CHAPTER 136

High Reynolds Number Oscillating Flow by Cylinders

by

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and

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ABSTRACT

In order to determine the added mass, drag and lift coefficients of a smooth cylinder at various distances from a plane boundary, the forced cylinder oscillation tests at high Reynolds number $10^5$ to $10^6$ and the wave force tests at moderate Reynolds number $10^4$ to $10^5$ have been carried out at the Wave Research Facility at Oregon State University.

It is found that the drag, lift and added mass coefficients are all strongly Reynolds number dependent. The effect of the near by plane boundary is to increase all of the force coefficients two to four times as compared to the free stream flow values. This is a most important factor to be aware of for design purposes.

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INTRODUCTION

Fluid forces on submerged cylinders are important to ocean engineers who have to design pipe-like structures which are subjected to environmental fluid-dynamic loads. Some typical structures are submerged oil pipelines, ocean outfalls, offshore drilling rigs, submerged oil storage tanks and mooring cables. Although this subject has been investigated by many investigators for several years, the problem has not been completely understood because of the very complicated nature of the flow. These forces are influenced by Reynolds number, cylinder roughness, angle of skew, free stream turbulence, formation and collapse of the wake and the proximity of the free surface, the ocean bottom and other cylinder members.

This subject is a continuing study at Oregon State University. Rigid cylinders subjected to flow that is perpendicular to the central axis are considered as a beginning investigation to more general conditions. An extensive literature review was made early in the study (8). The influence of a plane boundary was first treated analytically in (8, 9). The theory has been extended to a group of cylinders (7) and applied to various types of engineering problems (10, 11).

Experimental verification of the theory was made for the simple case of a cylinder near a plane boundary (3, 4, 7, 8, 11). An excellent agreement between theory and experiment was obtained for the case when wake formation is small (A/D<1.5 where A = the double amplitude of water particle displacement relative to the cylinder, and D = cylinder diameter). For the cases of large wake formation (A/D>1.5) the experimental results deviate considerably from potential flow theory.

The above results suggest that the real fluid oscillatory flow around rigid cylinders can be classified into two major flow situations; 1) the potential flow, and 2) wake flow. The classification of these two flow situations is discussed briefly in the following.

Potential Flow. If the thickness $\delta$ of the boundary layer on the cylinder is small compared to the cylinder diameter and if the boundary layer does not separate, the flow around the cylinder can be essentially modeled with the potential flow theory. The thickness of the laminar boundary layer on the cylinder may be given by

$$\delta = C \sqrt{\frac{\nu}{\omega}}$$  \hspace{1cm} (1)

in which

$C$ is a constant slightly dependent on the diameter of cylinder (for a flat plate $C = 2\pi$, Ref. 12)
\( \nu = \text{kinematic viscosity} \)

\( \omega = \frac{2\pi}{T} \) where \( T = \text{period of oscillation} \).

The relative boundary layer thickness may be defined as

\[
\delta^* = \frac{\delta}{D/2} = 2C \sqrt{\frac{\nu}{\omega D/2}} = \frac{4\pi}{D} \sqrt{\frac{\nu}{\omega}}
\]

(Sarpkaya (4) claims that \( D^2 \omega/\nu \) (which he calls the \( \beta \) parameter) is very important in oscillatory flow, without physical explanation. From Eq. (2), \( \beta \) is related to \( \delta^* \) as

\[
\delta^* = 4\pi \sqrt{\frac{1}{\beta}}
\]

The ranges of the oscillation period for ocean waves and for earthquake ground motions are about 1 to 15 sec. and .1 to 1 sec., respectively. According to Eq. 1, the boundary layer thickness ranges from 2.43 to 9.35 mm for ocean waves and .76 to 2.42 mm for earthquakes. This is small compared to the diameter of structural members for offshore structures which range from about 100 to 2000 mm. However, the boundary layer effect may not be negligible for small size models of offshore structures in laboratory experiments.

