

CHAPTER 77

STABILITY AND IMPULSE RESPONSE OF EMPIRICAL EIGENFUNCTIONS

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ABSTRACT

The statistical method of empirical eigenfunctions has been applied to four years of beach profile data. The eigenfunctions associated with the three largest eigenvalues are shown to be stable for data sets of one, two, three, and four years length, and they correctly describe beach changes caused by storm activity. The usefulness of the eigenfunction representation is confirmed as a concise means of representing beach profile variability.

INTRODUCTION

The onshore-offshore movement of nearshore sediments in response to changing incident wave energy makes an important contribution to coastal zone variability. The basic motivation for this paper is to concisely describe the nearshore variability associated with selected beach profiles by means of empirical orthogonal eigenfunctions. In this manner the seasonal onshore-offshore movement of sediment can be distinguished from shorter-term changes in beach profiles. In addition, the method has the advantage of describing all of the variability of the beach profiles by means of just a few simple functions.

The data set consists of four years of beach profile data taken at Torrey Pines Beach, California, at monthly intervals. These profiles were taken by the method described by Nordstrom and Inman (1975), and include measurements from the backshore out to a depth of 20 meters. The study location is a fine-grained sand beach approximately 3 km north of Scripps Institution of Oceanography. The beach is straight with uncomplicated offshore bathymetry exposed to wave energy coming from all offshore quadrants.

Figures 1 and 2 illustrate the magnitude of the seasonal changes in Southern California. Figure 1 shows a portion of the beach in La Jolla at the end of winter, 1975, when the "winter" beach was fully developed. The rocks are exposed approximately one meter. Figure 2 shows the same beach at the end of summer, 1974, when the "summer" beach was fully developed; the rocks are completely covered with sand.

More quantitatively, seasonal changes at North Range, Torrey Pines Beach, are shown in Figure 3 for two successive winter and summer profiles. The summer berm is built up about one meter above the sand level present at the same place during the winter. Similarly, the bar at a depth of 5 meters builds up approximately one meter. These seasonal trends can be obscured by short-term events, such as storms or periods of extremely low wave energy. The method of empirical eigenfunctions can be used to distinguish between variability on these different time scales.



Figure 1. Beach in La Jolla in winter of 1975 with rocks exposed one meter.



Figure 2. Beach in La Jolla in summer of 1974 with rocks fully covered with sand.

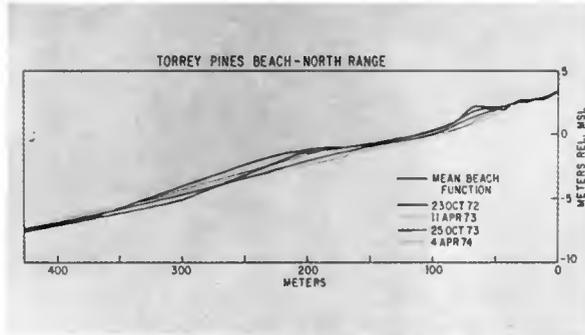


Figure 3. Comparison of winter and summer beach profiles showing seasonal erosion and accretion of sand.

An earlier attempt to depict the seasonal variability of beach profiles using an objective statistical technique was described in Winant, Inman and Nordstrom (1975). The method of empirical eigenfunctions was used to describe the first few modes of variability in two and one-half years of beach profile data taken at monthly intervals. This data set was subsequently expanded to four years (Aubrey, Inman, and Nordstrom, 1976). Over 99.75% of the variability of the data can be accounted for by the three eigenfunctions associated with the three largest eigenvalues. Figure 4 shows the spatial and temporal variation of these eigenfunctions. The first eigenfunction is the mean beach function which reflects the mean beach level. Its time dependence is nearly constant. The second function is the bar-berm function, which shows a large maximum at the location of the summer berm as well as a minimum in the area of the winter bar. The time dependence of this function shows a one year periodicity. The third function is the terrace function, which shows a broad maximum over the low-tide terrace. Its time dependence cannot be simply interpreted.

