

CHAPTER 54

Generalized Model for Storm Surges

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Abstract. A two-dimensional numerical simulation model of storm surges based on the vertically integrated hydrodynamic equations of continuity and momentum is adopted. The model embodies the inundation over the low-lying land surface with moving water-land interfaces. Wind stress coefficient is considered not only function of wind speed but also dependent on the temperature differential between air and water. Bottom stress is treated by an evaluation of convection integrals over the surface slope pressure depression and wind shear. The contribution of momentum by river inflows is taken into account. Particular attention is directed to the treatment of nonlinear terms in the governing equations to insure the improvement of numerical stability and accuracy. The model is applied to the New Jersey Coastal area and reproduces the historical storm surges quite well both at Atlanta City and Sandy Neck.

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INTRODUCTION

Reliable estimates of water-level changes under storm conditions are essential for the planning and design of coastal engineering works. Determination of design water elevations during storms is a complex problem involving interaction among the bathymetry and geometry of the water body, the forces of the wind field and atmospheric depression, the resistance of the bottom, the momentum generated in the water body, and the effects caused by other mechanisms unrelated to the storms, such as astronomical tides, earth rotation, etc.

The development of numerical models for predicting storm surges has been advanced rapidly during the past decade, from simple one-dimensional bathystropic theory (Bretschneider and Collins, 1963) to more complex two-dimensional simulation of arbitrary water bodies (Reid and Bodine, 1968; Pearce, 1972; Tsai and Chang, 1974; Pearce and Pagenkopf, 1975; Wanstrath, 1975; Damsgaard and Dinsmore, 1975). Most of the models are found to be useful only for specific localities to where one must approximate its underlying assumptions. In view of the various deficiencies in the existing storm surge models, additional considerations to improve the accuracy of the results and the generalization of the applications are listed as follows:

1. Inundation boundary conditions over the low-lying land surface are incorporated automatically with the rationale of both mathematical and physical justification rather than based on the empirical or a weir-type formula.
2. Bottom stress is time varying and may be obtained by an evaluation of convolution integrals over the surface slope, pressure depression, and wind stress. Only with this wind dependent bottom stress, it seems possible to facilitate the computational scheme over the low-lying land surface.
3. Wind stress coefficient is not a unique function of wind speed but also a function of temperature differential between the air and water. This inclusion of temperature effects is highly significant for the estimation of wind stress coefficients in the Northern United States, and should greatly enhance the predictability of storm surges in the Great Lake of Northern America where the temperature has significant change during storms.
4. The surge elevations at the lateral open boundaries perpendicular to the coastline are obtained by assuming zero gradient of total water depth. This would allow the selection of the locations of lateral boundaries with less restriction.

5. The stream inflows in estuarine areas are included in the momentum equations. Conventional assumption by neglecting the stream momentum but adding only the stream mass flux into the continuity equation is not totally true for many estuarine conditions.
6. All nonlinear terms such as advections are not linearized in order to account for the bathymetries involving irregular coastlines and bay complexes.

The numerical techniques used for computing long wave equations generally fall into one of the three schemes: (i) Explicit Finite Difference Method, (Reid and Bodine, 1968; Pearce, 1972; Wanstrath, 1975), (ii) Implicit Finite Difference Method (Leendertse, 1967; Tsai and Chang, 1974), and (iii) Finite Element Method (Pearce and Pagenkopf, 1975; Gallagher, etc., 1973; Gallagher and Chan, 1973). Engineering practice has indicated that the explicit scheme having to satisfy the Courant stability condition is too strenuous. Although the finite element method offers smooth treatment on irregular boundaries, it has serious drawback in the requirements of computing time and computer storages. Furthermore, the lack of provability on the stability of the method often leads to obscure trial and error. The alternate direction (ADI) implicit scheme has gained the popularity. It is unconditionally stable to the long wave equations without nonlinear advection terms (Leendertse, 1967). The authors have been unable to make practical use of this advantage for the cases when the nonlinear terms are included in the long wave equations. It is surmised that this may be due to the way these nonlinear terms are approximated as part implicit and part explicit. A modified ADI scheme, which considers all terms implicit, is therefore adopted.