Due to the oscillatory nature of the flow, the boundary layer on the cylinder surface does not separate (or the wake formation after separation is small) if the amplitude of water particle displacement to the cylinder diameter is \( A/D < 1.5 \).

The authors have derived a closed form analytical solution for the hydrodynamic forces on any number of cylinders which may move in any manner as shown in Ref. (7). Several practical examples were solved (9, 10, 11).

Wake Flow. If the amplitude of the water particle displacement is large compared to the cylinder diameter, or \( A/D > 1.5 \), the boundary layer separates from the cylinder surface and a wake forms. The characteristics of the wake depend on the maximum Reynolds number, \( U_m D/\nu \), as well as the wake parameter, \( A/D \), where \( U_m = \text{maximum water particle velocity relative to the cylinder} \). Since no complete theory is available for the prediction of hydrodynamic forces on cylinders in the wake flow situation, they must be determined experimentally.

The data in the open literature to date are limited to the lower values of \( U_m D/\nu \) and \( A/D \) and only for the free stream flow condition (1,6). The present investigation considerably extends the range of the parameters to higher values and generates information for various values of \( e/D \) for the first time (\( e = \text{the gap between the cylinder and a near by plane boundary} \)).
An experiment specially designed to generate the hydrodynamic force coefficients at high Reynolds numbers (1x10^5 to 1.05x10^6) and large wake parameters is reported on herein. Newly generated wave force data with moderate Reynolds numbers (1x10^4 to 1x10^5) are also presented.

OSCILLATING CYLINDER EXPERIMENTS

In order to investigate the oscillatory flow around cylinders with a large wake formation at high Reynolds numbers, the experiments were conducted at the Wave Research Facility at Oregon State University. The facility has a wave and towing basin which is 104m long, 3.66m wide and generally 4.57m deep. The overall view of the experimental setup is shown in Fig. 1. The 30cm (12 in) diameter test cylinder, which nearly reaches across the width of the wave basin, (3.66m) was forced to oscillate in a large mass of water which is otherwise still. The wave board, with a 150 horsepower motor and piston, was used to move the cylinder through a cable linkage as shown. A dam was constructed to keep the wave board dry. The motion of the wave board was amplified and transmitted to the test cylinder through the wire ropes and sheaves. Wire ropes were anchored on the amplifier sheaves to avoid any possible slips and were tensioned by spring pulleys. The maximum amplitude of cylinder motion was 6m. The cylinder was located 2m from both the basin bottom and the free surface and 12m from the dam, to best approximate the free stream flow condition.

To investigate the effect of a near by plane boundary, a movable concrete false bottom covering the 12m section below the cylinder was located at various distances from the cylinder. The test cylinder is detailed in Fig. 2. The 76cm long test section is suspended between two dummy sections by two elastic bars on which strain gages were attached to measure the horizontal and vertical component of forces. Two accelerometers were attached to the inner cylinder to measure the horizontal acceleration of the cylinder. The total amplitude, A, of cylinder displacement was measured by eye.

Four amplitudes were studied, i.e. A/D = 20, 15, 10 and 5. For each amplitude, the frequency of oscillation was varied to cover the maximum cylinder velocity, U, from 0.3 to 4.27 m/sec. The horizontal accelerations together with the wave board displacement were simultaneously recorded on a photosensitive strip chart recorder.

It should be noted that the flow past a stationary object is hydrodynamically equivalent to the flow due to the same object moved in the otherwise still fluid as long as the fluid is incompressible. The only difference is that the object in the moving fluid experiences an extra force, which is equal to the displaced mass of the fluid times the ambient fluid acceleration, due to the pressure field necessary to accelerate the fluid.
WAVE FORCE EXPERIMENTS

In order to investigate the oscillatory flow around cylinders in a large wake flow situation at moderate Reynolds numbers (1x10^4 to 1x10^5), wave force experiments were also made at theWave Research Facility. A 76mm diameter horizontal cylinder was instrumented and mounted at various distances from a false bottom in 2m water. The horizontal and vertical forces, together with the water surface fluctuation, were recorded simultaneously. Since the detail of similar experimentation has been reported in our previous paper (Ref. 4), it will not be repeated here again.

