CHAPTER 30

THREE-DIMENSIONAL CONDITIONS OF SURF

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ABSTRACT

Wave height variability along the crest of breaking waves is shown to be a significant factor in the assessment of surf zone dynamics. Variations in excess of 50 percent of the maximum wave height can occur along a single crest without significant variations in bathymetry. The horizontal scale of this longshore variability in crest height corresponds to the wave length of incident breaking waves. Four possible mechanisms for this variability are postulated and then evaluated individually on the basis of field observations. A major result of these evaluations is that two-dimensional shallow-water wave equations appear to be inappropriate for expressing natural surf zone wave transformations and water motions even under the condition of waves encroaching on a plane sloping bottom. Consequently, three-dimensional equations of surf should be used for describing most natural surf zone dynamics.

INTRODUCTION

Conventional techniques for estimating dynamic conditions in the surf utilize linear or weakly non-linear transformations of a set of deep water wave parameters to predict a set of shallow water wave parameters assumed to be incident at the outer limits of the surf zone. Once this set of shallow water wave parameters has been determined it is then used to establish initial and seaward boundary conditions for calculating two-dimensional wave transformations and water motions within the surf zone. To a first approximation two-dimensional irrotational motion of inviscid fluid on a shoaling beach satisfies nonlinear shallow-water equations (Meyer and Taylor, 1963; Ho, et al., 1963). However, most natural surf conditions are not two-dimensional thus, it is necessary to determine if longshore variations in the initial and seaward boundary conditions are of significant magnitude to invalidate the use of two-dimensional theory.

The three-dimensional time dependent shallow water equations of motion show that longshore variations in surf are caused primarily by longshore variations in the wave field approaching the coast and by longshore variations in bottom geometry within the surf zone (Stoker, 1957). Meyer and Turner (1967) have shown, for initial and seaward boundary conditions independent of distance parallel to shore, that when the longshore bottom profile slope is gentler than the normal bottom profile slope "weakly three dimensional" surf can still be analyzed directly by two-dimensional theory. However the effect of longshore variations in the incident wave field at the outer surf zone has not yet been investigated.

Therefore, the primary purpose of this study was to measure three-dimensional variations in surf zone wave conditions and to relate them to incident wave conditions at the outer surf zone. An analysis of longshore variations in wave height at the outer surf zone was carried out in order to evaluate uniformity of the incident wave field. Wave height probability distributions were calculated from the observed data in order to show the distributional changes related to wave breaking and transformation through the surf.

THREE-DIMENSIONAL EQUATIONS OF SURF

Derivation of a shallow water wave theory from exact hydrodynamic equations utilizing a formal perturbation procedure, where all quantities are expanded in powers of $\sigma = kh$, (h is the depth and k is the maximum initial curvature of the free surface) has been carried out by Friedrichs (1948) and Stoker (1957). Defining the x, z plane as the undisturbed water surface with the y-axis positive upward, the free surface and bottom position are specified by $y = \eta\ (x, z, t)$ and $y = -h\ (x, z)$ respectively. The general three-dimensional problem is then formulated in terms of the equations of motion

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g \tag{2}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{3}$$

and continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{4}$$

It is assumed that the flow is irrotational with kinematic boundary conditions at the bottom and free surface given by

$$u\frac{\partial h}{\partial x} + v + w\frac{\partial h}{\partial z} = 0 \quad \text{at } y = -h \tag{5}$$

$$\frac{\partial \eta}{\partial t} + u\frac{\partial \eta}{\partial x} + w\frac{\partial \eta}{\partial z} = v \quad \text{at } y = \eta \tag{6}$$

respectively and a dynamic boundary condition at the free surface given by

$$p = 0 \quad \text{at } y = \eta. \tag{7}$$

Introduction of dimensionless variables through the use of a typical depth h and a typical horizontal length k into equations (1) through (7) and expanding u, v, w, η, and p in a power series of σ provides the opportunity to equate coefficients of like powers of σ. The first order approximation to u, w, and η are then given by the "three-dimensional shallow-water equations" (Stoker, 1957).

