.51	1.2	7.90	20.20	377

Table 2: Peak Velocity Data of Selected Waves from Goda (1964)

Test No.	d/L_{Airy}	$u_{c_{Airy}}$ (cm/sec)	$u_{c_{s.f.}}$ (cm/sec)	$\bar{U}_c$ (cm/sec)	s_u (cm/sec)	$\bar{U}_c/u_{c_{Airy}}$	r_{HU}	$s_{U/c_{Airy}}$
44	0.20	30.0	31.8	39.0	1.2	1.30	0.20	0.0039
45	0.20	43.4	44.7	49.7	2.1	1.14	0.17	0.0067
46	0.19	49.7	50.6	55.7	1.7	1.12	0.33	0.0055
47	0.15	34.3	36.0	37.4	1.7	1.09	-0.58	0.0051
48	0.14	57.1	57.9	66.8	3.3	1.17	-0.41	0.0096
49	0.10	32.3	37.9	35.7	1.6	1.11	0.04	0.0045
50	0.10	53.2	62.1	62.9	2.9	1.18	0.36	0.0081
51	0.10	73.1	78.8	80.0	2.1	1.10	0.56	0.0057
52	0.07	68.9	84.4	73.8	5.2	1.07	-0.29	0.0139
53	0.05	79.6	98.6	84.0	4.3	1.06	-0.06	0.0113
54	0.05	95.6	111.4	99.7	2.4	1.04	0.10	0.0063
Average	--	--	--	--	--	1.12	0.04	0.0073

Goda's data appear to show clearly that the predictions of the stream function theory [Dean (1965, 1974), Dalrymple (1974)] are too high when long waves are involved. Le Méhauté et al (1968) have concluded, on the basis of laboratory experiments, that the Airy theory provides the best estimate of near-bottom peak velocity of a number of classical theories.

BACKGROUND TO DATA

The pulse output of the velocity sensor and the stepped history of the wave staff were recorded on the same d.c. chart recorder. We found it easier to calibrate the staff against the graduated wave mast than by noting readings on the digital readout which was updated at a rate of ten per second. There were various problems with the wave staff but by and large these were compensated for in some way - e.g. by using the staff in a potentiometer arrangement, by reading trough values from the mast in cases when there were aberrations in wave staff behavior, and in reading crest levels from the mast when a particularly large wave went above the top of the staff.

The velocity history, a series of pulses whenever one blade magnet of the three-bladed impeller passed a reed switch enclosure on the side of the duct, was translated into a velocity history as follows. A sequence of between-pulse times was first lightly smoothed. The starting point for the translation of pulse history into a velocity trace was at the shortest interval or intervals within a trough or crest where the steady-state calibration of the sensor should be best represented. The rectangular-area rule was used until the between-pulse time changed. From then on the trapezoidal-area rule was followed - until the last (or first) pulse before (after) the change in flow direction. Points worked forward in time from a trough and backwards in time from a following crest were joined with a smooth curve whose maximum slope provided our best estimate of the peak acceleration under the wave.

The major advantage to the ducted meter is that it works - and one can see easily whether it is working or not. However, one pays a price for this. First, there is the time-consuming translation of the pulse history into a smooth velocity trace as outlined above. A second problem could concern off-angle use of the ducted meter. However, we oriented our meter directly into the approaching swell (as judged by the feel of the water motion and the movement of sand along the sea floor) and so there should be a minimal problem of this nature.

There are mixed emotions about the electromagnetic current meter. Some researchers feel that it is a first-class measuring device and various others are diametrically opposed. On one occasion we borrowed an electromagnetic sensor (spherical) and its owner-operator in order to compare the output of the ducted and electromagnetic types. I mounted these two meters side by side myself. Subsequently, the ducted meter ran perfectly; the electromagnetic meter yielded a trace that was very much in step with the output of the ducted meter but one that had nothing to do with the supposed calibration of the instrument.