The present paper addresses two problems not considered in previous work. In order to usefully represent beach variability, the spatial and temporal behavior of the eigenfunctions must be stable with respect to various data sample lengths. Data sets of one, two, three, and four years of beach profiles are examined, and their eigenfunctions are compared. In addition, the first few eigenfunctions should correctly yield the magnitude of beach erosion due to storm waves. This point is investigated by analyzing the first winter storm in October 1974, and its effect on the beach.

STATISTICAL METHOD

One seeks to represent the variability of the beach profile data in terms of a set of orthogonal functions. Obviously any one of a number of such orthogonal series can be generated, one example being a Fourier series. One could then ascribe the variability associated with a one year period to the seasonal variations. Unfortunately, there is no *a priori* reason to suspect that the seasonal dependence of onshore-offshore sand movement is sinusoidal. In fact, there is evidence to the contrary, as when the beach erodes rapidly as the first winter storm waves erode the "summer" beach.

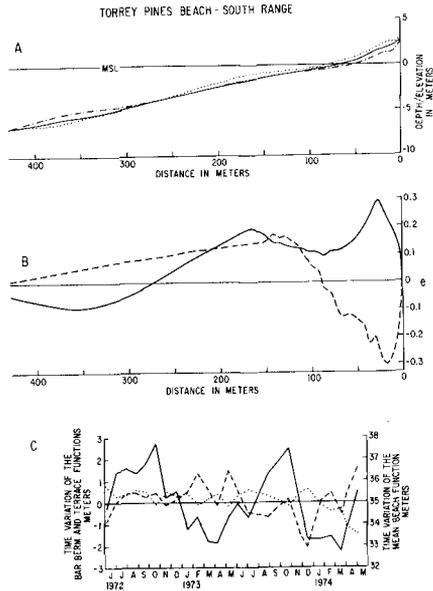


Figure 4. Spatial and temporal dependence of the first three eigenfunctions for a two year period. (a) solid line, mean beach function; dash-dot line, profile of 11 April 1973; and dotted line, profile of 23 October 1972. (b) solid line, bar-berm function; and broken line, terrace function. (c) time variation of bar-berm function (solid line), terrace function (broken line), and mean beach function (dotted line).

A better choice is a set of empirical eigenfunctions which most concisely describe the beach profile variability. The properties of these functions have been summarized by Davis (1976):

(a) They provide the most dense representation of a data set in the sense that the first n terms in the expansion represent more of the data variability than the first n terms of any other orthogonal expansion.

(b) Both the spatial and temporal eigenfunctions are orthonormal sets, so that each corresponding set (λ_n, c_n, e_n) may be regarded as representing a mode of variability which is uncorrelated with any other mode.

The method has been described by Winant, et al (1975). In brief, one seeks an eigenfunction expansion in the form

$$h_{xt} = \sum_n c_{nt} e_{nx} (\lambda_n n_t)^{\frac{1}{2}}$$

where h_{xt} are the beach profile data, c_{nt} represent the temporal eigenfunctions,

e_{n_x} represent the spatial eigenfunctions, n_x represents the number of data points per profile, n_t represents the number of different profiles, and the λ_n are the eigenvalues. In this study, $n_x = 51$ and $n_t = 46$.

The spatial correlation matrix A is formed by the elements

$$a_{ij} = \frac{1}{n_t} \sum_{t=1}^{n_t} h_{it} h_{jt}$$

Similarly, a time correlation matrix B is formed with the elements

$$b_{ij} = \frac{1}{n_t} \sum_{x=1}^{n_x} h_{xi} h_{xj}$$

Both A and B are real symmetric matrices with only positive real eigenvalues. In fact, A and B can be easily shown to have the same non-zero eigenvalues λ_n . The functions e_n then are the eigenvectors of A and the functions c_n are the eigenvectors of B:

$$Ae_n = \lambda_n e_n$$

$$Bc_n = \lambda_n c_n$$

The physical interpretation of the first few of these eigenfunctions has been previously discussed by Winant, et al (1975).