ANALYSIS OF EXPERIMENTAL DATA

The analysis of the data from the oscillating cylinder tests and the wave force tests was based on the Morison equation. The horizontal force \( F_x(t) \), on a cylinder undergoing sinusoidal motion may be given in the form of the Morison equation as

\[
F_x(t) = C_D \frac{1}{2} \rho DL \left| \dot{x}(t) \right| \dot{x}(t) + (C_M \frac{1}{4} \rho \pi D^2 L + M) \ddot{x}(t)
\]

wherein \( x(t) \) is the cylinder displacement from the neutral position and \( \dot{\cdot} \) and \( \ddot{\cdot} \) on \( x \) designate \( \frac{d}{dt} \) and \( \frac{d^2}{dt^2} \) respectively, \( \rho = \) density of fluid, \( C_D = \) drag coefficient, \( C_M = \) added mass coefficient and \( L = \) the length of test cylinder, \( M = \) mass of test cylinder.

For a sinusoidal motion, \( x(t) \), \( \dot{x}(t) \) and \( \ddot{x}(t) \) are given as

\[
x(t) = \frac{A}{2} \sin \omega t
\]
\[
\dot{x}(t) = \frac{A \omega}{2} \cos \omega t
\]
\[
\ddot{x}(t) = -\frac{A \omega^2}{2} \sin \omega t
\]

From Eq. 6, the maximum velocity \( U_m \) is

\[
U_m = \frac{A \omega}{2}
\]

The maximum acceleration \( \ddot{U}_m \) is given as

\[
\ddot{U}_m = \frac{A \omega^2}{2}
\]

For a sinusoidal motion \( \dot{x}(t) \) vanishes when \( \ddot{x}(t) \) is maximum and vice versa. In a very simple analysis, \( C_D \) and \( C_M \) is often evaluated by measuring the horizontal force \( F_D \) at the instant of maximum ambient velocity and the force \( F_M \) at the instant of maximum acceleration.
For the simple approach,

\[ C_D = \frac{F_D}{\frac{1}{2} \rho D L U_m^2} \]  \hspace{1cm} (10)

and

\[ C_M = \frac{F_M}{\frac{1}{2} \rho \pi D^2 L U_m^2} - \frac{M}{\frac{1}{2} \rho \pi D^2 L} \]  \hspace{1cm} (11)

The measured cylinder motion was fairly sinusoidal. The maximum horizontal force at A/D = 15 and 20 occurred at the instant of maximum velocity. This indicates that there is no phase lag between the drag force and the ambient velocity. Thus, the simple approach was used in this paper. Some data from the forced cylinder oscillation tests were also recorded in digital form on magnetic tapes. If time and opportunity permit in the future, the least squared time average method used in Refs. 1 and 6 will be applied to the digital data to see the difference in the values \( C_D \) and \( C_M \) determined by the two methods.

The lift coefficient, \( C_L \), is defined from the maximum vertical force, \( F_L \), and the maximum velocity, \( U_m \), as

\[ C_L = \frac{F_L}{\frac{1}{2} \rho D L U_m^2} \]  \hspace{1cm} (12)

In Eqs. 10, 11 and 12, \( U_m \) was computed from the measured value of \( A \) and \( \omega \) and Eq. 8, and \( \dot{U}_m \) was directly obtained from the measurement. The error between the measured value of \( \dot{U}_m \) and the computed value from Eq. 9 using measured values of \( A \) and \( \dot{U}_m \) was usually small. For each run (which included several oscillations) the ensemble average of four to six samples was used to determine the force coefficients.

From wave force data, \( C_D \) and \( C_L \) were also evaluated based on Eqs. 10 and 12 utilizing the Airy wave theory and measured wave height and frequency.