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} + \frac{\partial \eta}{\partial x} = 0 \tag{8}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} + \frac{\partial \eta}{\partial z} = 0 \tag{9}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial[u(\eta + h)]}{\partial x} + \frac{\partial[w(\eta + h)]}{\partial z} = 0. \tag{10}$$

TWO-DIMENSIONAL EQUATIONS OF SURF

It is easily seen that if w is assumed to be zero and all other quantities are assumed to be independent of z that equations (8) through (10) become the "two-dimensional shallow-water equations" [Stoker, 1947; Friedrichs, 1948; Stoker, 1957; Meyer and Taylor, 1963; and Meyer and Turner, 1967]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial \eta}{\partial x} = 0 \tag{11}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial[u(\eta + h)]}{\partial x} = 0. \tag{12}$$

Meyer and Turner (1967) also show these equations to be applicable to "weakly three-dimensional surf". Equations (11) and (12) are further simplified by assuming that u and η and their derivatives are small quantities whose squares and products are negligibly small compared to the linear terms. The resulting equations are the "linear shallow-water equations"

$$\frac{\partial u}{\partial t} + \frac{\partial \eta}{\partial x} = 0 \tag{13}$$

$$\frac{\partial \eta}{\partial t} + \frac{\partial (uh)}{\partial x} = 0 \tag{14}$$

The preceeding derivation depends upon three important simplifying assumptions with respect to longshore water motions and bottom variability: that the longshore velocity component is zero; that the incident wave field at the seaward boundary of the surf zone is uniform in the longshore direction; and that the bottom slope in the longshore direction is zero. Meyer and Turner (1967) showed analytically that unless $\beta/\varepsilon \to 1$, where ε is the beach slope normal to shore and β is the beach slope parallel to shore, the first and third assumptions above are valid approximations. However, they accepted the second assumption implicitly, asserting it was, in fact, valid whenever initial and seaward boundary conditions were primarily dependent upon beach slope ε. This assumption that uniform wave crests are a reasonable expectation for an incident wave field at the outer surf zone is one of the most important assumptions in formulating the two-dimensional equations of surf. A field evaluation of this assumption forms the primary context for this paper.

FIELD EXPERIMENT

In order to evaluate three-dimensional conditions of surf a three by three wave gauge array was positioned in the surf zone with its longshore axis parallel to the crests of incident breaking waves, Figure 1. Separation distance between individual wave monitoring positions was approximately one half the incident wave length. The three outer most stations were located seaward of the zone of active breaking. The six inner stations were located within the breaker zone and bracketed the region of most active breaking. Approximately eighty percent of initial breaking occurred within the monitoring array.

The bottom in the study area was characteristic of a sandy barred coastline with isobaths parallel or slowly varying in the longshore direction. Positioning of the monitoring array was such that ε was large (normal beach slope 1:20) and $\beta/\varepsilon \to 0$ (to correspond to the conditions suggested by Meyer and Turner, 1967).

Wave monitoring was carried out continuously for fifteen minute periods separated by fifteen minute intervals. Wave breaking and transformation in the surf zone was monitored using a super 8 mm motion picture camera. The motion picture film records were taken in order to analyze the individual crests behavior as they passed through the monitoring grid. The only conditions monitored were those when waves broke within the monitoring grid. Conditions resulting in breaking or breaking and reforming seaward of the grid were not measured. Conditions during which waves appeared to be arriving from more than one direction, at the outer surf zone, were also excluded from the monitoring periods.