STABILITY OF THE EIGENFUNCTION REPRESENTATION

In order to evaluate the stability of the eigenfunction representation, the spatial eigenvectors were calculated for data sets of one, two, three, and four years length, while the temporal eigenvectors were calculated for data sample lengths of two years and four years. If the eigenfunctions are to accurately represent the beach variability, they should exhibit similar features for the different data sets.

The results of the analysis for the mean beach function are shown in Figure 5. The spatial eigenvectors for the four different data sets are nearly indistinguishable, so this function has no dependence on the length of the data sample. Figures 6 and 7 show the time dependence of the first three eigenfunctions. Figure 6 is for a two year data set; Figure 7 is for a four year data set. The time dependence of the mean beach function is almost constant and is independent of the length of the data set.

Table 1 shows the percentage of the total mean square value of the data associated with each of the first five eigenvalues. The percentage represented by the eigenvalue associated with the mean beach function is essentially independent of the length of the data set. This suggests that the mean

beach function is constant at North Range. No appreciable net erosion or accretion is seen in this function for the four year period.

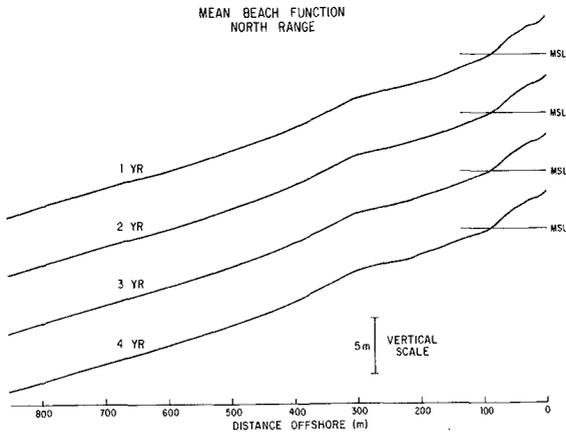


Figure 5. Spatial dependence of mean beach function for data sets at one, two, three, and four years length.

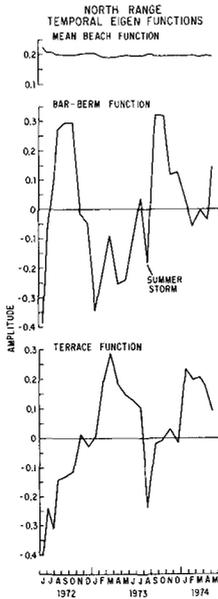


Figure 6. Temporal dependence of first three eigenfunctions for a two year data set.

The results for the spatial dependence of the bar-berm function are shown in Figure 8 where this function is plotted for each of the four data sets. The shape of the function is the same, except for the tendency of the extrema to broaden. This broadening occurs because the summer berm and winter bar do not form in the identical locations every year. The magnitude of this function also varies somewhat, reflecting the variability in heights of the berms and bars. Figures 6 and 7 show the time dependence of this function for the two data sets. In the two-year overlap in these graphs, the time dependence is nearly identical. The time dependence has a distinct seasonal trend. Table 1 shows that, for the spatial dependence of this function, the same amount of variance is accounted for by the associated eigenvalue in all four data sets.

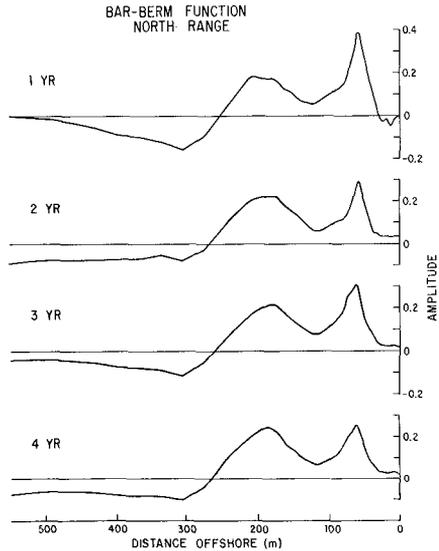


Figure 8. Spatial dependence of bar-berm function for data sets of one, two, three, and four years length.