MODEL FORMULATION

Basic Equations and Assumptions

The mathematical equations describing storm driven surges can be obtained by integrating vertically the Navier-Stokes equations for fluid motions. In this development, it is assumed that density over the depth is constant, pressure variation with the vertical coordinate is hydrostatic, and the variation of momentum transport with vertical coordinate dominates those with horizontal coordinates. The vertically integrated forms of the conservation equations in a Cartesian Coordinate system with x and y on the horizontal plan can be written as

$$\frac{\partial H}{\partial t} + \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = qH \quad (1)$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left(\frac{U^2}{H} \right) + \frac{\partial}{\partial y} \left(\frac{UV}{H} \right) - 2qU - fV = - \frac{g}{2} \frac{\partial H^2}{\partial x} + gH \left(\frac{\partial h}{\partial x} - \frac{1}{\rho g} \frac{\partial p_s}{\partial x} \right) + \frac{\tau_x^w - \tau_x^b}{\rho} \quad (2)$$

$$\frac{\partial V}{\partial t} + \frac{\partial}{\partial x} \left(\frac{UV}{H} \right) + \frac{\partial}{\partial y} \left(\frac{V^2}{H} \right) - 2qV + fU = - \frac{g}{2} \frac{\partial H^2}{\partial y} + gH \left(\frac{\partial h}{\partial y} - \frac{1}{\rho g} \frac{\partial p_s}{\partial y} \right) + \frac{\tau_y^w - \tau_y^b}{\rho} \quad (3)$$

The symbols used in Equations (1), (2), and (3) are defined as follows:

- H = total water depth
- h = undisturbed water depth
- U, V = flux density in x- and y-directions, respectively
- g = gravitational acceleration
- f = coriolis coefficient
- τ_x^w, τ_y^w = wind stress components in x- and y-directions, respectively
- τ_x^b, τ_y^b = bottom stress components in x- and y-directions, respectively
- p_s = atmospheric pressure
- x, y = horizontal orthogonal coordinates
- t = time
- ρ = water density

The terms included in the momentum equations are from the left to the right representing inertia term; nonlinear longitudinal and lateral advection momentum terms; the momentum attributed to the artificial discharges or river inflows and outflows; coriolis acceleration; nonlinear gravity terms; forces due to bottom slope and atmospheric pressure gradient; and wind and bottom stresses. It is interesting to note that the atmospheric depression column and the water depth column are equivalent. Therefore, the accurate reading of the undisturbed water depth is more important than the consideration of atmospheric pressure depression.

Variables in Equations (1), (2), and (3) are H, U, V, h, q, p_s , τ_x^w , τ_y^w , τ_x^b , and τ_y^b . Among these variables, p_s , τ_x^w , and τ_y^w are the forcing functions

depending on the atmospheric pressure and wind field distributions of the storms; h and q are given functions of x and y . Thus, if bottom stress components, τ_x^b and τ_y^b , are related to H , U and V , Equations (1), (2) and (3) will constitute three simultaneous partial differential equations for three unknowns, H , U , and V .

Wind and Bottom and Stresses

The most important and sensitive parameter in storm surge modeling is the wind shear stress since it is the primary driving force. In general, the wind shear stress is related to the wind speed, w , through the following expression

$$\tau^w = \rho_a K w^2 \quad (4)$$

where ρ_a is the density of air and K is the wind stress coefficient. The values of K should be a function of other parameters involving wind speed, surface roughness, and stability and turbulence of the atmosphere. A model treating K as function of wind speed has been presented elsewhere (Van Dorn, 1953; Reid and Bodine, 1968),

$$K = K_1 \quad \text{for } W \leq W_c \quad (5a)$$

$$K = K_1 + K_2 (1 - W_c/W)^2 \quad \text{for } W > W_c \quad (5b)$$

in which the constants, K_1 , K_2 and the critical wind speed have been taken as

	K_1	K_2	W_c
Van Dorn (1953)	$1.2 \cdot 10^{-6}$	$1.8 \cdot 10^{-6}$	6.7 m/sec
Reid & Bodine (1968)	$1.1 \cdot 10^{-6}$	$2.5 \cdot 10^{-6}$	7.2 m/sec