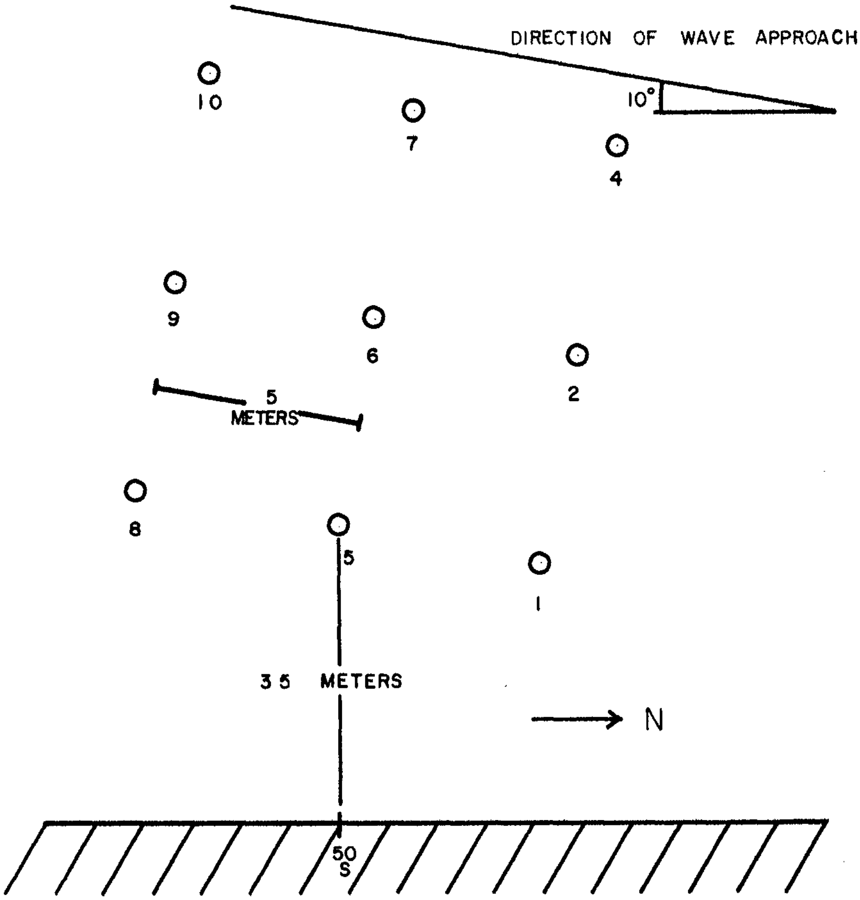


FIGURE I. STATION CONFIGURATION FOR EXPERIMENT

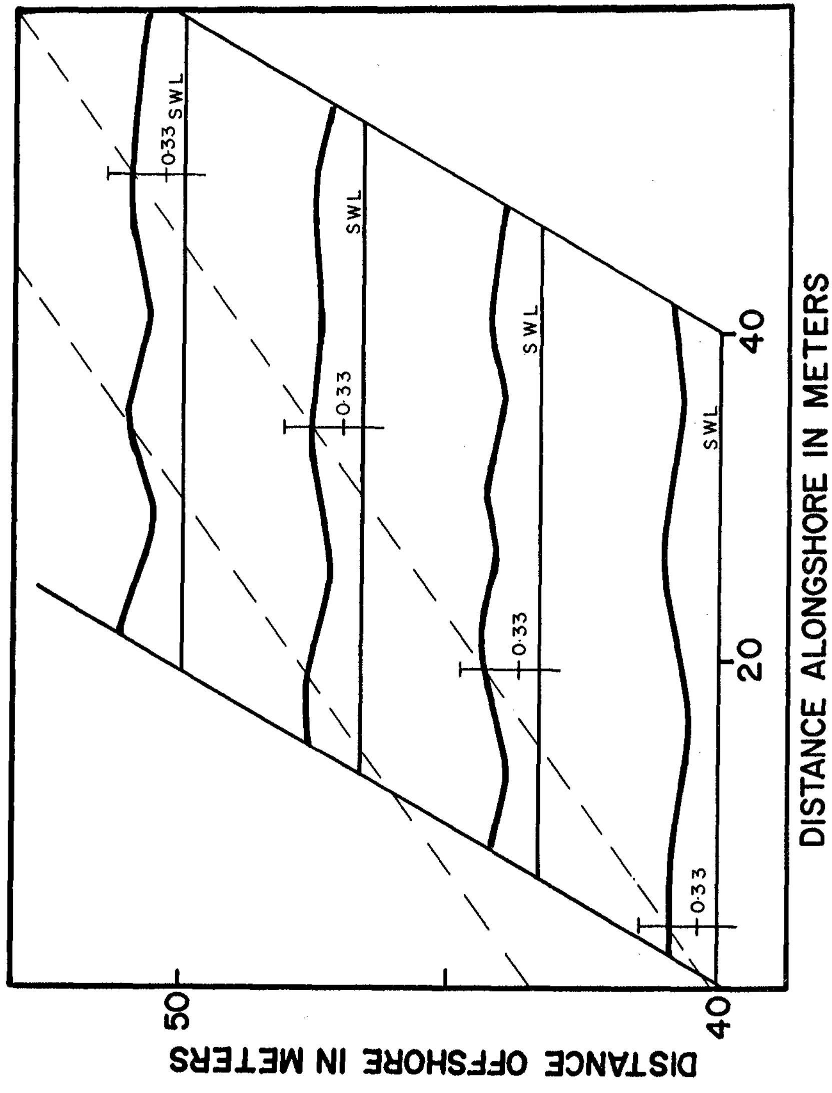
EXPERIMENTAL RESULTS

Initially the movie film records were analyzed using the in situ grid, provided by the wave monitoring stations, as a reference base. Wave height along a single crest was observed to vary noticeably over short horizontal distances. Figure 2 shows successive wave crest height profiles for a single wave from approximately five meters seaward of the breaker zone to a position where fully developed breaking is occurring along the entire crest. The primary result from these film records is that significant variation in wave crest height is observed seaward of the breaker zone and prior to the development of major instability in the wave crest.

Nine free surface time histories from each monitoring period were evaluated directly and analyzed in two separate statistical contexts. First, wave height probability distributions were generated for each of the monitoring positions from the time history data and second, wave spectra were computed from each of the time series. The use of probability distributions was extremely convenient for understanding individual crest transformations in the surf zone.

Figures 3, 4 and 5 show five successive waves, measured at three positions along the crest, at the outer, middle and inner lines of the array (see Figure 1 for station locations). This series of waves is representative of the longer time histories and clearly shows the characteristic variation in wave height along the crest. Seaward of the surf zone, variations in excess of 50 percent of the maximum wave height occur along an