Figure 9 shows the variation in the spatial dependence of the terrace function for the four data sets. The general shape of the curves is conserved, as are the relative magnitudes of the extrema. The location of the extrema vary slightly in response to the fact that sand erosion and deposition occur in slightly different locations along the profiles in response to different wave conditions. The dominant feature in the spatial dependence is the broad maximum across the low tide terrace. Figures 6 and 7 show the temporal dependence of this eigenvector. Table 1 shows that approximately the same percentage of the total variability is explained by the eigenvalue associated with this eigenvector.

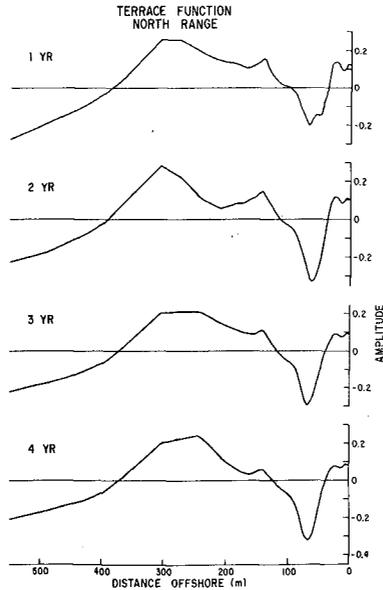


Figure 9. Spatial dependence of terrace function for data sets of one, two, three, and four years length.

IMPULSE RESPONSE OF THE EIGENFUNCTIONS

The response of these empirical eigenfunctions to impulses in the form of storm waves has been examined. Several examples demonstrate that the first few eigenfunctions are sensitive to short-term events. Two qualitative results are shown in Figure 9. On 27 August 1973, the time dependence of the bar-berm function became more negative instead of increasing according to the normal seasonal trend. This reflects the occurrence of a major summer storm on 23 August 1973 which eroded the developing summer profile. Similarly, the large positive values for February and March of 1976 reflect the anomalous occurrence of long, low waves which began to build the beach toward its summer configuration before it was eroded again by the more energetic waves more typical of winter conditions.

In October of 1974 the first large winter storm to hit the Southern California coast was generated off the low pressure center in the Gulf of Alaska. Storm front positions on 27, 28 and 29 October 1974 are shown in Figure 10. Significant environmental parameters measured at or near Scripps Institution of Oceanography during this time period are shown in Figure 11. The barometric pressure dropped from 1017 to 1005 mb and wind speeds reached 15 ms^{-1} . Coincidentally the rms wave amplitude increased to 0.5 m in 10 m of water. Wave energy spectra for the period just preceding and during the

storm are shown in Figure 12. The storm spectra are characterized by an order of magnitude higher spectral peak as well as a broader frequency band.

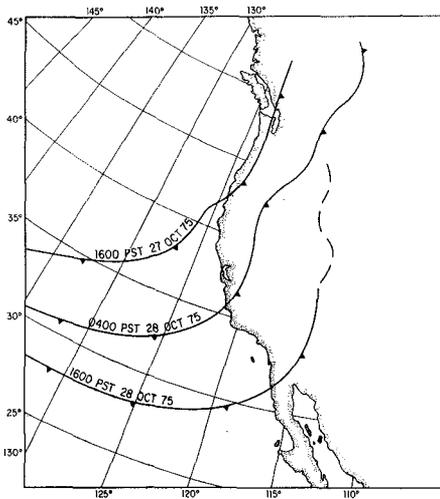


Figure 10. Storm front positions for first winter storm in late 1974.

The increased wave energy was coincident with spring tides of 2 m amplitude. These combined occurrences maximized the erosion on the beach. Figure 13 shows a comparison of a series of beach profiles at North Range taken before, during, and after the storm. The well-developed "summer" profile on 24 October was rapidly eroded as the storm passed through the area. At a distance of 70 m from the profile benchmark where the berm was located, 46 cm of sand was eroded.

This rapid erosion is reflected in the behavior of the temporal dependence of the bar-berm function shown in Figure 14. The 31 October profile shows an erosion in the beach on the order of 15 cm. For this particular storm, the terrace function showed an erosion of 17 cm, while the fourth eigenfunction showed an erosion of 13 cm. The bar-berm function correctly responds to the wave energy impulse, while the magnitude of the beach change can be fully accounted for in the first few eigenfunctions.