A slightly different formula for describing K has been proposed (Tsai and Chang, 1974) as follows:

$$K = \begin{cases} 1.25 \cdot 10^{-6} & \text{for } W \leq W_1 \\ 1.25 \cdot 10^{-6} + 1.75 \cdot 10^{-6} \sin\left(\frac{\pi}{2} \frac{W-W_1}{W_2-W_1}\right) & \text{for } W_1 < W < W_2 \\ 3.0 \cdot 10^{-6} & \text{for } W \geq W_2 \end{cases} \quad (6)$$

in which $W_1 = 5.1$ m/sec and $W_2 = 15.4$ m/sec. Most of the investigators has not included the atmospheric stability in the formulation of K . However, it has been demonstrated that the temperature differential between the water and air has a significant effect on the value of K (Gillies and Punhani, 1971). It is, therefore, proposed in this paper that K be given by the following equation:

$$K = \begin{cases} K_1 + K_3 \Delta T & \text{for } W \leq W_c \\ K_1 + K_2 (1 - W_c/W)^2 + K_3 \Delta T & \text{for } W > W_c \end{cases} \quad (7)$$

The values of K_1 , K_2 and K_3 will have to be determined from further numerical experiments on storm surges.

The relationships describing the bottom stress have been taken for granted by many authors (Reid and Bodine, 1968; Dansgaard and Dinsmore, 1975) as simply given by the following formula:

$$\tau_x^b = \frac{\rho g \sqrt{U^2 + V^2} U}{C^2 H^2} \quad (8a)$$

and

$$\tau_y^b = \frac{\rho g \sqrt{U^2 + V^2} V}{C^2 H^2} \quad (8b)$$

where C is the Chezy coefficient. Bottom stress formulation based on this line of approach may sometimes yield large error, since there are possible flows in which the transport is small but there still may be a significant bottom shear layer. This problem can be avoided by calculating the drag from the flow profile calculations based on linear theory of long wave equations, as done by Jelesnianski (1970) and Forristall (1974). Accordingly, the bottom stress is given by

$$\vec{\tau}^b = \frac{2u}{H^2} \int_0^t \left(\frac{\vec{t}}{\rho} - \frac{1}{\rho} \vec{v}_p^s \right) K_F(t-\tau) d\tau - \frac{2u}{H^2} \int_0^t g \vec{v}_\eta^s K_Q(t-\tau) d\tau \quad (9)$$

in which ν is the eddy viscosity, $\eta = H - h$, and K_P , K_Q , ∇_p^s , ∇_η^s , τ^w , and τ^b denote

$$K_P = \sum_{n=0}^{\infty} (-1)^n (n + 1/2)\pi \text{Exp}\{-\nu(n + 1/2)^2 \pi^2 t/H^2 - (if - 2q)t\} \quad (10a)$$

$$K_Q = \sum_{n=0}^{\infty} \text{Exp}\{-\nu(n + 1/2)^2 \pi^2 t/H^2 - (if - 2q)t\} \quad (10b)$$

$$\nabla_p^s = \frac{\partial p_s}{\partial x} + i \frac{\partial p_s}{\partial y}; \quad \nabla_\eta^s = \frac{\partial \eta}{\partial z} + i \frac{\partial \eta}{\partial y}; \quad \tau^b = \tau_x^b + i \tau_y^b; \quad \tau^w = \tau_x^w + i \tau_y^w \quad (10c)$$

respectively. The adoption of Equation (9) is more physically appealing and consistent with the overall method than Equation (8). The derivation of Equation (8) involves more assumptions than that of Equation (9).