VELOCITIES: DETERMINISTIC APPROACH

The comparisons between theoretical and measured kinematics data are shown in Figures 1 through 4. The sample size for the velocity data is 236, for the acceleration data 171.

During the final month and a half of the project, much of the higher velocity information in Figures 1 and 2 was obtained. Before these data were gathered and processed, it appeared that Airy theory provided predictions that were not only very good but also superior to those of the stream function theory. However, it is clear from Figures 1 and 2 that Airy theory tends to underestimate the high speeds while the stream function theory overestimates them. The Airy theory still provides the lower standard error of estimate, s_e (0.43 versus 0.54 fps), for the data, but such a measure of fit is of course biased by whatever data constitute the sample - i.e. high or low speeds.

Even if the data for higher flow speeds (Figure 1) were to lie along the line as for the lower speeds, it is still obvious that the peak flow speed for any particular wave can exceed that predicted by Airy theory by approximately up to 40%. In the past it was suggested [e.g. Grace and Rocheleau (1973)] that a powerful approach to the prediction of extreme peak flow speeds would be to use the Airy theory predictions and then to tack on a probabilistically-chosen residual. It was suggested that the distribution of residuals be considered Gaussian with zero mean and a standard deviation given by 0.007 times the Airy theory celerity of the wave. Although the number 0.007 was based on field work, the same figure applies in an average sense to the data of Goda (1964) in Table 2.

Figure 1 indicates, however, that for ocean swell approaching those used in design the above approach may be unworkable. For this reason, since the stream function theory predictions provide a vague upper boundary for the velocity data (Figure 2), it is suggested that the predictions of this theory be used as the best theoretical estimate of $U_{max_{max}}$, the value near the top of the distribution for the true peak horizontal, near-bottom flow speed. An alternate approach would be to use 1.4 times $u_{max_{Airy}}$.

ACCLERATIONS

The standard errors of estimate for the acceleration predictions of Airy theory (Figure 3) and the stream function thoery (Figure 4) are respectively 0.56 and 0.48 ft/sec^2. It is clear from the Figures that both theories underestimate $\dot{U}_{max}$ for the higher waves, a failing of some considerable import in engineering design situations. A line given by the equation $\dot{U}_{max} = 2.5\ \dot{u}_{max_{Airy}}$ provides an upper envelope to the Figure 3 data, and it is provisionally suggested that non-breaking wave near-bottom accelerations for design ($\dot{U}_{max_{max}}$) be adopted by using this equation when $0.05 \leqslant d/L_{Airy} \leqslant 0.10$.

Graphical differentiation of the near-bottom (S/d = 0.05) velocity plots in Iwagaki and Sakai (1970) have resulted in the data shown in Table 3.* There is substantial scatter in these data due to the double-humped nature of some of the waves and conceivably also to the technique used by the researchers in linking forward-flow and rearward-flow parts of the hot film anemometer velocity traces. But it is fairly clear that there is a general tendency for R to grow with increasing d/L_{Airy} and that the numbers obtained largely mirror those found in this field investigation.

VELOCITIES: STOCHASTIC APPROACH

Consider three ocean waves with the same gross characteristics of height, period and water depth. Classical wave theory predicts the same near-bottom peak flow velocity and acceleration for all three waves.

There is virtually an infinity of possible water surface configurations that could exist between vertical constraints (wave height) and horizontal constraints (wave length, inferred from wave period and water depth), and there is no reason to suppose that three real waves of identical gross characteristics would have the same near-bottom maximum flow velocity and acceleration. See Figure 5.

A sample of n waves with the same H, T and d would give n values of U_{max} and n values of $\dot{U}_{max}$. This dispersion can be accounted for with a deterministic wave model either by taking account of the actual surface profile (rare, and not applicable in design) or by using the theoretical prediction as an initial estimate and then adding on a probabilistically-chosen residual as remarked earlier.