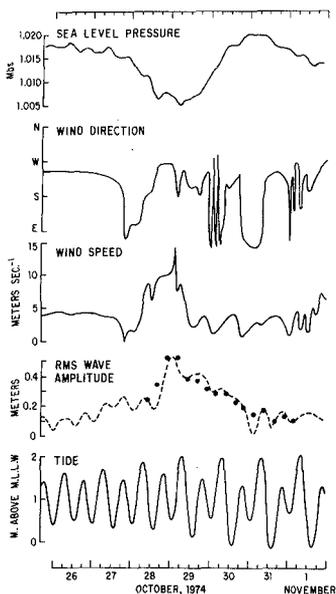


Figure 11. Environmental parameters measured at or near Scripps Institution of Oceanography during first winter storm in late 1974.

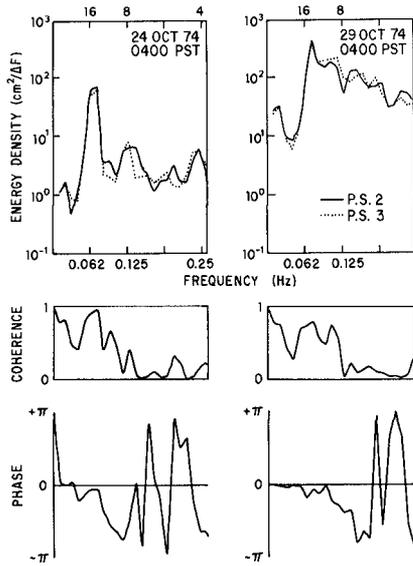


Figure 12. Frequency spectra of ocean surface waves measured before and during the first winter storm of late 1974.

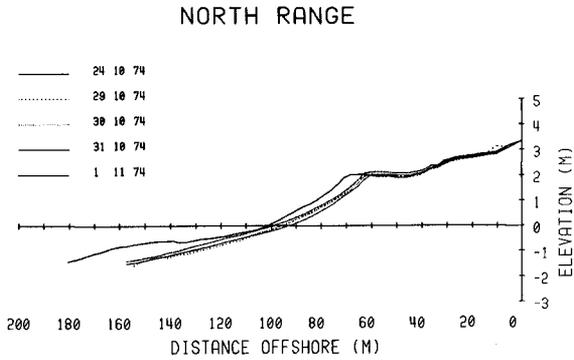


Figure 13. Beach profiles measured at North Range before, during, and after the first winter storm of late 1974.

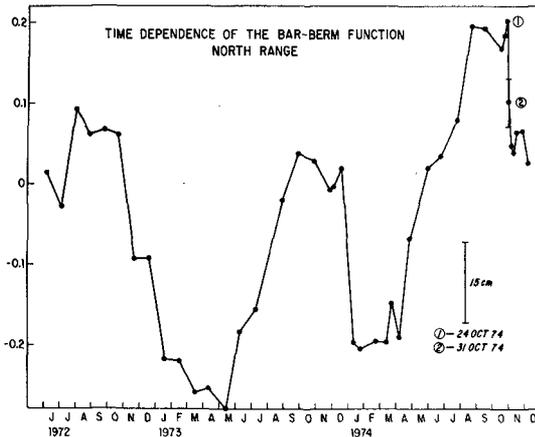


Figure 14. Time dependence of the bar-berm function for a two and one-half year data set at North Range.

CONCLUSIONS

The method of empirical eigenfunctions has been shown to be of great value in analyzing beach profile data. Using no a priori assumptions on the structure of the orthogonal functions, the most concise orthogonal set is generated by the data. The first three eigenfunctions describe over 99.75% of the variability in the data, and can be used instead of the data themselves to quantify the variability. The functions are stable with respect to the length of the data set examined, so an analysis of one year of data will indicate significant trends in the eigenfunctions. Erosion or accretion caused by short term events such as storms can be accurately predicted by the behavior of the temporal dependence of the bar-berm function, while the magnitude of this impulsive change is correctly given by the first four eigenfunctions.

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