Finite Difference Equations

A space-staggered scheme is used to approximate the differential equations with finite difference equations (Leendertse, 1967). The scheme describes the flux density, total water depth, and undisturbed water depth at different grid points as shown in Figure 1.

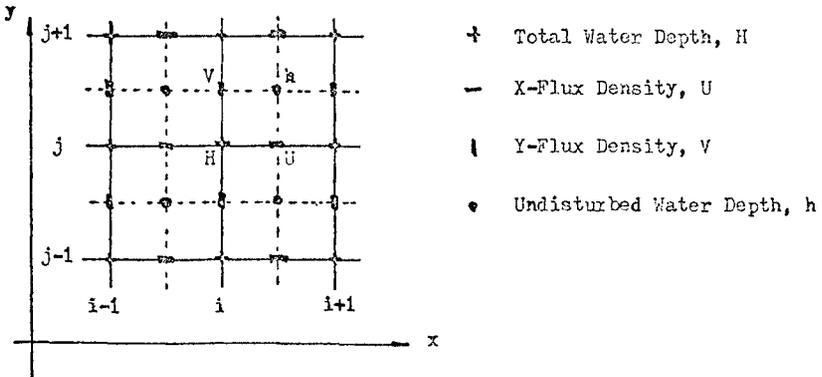


Figure 1

To simplify the discussion on the formulation of finite difference equations, Equations (1), (2), and (3) can be written as

$$\frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{F}}{\partial x} + \frac{\partial \vec{G}}{\partial y} - I = 0 \tag{11}$$

where

$$\vec{V} = \begin{bmatrix} H \\ U \\ V \end{bmatrix}; \quad \vec{F} = \begin{bmatrix} U \\ \frac{U^2}{H} + \frac{gH^2}{2} \\ \frac{UV}{H} \end{bmatrix}; \quad \vec{G} = \begin{bmatrix} V \\ \frac{UV}{H} \\ \frac{V^2}{H} + \frac{gH^2}{2} \end{bmatrix} \tag{12}$$

and

$$\vec{I} = \begin{bmatrix} qH \\ gH\left(\frac{\partial h}{\partial x} - \frac{1}{\rho g} \frac{\partial p_s}{\partial x}\right) + fV + 2qU + \frac{\tau_x^w}{\rho} - \frac{\tau_x^b}{\rho} \\ gH\left(\frac{\partial h}{\partial y} - \frac{1}{\rho g} \frac{\partial p_s}{\partial y}\right) - fU + 2qV + \frac{\tau_y^w}{\rho} - \frac{\tau_y^b}{\rho} \end{bmatrix} \tag{13}$$

Let

$$\vec{V}_{i,j}^{(n+1)} = \begin{bmatrix} H(i\Delta x, j\Delta y, (n+1)\Delta t) \\ U(i\Delta x, j\Delta y, p\Delta t) \\ V(i\Delta x, j\Delta y, q\Delta t) \end{bmatrix}$$

where p and q will be replaced by (n+1) or (n) as demanded by the ADI algorithm, then the finite difference approximation of Equation (11) is

$$\frac{\vec{V}_{i,j}^{(n+1)} - \vec{V}_{i,j}^{(n)}}{\Delta t} + \frac{1}{2\Delta x} \left[\vec{F}_{i+1,j} - \vec{F}_{i-1,j} \right] + (1-\theta) \left[\vec{F}_{i+1,j} - \vec{F}_{i-1,j} \right] + \dots = O(\Delta t, \Delta x^2) \tag{14}$$

where θ is a weighting factor ranging from 0 to 1, and $\vec{F}_{i,j}^{\sim}$ is given by

$$\vec{F}_{i,j}^{\sim} = \frac{\vec{F}_{i+1,j} + \theta \vec{F}_{i,j} + \vec{F}_{i-1,j}}{4} \tag{15}$$