The standard modern method for accounting for dispersion, however, involves the use of a probabilistic approach, the so-called Gaussian wave model. An infinity of independent, infinitesimal-mean-square sinusoids is assumed to be propagating in the same direction. A Gaussian distribution then applies to the overall surface wave ordinate; the same probability density function also applies to the (horizontal, near-bottom) flow speeds and accelerations due to the linear relationship between such quantities and the wave ordinate according to the Airy theory which applies to sinusoidal waves.

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*Other laboratory data such as those of Elliot (1953) (referenced in Wiegel (1964)), Le Méhauté et al (1968) and Tsuchiya and Yamaguchi (1972) do not yield near-bottom acclerations.

Table 3: Iwagaki and Sakai (1970)

Acceleration Data and Airy Theory Predictions

(d = 16.0 cm, S = 0.8 cm)

H (cm)	T (sec)	H/d	d/L_{Airy}	U_{max} (cm/sec^2)	$u_{max_{Airy}}$ (cm/sec^2)	R**
6.72	1.03	0.42	0.138	132.7	127.7	1.04
6.86	1.30	0.43	0.104	144.5	113.3	1.28
6.04	1.58	0.38	0.084	164.0	85.9	1.91
7.21	1.83	0.45	0.072	105.3	90.8	1.16
7.05	2.08	0.44	0.063	133.8	79.4	1.69
4.89	2.39	0.31	0.055	47.6	48.3	0.99
5.06	2.63	0.32	0.049	104.3	45.8	2.28
4.13	2.84	0.26	0.045	101.0	34.9	2.89
4.11	3.07	0.26	0.042	69.2	32.0	2.16
3.66	3.40	0.23	0.038	86.3	25.9	3.33

** $R = U_{max} \Big/ u_{max_{Airy}}$

The mean-square spectral density, commonly called the spectrum, of surface wave ordinate displays the absolute amounts of mean-square value contributed by the different-frequency components in the sea surface. Such a measured spectrum can be transformed analytically into one for the near-bottom water particle flow speed by using the (frequency-dependent) Airy theory transformation factor.

In Figure 6 a surface wave ordinate spectrum is presented. In addition, there are two near-bottom flow speed spectra shown, one theoretical and the other measured. It is clear that the theoretical curve has only about half the mean-square content of the measured one; in addition the theoretical peak is about 25% less than that for the measured data. Thus, although the probabilistic wave model yields dispersion, its predictions for relatively shallow water swell are out of line with reality, at least according to our results.

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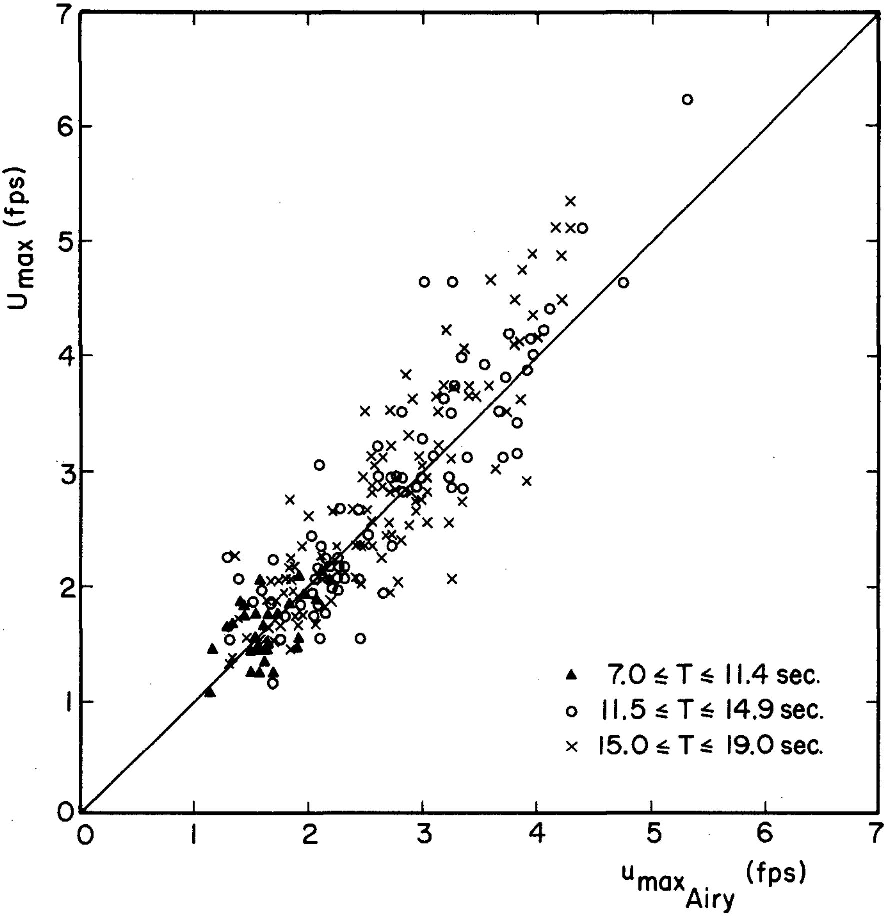