**EXPERIMENTAL RESULTS**

Experiments were carried out for the following combinations of the parameters, \( e/D \), \( A/D \) and \( U_m D/v \):

**Oscillating Cylinder Tests (D=30cm)**

\( e/D \) : 6, 1, 0.5, 0.25, 0.083
Oscillating Cylinder Tests (D=30cm) continued

\[
\frac{U_m}{v} x 10^{-5} : 10.5, 7.88, 5.25, 2.63, 1.31
\]

Note: The maximum value of \( \frac{U_m}{v} \) depends on A/D.

Wave Force Tests (D=7.62cm)

e/D : 3, 1, .5, .33, .063

A/D : 3.5 to 33

\[
\frac{U_m}{v} : 10^{4} \text{ to } 10^{5}
\]

Example Data. Typical force, acceleration and displacement records from the cylinder oscillation tests are shown in Fig. 3 for e/D = 6, A/D = 10 and \( \frac{U_m}{v} = 5.25 \times 10^{2} \), and in Fig. 4 for e/D = 0.083, A/D = 10 and \( \frac{U_m}{v} = 5.25 \times 10^{5} \). The cylinder acceleration is fairly sinusoidal and has practically no phase lag with the wave board displacement. This is a good indication that the actual cylinder motion was approximately sinusoidal. (Because of small cable slack, the motion of the cylinder cannot be exactly sinusoidal.) The high frequency noise at about 8 Hz is due to the natural vibration frequency of the test cylinder in water.

Compare the vertical forces for e/D = 6 and .083. For the free stream flow condition, e/D = 6, the lift force fluctuates equally up and down at six times the oscillation frequency for this particular condition. This is due to the alternating vortex shedding. However, for the near-bottom flow condition, e/D = .083, the lift force is toward the boundary or downward during a very short period where the velocity is small, but for the remainder of the flow cycle, where the velocity is significantly large, the lift force is away from the boundary, or upward. The explanation for this is that due to the oscillatory nature of the flow, there is a moment where the wake does not clearly exist, so the force is toward the boundary as for the potential flow condition. But as soon as the wake becomes clear, the force becomes upward, due to the assymetric flow due to the near boundary. For this case, the lift force always fluctuates at twice the flow oscillation frequency.

This effect of a near boundary may be important with regard to the design of submerged pipelines.
The force coefficients, $C_n$, $C_M$, and $C_p$, are plotted versus the Reynolds number, $U_D/v$, with the wake parameter, $A/D$, and the relative gap, $e/D$, as parameters, and are discussed in the following section.

**Drag Coefficient. The plots of the drag coefficient versus the Reynolds number for the "free stream" flow condition are shown in Fig. 5. Actual values of $e/D$ are 6 and 3 for oscillating cylinder tests and wave force tests, respectively. The general trends of the oscillatory cylinder data and the wave force data match continuously. This may be a good indication of the physical similarity between the two flow conditions. The oscillatory flow data (1,6) and the steady flow data (5) available in the literature are also shown for comparison.**

The oscillatory flow results from the three investigations agree fairly well. For the range of the wake parameter studied, i.e., $A/D$ from 3.5 to 33, $C_D$ from oscillatory flow varies gently as $U_D/v$ varies from $10^4$ to $10^6$. This is distinguishably different from the well known steady state data which has an abrupt transition of the high subcritical value of $C_D$ to the low supercritical value.

Does the oscillatory flow value of $C_D$ approach the steady state value as $A/D$ approaches infinity? This question still remains to be answered.

The plots of $C_D$ vs. $U_D/v$ for the near boundary flow condition ($e/D = .083$ for the oscillating cylinder tests and $e/D = .063$ for the wave force tests) are shown in Fig. 6. The general tendency is similar to that of the free stream flow case except that the value of $C_D$ for this case is considerably higher than that for the free stream flow. The steady flow data by Jones (2) happens to give a lower limit of the oscillatory flow data shown.

Similar plots for three intermediate values of $e/D$ were made but are not shown here because of space limitation. The general trends are similar to those in Figs. 5 and 6 but the value of $C_D$ varies depending on $e/D$.