Expansion of Equation (14) in Taylor series, it becomes,

$$\frac{\partial V}{\partial t} + \frac{\partial F}{\partial x} + \frac{\Delta x^2}{4}(1-\theta) \frac{\partial^3 F}{\partial x^3} + \dots = 0 \tag{16}$$

The third term which has the magnitude of the order of Δx^2 is the numerical filter term to screen the short wave number. For complete implicit scheme, $\theta = 1$, and therefore, no filter term is encountered. This would probably explain the frustration experienced by many model practitioners when Equation (14) with $\theta = 1$ is applied to the solution of circulation problems. Under such circumstances, the noise caused by the short wave would amplify very fast and blow out the computations. Instead of $\vec{F}^{(n)}$, if $\vec{F}^{\sim(n)}$ is used in Equation (14), similar problems will also arise since no filter term is involved to dampen the short wave noise.

Since \vec{F} , \vec{G} and \vec{I} are nonlinear functions of H , U , and V , Equation (14) is a system of nonlinear algebraic equations. These equations can be solved directly by iterative method. However, iterative solution to a large number of simultaneous equations at each time step is not only time consuming but also causes convergent problems in many areas. Hence, linearization techniques must be used to render these equations to a system of linear algebraic equations that direct solution can be obtained. In general, Taylor series expansion is adopted, i.e.,

$$\vec{F}^{(n+1)} = \vec{F}^{(n)} + \frac{\partial \vec{F}}{\partial H}(H^{(n+1)} - H^{(n)}) + \frac{\partial \vec{F}}{\partial U}(U^{(n+1)} - U^{(n)}) + \dots$$

For example, the term U^2 in the second component of \vec{F} in Equation (12) can be expanded as

$$U^2(n+1) = U^{(n)}(U^{(n+1)} + U^{(n+1)} - U^{(n)}) + \dots \tag{18}$$

Most of the numerical modelers would linearize this term by treating it as part explicit and part implicit (Leendertse, 1967; Tsai and Chang, 1974), i.e.,

$$U^2(n+1) = U^{(n)}U^{(n+1)} \tag{19}$$

It is seen from Equations (18) and (19) that the former will render the latter as its special case if $U^{(n+1)} \rightarrow U^{(n)}$. The implicit scheme, Equation (14), has been proved unconditionally stable when it is applied to linear partial differential equations (Leendertse, 1967). However, frustrations have been experienced more than often when it is applied to nonlinear equations in along with Equation (19), unless the time step is smaller than that given by Courant stability criteria. This fact may be explained by comparing Equations (8) and (19). The comparison indicates that the time step has to be small for the two to agree.

Boundary Conditions

There are two types of boundaries in the numerical simulation of storm surges. One is the water-land interface while the other is the fictitious open boundaries which are artificial termination of the flow field. During a storm, the wind generated surge close to shoreline is most prominent. For locations further seaward, the magnitude of the wind setup (or setdown) becomes progressively insignificant for several reasons. The first effect occurs due to decreased hindrance of the boundary. The increase in water depth accounts for the second reason as the water mass would be more difficult to set in motion at a location of greater depth by the water surface shear forces.

For locations beyond the Continental Shelf, the water elevation due to wind shear would be insignificant. Thus, at seaward boundary the setup (setdown) could be reasonably assumed equal to the barometric and astronomical tides. The treatment of lateral open boundaries is still very controversial. Reid and Bodine (1968) proposed radiative boundary conditions. Pearse and Pagenkopf (1975) assumed zero onshore transport. Zero gradient of water surface slope has been used with the requirements that the lateral boundaries be chosen roughly perpendicular to the bottom contours lines (Stone & Webster, 1976). In this paper, this requirement will be removed by specifying the gradient of total water depth equal to zero at lateral boundaries.