individual wave crest, Figure 3. The regions of high crest height, stations 10 and 4, tend to maintain the same relative relationship to the region of low crest height, station 7, for successive wave passages. Many of the wave profiles at stations 10 and 4, appear to be near breaking waves. Film records verify, however, that no local breaking has occurred at or prior to reaching these stations. As this same group of waves pass into the breaker zone the crest height variability appears to be less clearly related, Figure 4. The low wave crests seaward of the breaker zone, station 7, have amplified and are steepening towards breaking limits, station 6. High wave crests seaward of the breaker zone, stations 10 and 4, have generally decreased in height due to breaking, stations 2 and 9. The result of these two inverse processes is to create a relatively uniform crest-wise wave height distribution. Near the shoreward limit of the breaker zone, these same waves again exhibit variations in excess of 50 percent of the maximum wave height along an individual wave crest, Figure 5. In contrast to the outer stations, Figure 3, the region of high crest height, station 5, is flanked by two regions of low crest height, stations 8 and 1. This result is, however, what would be anticipated due to wave height decay through breaking, at stations 8 and 1, and continued steepening or initial breaking at station 5. Thus differential breaking seems to be a reasonable explanation for the systematic changes along the crest as an individual wave passes through the surf zone. It is not, however, a satisfactory explanation for the initial conditions observed at the seaward boundary of the surf zone.