The plots of $C_n$ versus $U_D/v$ at the largest $A/D = 20$ are shown in Fig. 7. As noted before, the general trends are the same for all $e/D$. The value of $C_n$ decreases until it reaches the minimum at $U_D/v = about 3 \times 10^5$, then it increases gradually as $U_D/v$ increases from $10^4$ to $10^6$. The absolute value of $C_D$ increases as the cylinder approaches the boundary; the value of $C_D$ for the near boundary flow ($e/D = .083$) is about two times as large as that for the free stream flow. This is probably due to the flow blockage effect of the plane boundary.

**Added Mass Coefficient. The added mass coefficient, $C_M$, obtained from the oscillating cylinder tests is plotted versus $U_D/v$ for the free stream flow condition ($e/D = 6$) in Fig. 8. The data from the present study are compared with the data by other investigators. Again, all**
of the data agree fairly well. The value of $C_M$ increases gradually until it reaches the maximum at about $U_D/\nu = 3 \times 10^5$, then it decreases gradually as $U_D/\nu$ increases from $10^4$ to $10^6$. The experimental value of $C_M$ is always smaller than the theoretical value ($C_M = 1.0$). This is due to the wake effect which strongly depends on the Reynolds number, $U_D/\nu$. The effect of $A/D$ on $C_M$ is not so clear.

Similar plots for the near boundary flow condition ($e/D = 0.083$) are shown in Fig. 9.

The plots of $C_M$ versus $U_D/\nu$ at $A/D = 20$ are shown for various values of $e/D$ in Fig. 10. The theoretical value of $C_M$, which is independent of $U_D/\nu$, increases from 1.0 to 2.3 as the cylinder approaches from the free stream to the plane boundary. The experimental value of $C_M$ increases by the same order of magnitude as $e/D$ decreases, but is strongly dependent on the Reynolds number.

Lift Force Coefficient. Plots of $C_L$ versus $U_D/\nu$ for the free stream flow condition ($e/D = 6$ for the oscillatory cylinder tests and $e/D = 3$ for the wave force tests) are given in Fig. 11. The OSU data are compared with the data in Ref. 6. The data from the two investigations agree very well at all values of $A/D$. The value of $C_L$ increases as $A/D$ decreases in the range of $A/D$ from 5 to 20. For a given $A/D$, $C_L$ decreases gradually as $U_D/\nu$ is increased. For the free stream flow condition the lift force fluctuates many times up and down in a half cycle of oscillation due to alternating vortex shedding as shown in Fig. 3. The fluctuation frequency depends on $A/D$.

The plots of $C_L$ versus $U_D/\nu$ for the near boundary flow condition ($e/D = 0.83$ for the oscillatory cylinder tests and $e/D = 0.063$ for the wave force tests) are shown in Fig. 12. As explained before, the lift force for this case is not due to the vortex shedding but due to the flow asymmetry. Therefore, the lift for this case is always at twice the oscillation frequency of the flow and independent of $A/D$.

The plots of $C_L$ versus $U_D/\nu$ at $A/D = 20$ are shown for various values of $e/D$ in Fig. 13. The positive lift force increases significantly as $e/D$ decreases. The value of $C_L$ for $e/D = 0.083$ is about four times larger than $C_L$ for the free stream flow condition for the entire range of $U_D/\nu$. The dependence of the negative lift force on $e/D$ is not as clear as that of the positive lift force.

**PRACTICAL APPLICATIONS**

Real design oscillatory flows such as ocean waves and earthquakes should be classified into either the potential flow situation ($A/D < 1.5$) or the wake flow situation ($A/D > 1.5$) according to the wake parameter.
For the potential flow situation, such as earthquake problems, a closed form solution for the hydrodynamic forces on any number of cylinders is available in Ref. 7. The calculated force coefficients for one, two and three cylinders near a plane boundary (11) and for a 4x4 cylinder arrays (10) are also available.