Figure 1: Comparison of Measured Velocity Data with Airy Theory Predictions

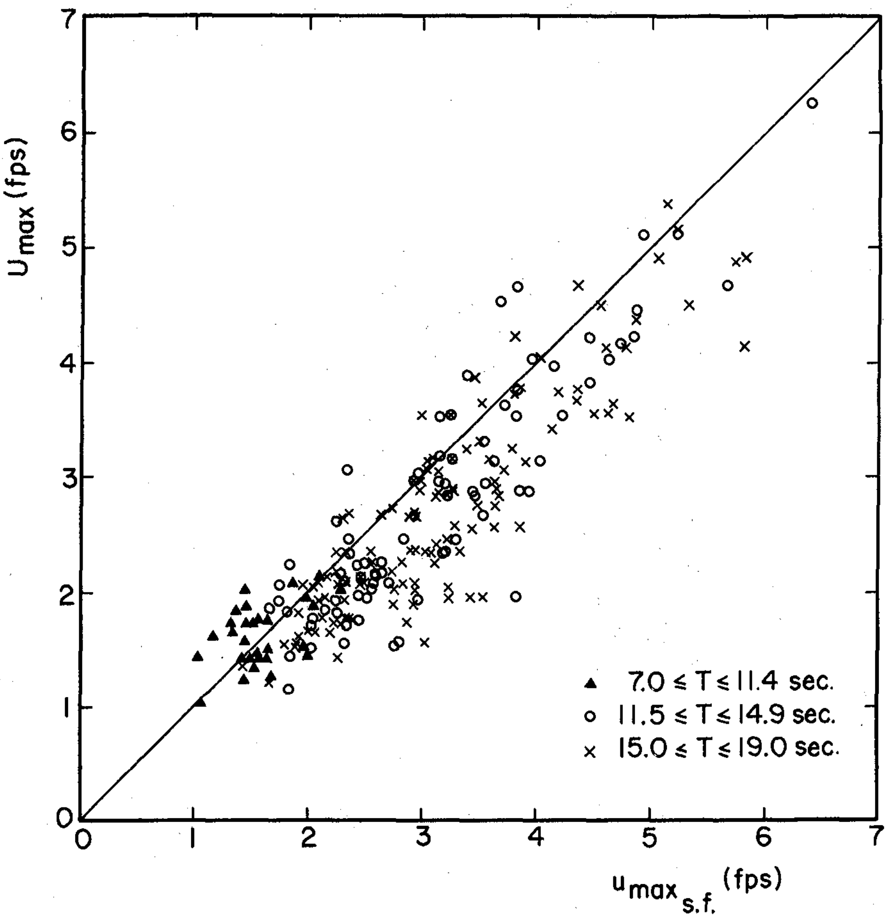

Figure 2: Comparison of Measured Velocity Data with Predictions of Stream Function Theory