Most of the existing storm surge models does not consider the moving boundaries at the water-land interfaces. General practice is to assume vertical wall interface that the normal flow is zero (Pearse and Pagenkopf, 1975). A few investigators model the inundation of low-lying land with weir type approach (Reid and Bodine, 1968; Dansgaard and Dinsmore, 1975). In the present model, the moving boundaries are accomplished by progressively advancing (retreating) the land-water interface as surges increase (decrease). Both continuity and momentum equations are actually utilized in tallying these moving inundation boundary grids.

MODEL APPLICATION

The storm surge model is applied to the New Jersey coastal area for 1944, hurricane, whose track parallels the east coast and lies well within the confines of the Continental Shelf. The track of the pressure center in the storm is taken from the report by Graham and Hudson (1960). The hurricane characteristics were given both near Hatteras, North Carolina, and Point Judith, Rhode Island (Graham and Hudson) as follows.

	<u>Hatteras, NC</u>	<u>Point Judith, RI</u>
Radiative of maximum winds, R	49 N. Mi	36 N. Mi
Center pressure index, p_o	27.88 in.	28.31 in.
Peripheral pressure, p_n	30.66 in.	29.39 in.
Maximum gradient wind, V_{gx}	113 mph	71 mph

Based on these hurricane parameters and the track, wind and pressure fields are constructed using the method suggested by Graham and Nunn (1959).

The New Jersey coastal area is constructed on a rectangular grid of 22 by 31 with grid size of 33,333 ft. It included the Continental Shelf to a depth of approximately 600 ft and is extended far north and south of the area of interest that the lateral boundary conditions are applicable.

Surge histories for 1944 hurricane are available at both Atlanta and Sandy Hook tidal gage stations in the southern and northern New Jersey coast, respectively (Harris, 1963). Both recorded and computed surges with the model at Atlanta City are shown in Figure 2. Figure 3 shows comparisons between the tidal gage measurements and the simulated results at Sandy Hook. The agreements at both Atlanta City and Sandy Hook are considered favorably. The peak surge as computed at Sandy Hook is higher than the measured.

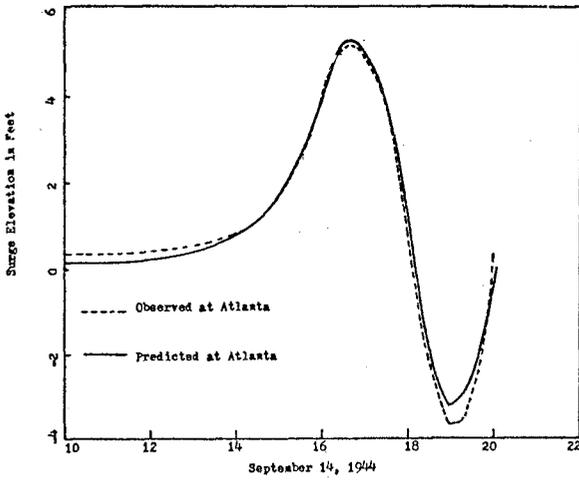


Figure 2

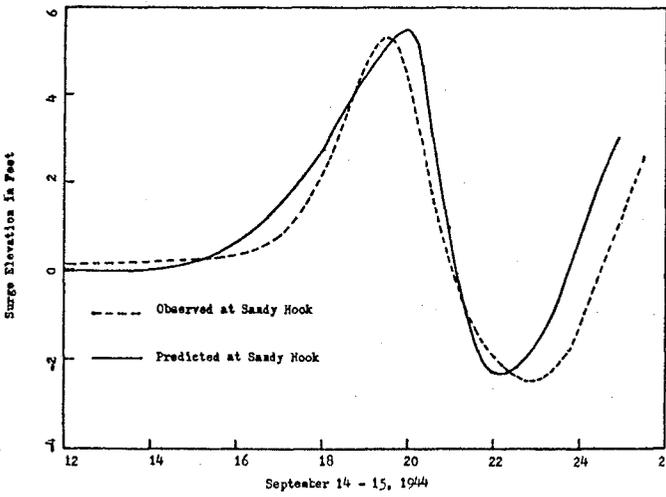


Figure 3

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