For the wake flow situation, such as wave force problems, the experimental data of drag, lift and added mass coefficients covering the Reynolds number, $U_D/v$, of from $10^4$ to $10^6$, the wake parameter, $A/D$, from 3.5 to 33, the relative gap between the cylinder and a plane boundary, $e/D$, from .065 to 6.0, are presented herein for the design of pipelines, offshore platforms and other pipe structures.

CONCLUSIONS

To determine the added mass, drag and lift coefficients of a smooth cylinder at various distances from a plane boundary, the forced cylinder oscillation tests at high Reynolds numbers $10^5$ to $10^6$ and the wave force tests at moderate Reynolds numbers $10^4$ to $10^5$ have been carried out at the Wave Research Facility at Oregon State University. The following conclusions are drawn from the results.

1. The data from the forced cylinder oscillation tests matched continuously with the data from the wave force tests. This indicates the physical similarity between the two flow situations.

2. The drag, lift and added mass coefficients are all strongly Reynolds number dependent. Unlike steady flow, $C_D$ from oscillatory flow for $A/D = 0$ to 20 decreases gradually from a high value until it reaches a minimum at about $U_D/v = 3x10^5$ and then increases gradually as the Reynolds number increases from $10^4$ to $10^6$. The value of $C_D$ monotonically decreases as the Reynolds number increases. The value of $C_m$ has a maximum at about $U_D/v = 3x10^5$. This is true for all values of $e/D$.

3. The drag and lift coefficients are generally higher for the smaller values of $A/D$. Both $C_D$ and $C_L$ become fairly independent of $A/D$ when this parameter becomes as large as 15 or 20. The influence of $A/D$ on $C_L$ is less clear. This tendency is true for all values of $e/D$.

4. The effect of a near by plane boundary is to increase all of the force coefficients. At $A/D = 20$, $C_L$ increased about two times, $C_D$ about four times and $C_m$ about two times as $e/D$ decreases from 6 to .083. This may be due to the blockage effect of the near by plane boundary. This is a most important factor to be aware of for design purposes.
5. The wake characteristics are completely different between the free stream flow and the near boundary flow conditions. In the free stream flow, many alternate vortexes shed during a half cycle of oscillation. The number of vortexes varies depending on the value of $A/D$. Near a plane boundary, the vortex shedding is suppressed. The flow changes from the potential flow situation to the wake flow situation during a half cycle. So the lift force also changes from negative to positive during the half cycle. This frequency characteristic of lift force near a plane are independent of the value of $A/D$ as long as $A/D > 1.5$.

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REFERENCES


FIG. 1. The experimental setup for the forced cylinder oscillation tests.
FIG. 2  Test cylinder for Forced Cylinder Oscillation Tests.
FIG. 3 Example Data for $e/D = 6$, $A/D = 10$ and $U_m D/\nu = 5.25 \times 10^5$, Oscillating Cylinder Tests.

FIG. 4 Example Data for $e/D = 0.083$, $A/D = 10$ and $U_m D/\nu = 5.25 \times 10^5$, Oscillating Cylinder Tests.
FIG. 5 Drag coefficient vs. Reynolds Number for Free Stream Flow condition, \( e/D = 6 \) for oscillating cylinder data and \( e/D = 3 \) for wave force data.

FIG. 6 Drag coefficient vs. Reynolds Number for Near Boundary Flow condition, \( e/D = 0.083 \) for oscillating cylinder tests and \( e/D = 0.063 \) for wave force data.
FIG. 7 Drag coefficient vs. Reynolds Number for various values of e/D at A/D = 20.

FIG. 8 Added Mass coefficient vs. Reynolds Number for Free Stream Flow condition.
FIG. 9 Added Mass coefficient vs. Reynolds Number for Near Boundary Flow condition.

FIG. 10 Added Mass coefficient vs. Reynolds Number for various values of e/D at A/D = 20.
FIG. 11 Lift Coefficient vs. Reynolds Number for Free Stream Flow condition.

FIG. 12 Lift Coefficient vs. Reynolds Number for Near Boundary Flow condition.
FIG. 13 Lift Coefficient vs. Reynolds Number for various values of $e/D$ at $A/D = 20$. 