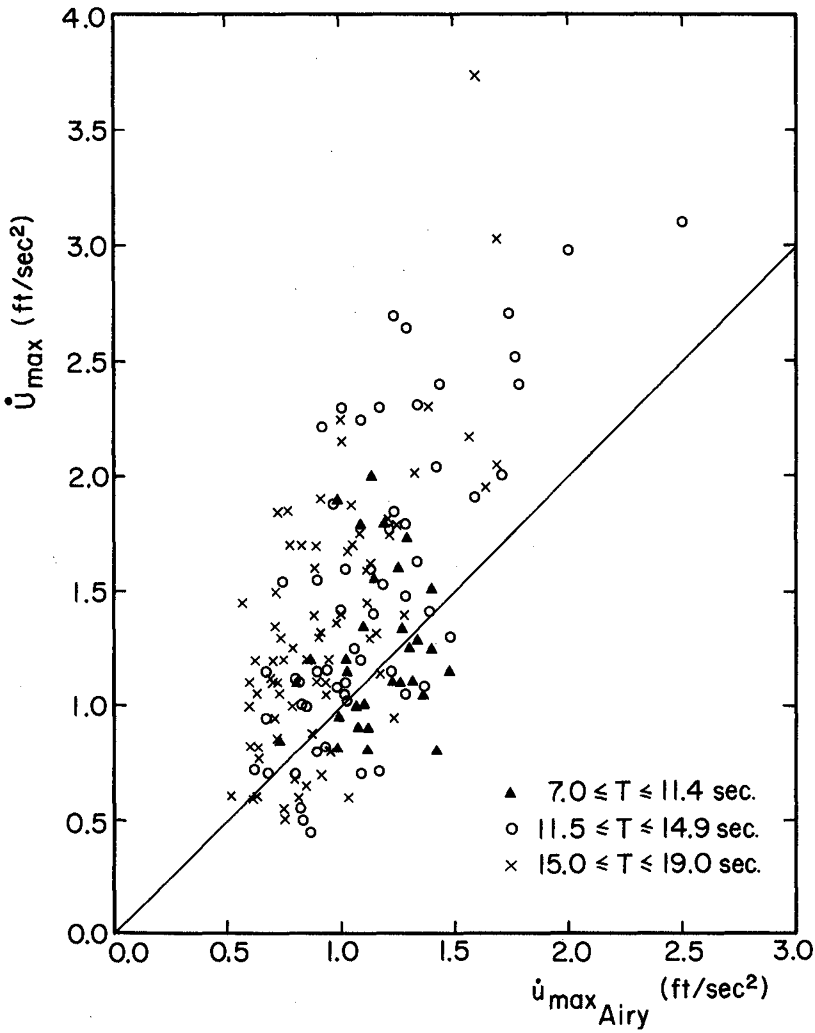

Figure 3: Comparison of Measured Acceleration Data with Airy Theory Predictions

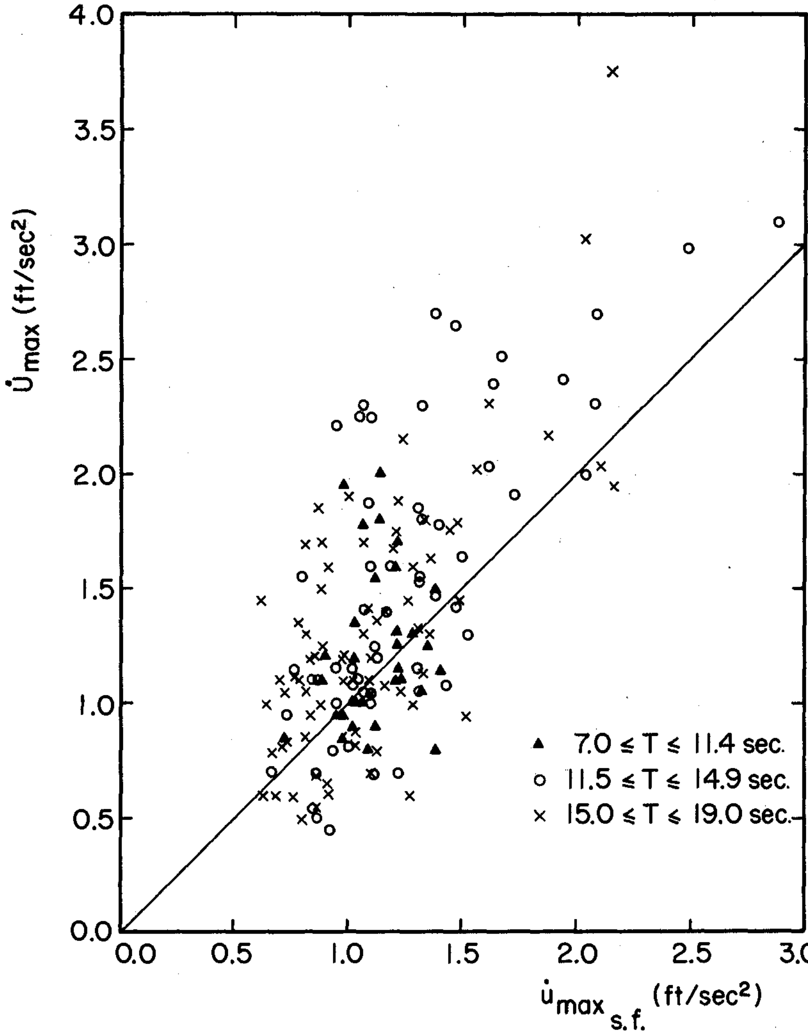

Figure 4: Comparison of Measured Acceleration Data with Predictions of Stream Function Theory

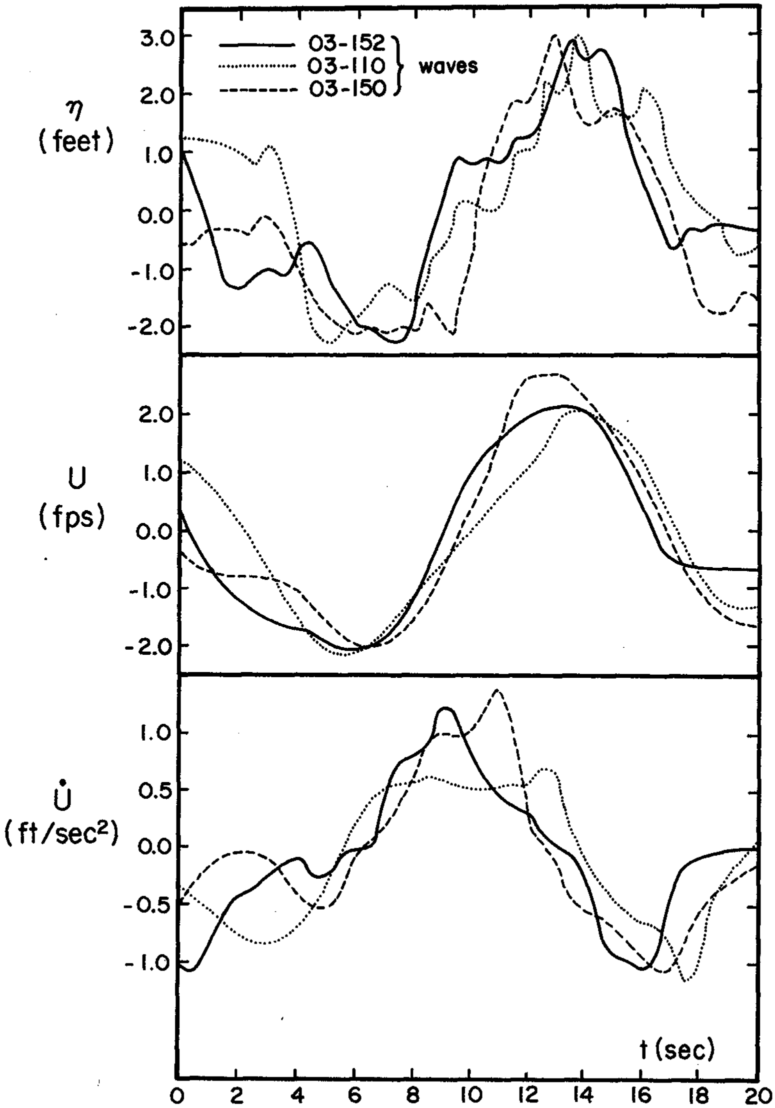

Figure 5: Measured Ordinate, Velocity and Acceleration Traces for Three 'Identical' Waves

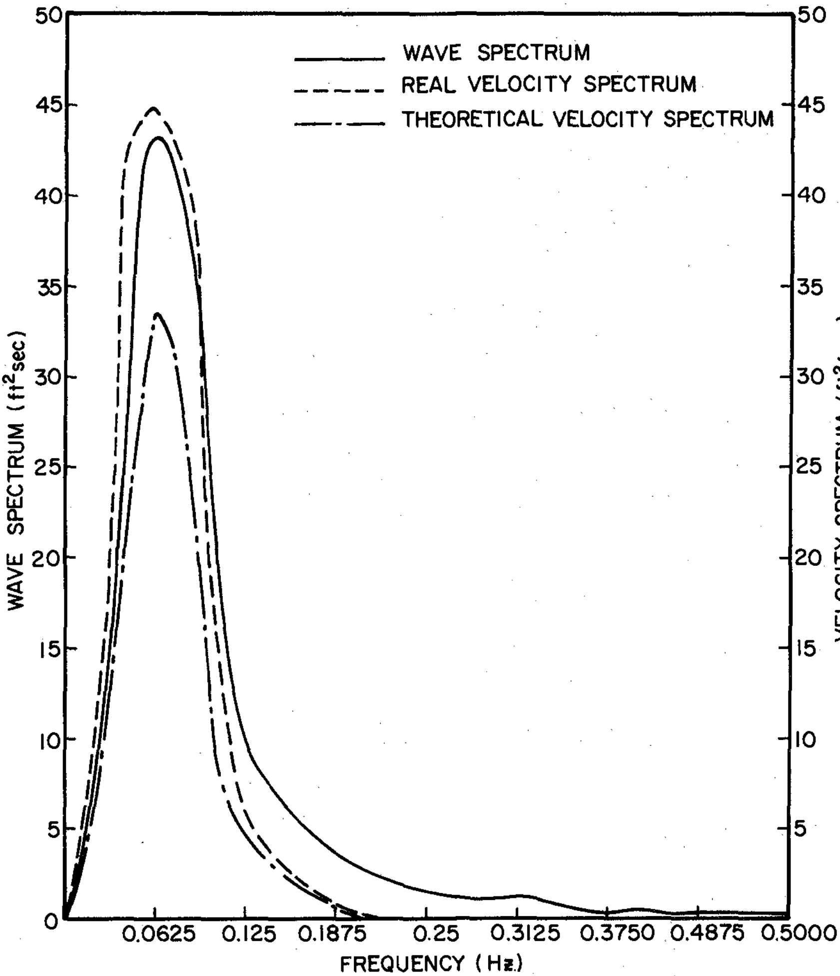

Figure 6: Example Comparison of Measured and Gaussian Wave Model Spectra