FIGURE 2. WAVE SURFACES FROM FILM RECORDS



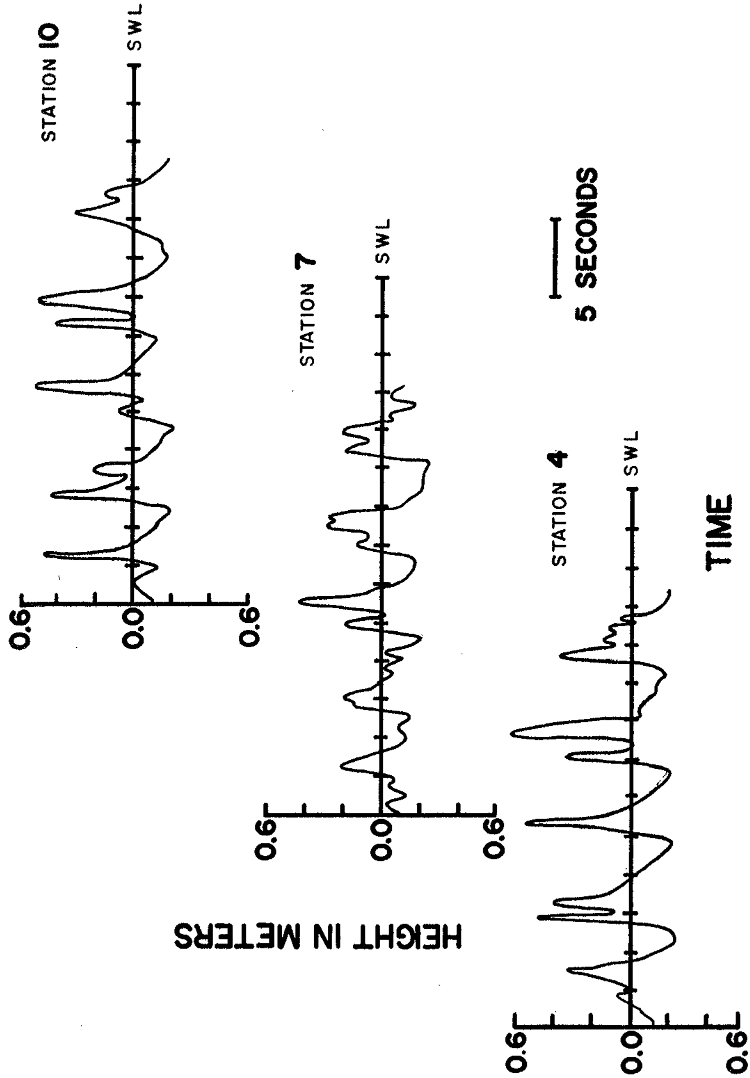


FIGURE 3. FREE SURFACE TIME HISTORIES AT OUTER LINE OF ARRAY

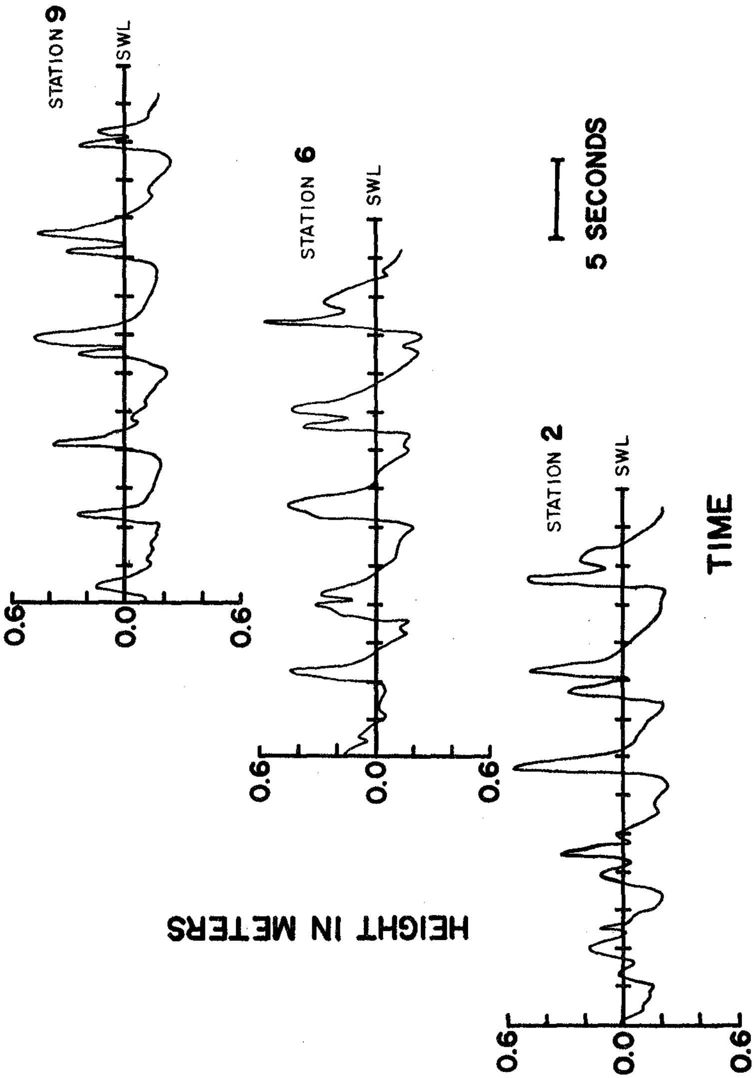


FIGURE 4. FREE SURFACE TIME HISTORIES AT MIDDLE LINE OF ARRAY

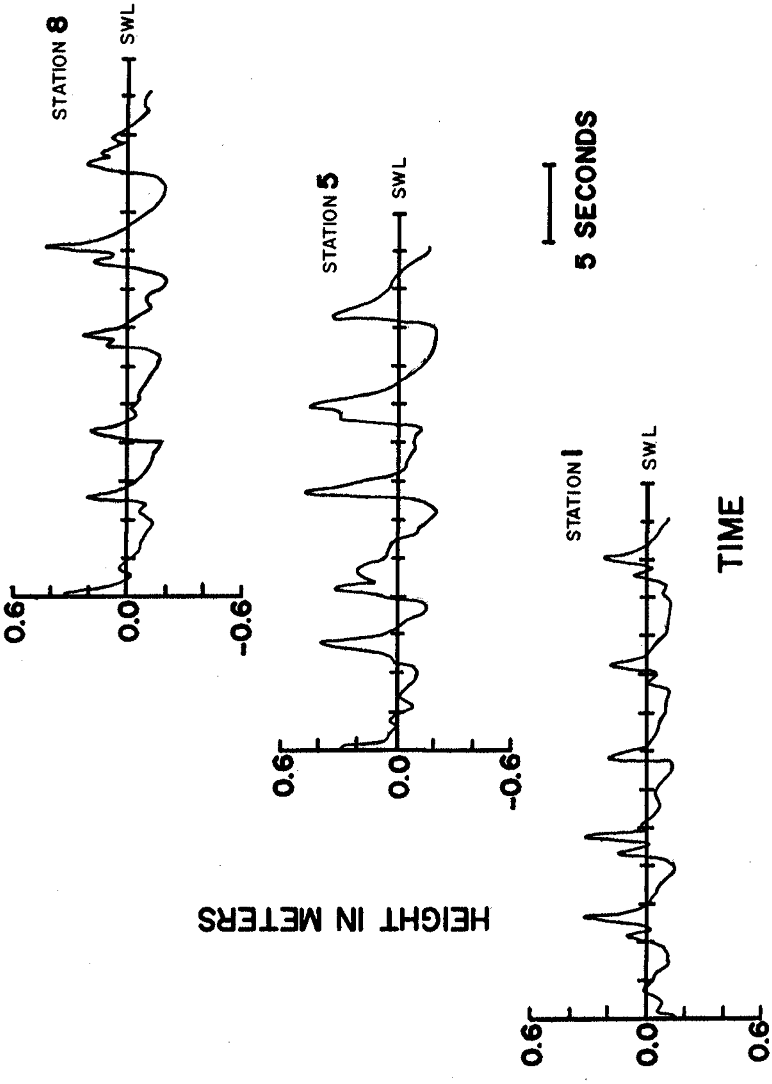


FIGURE 5. FREE SURFACE TIME HISTORIES AT INNER LINE OF ARRAY

In order to verify that the wave crest height relationships, suggested by a limited sample of individual waves, Figures 3, 4 and 5, were characteristic of the entire time series, probability distributions were computed for the entire 15 minute monitoring period (approximately 200 waves). Comparisons of the probability distributions from the outer and inner lines of the array support the relative wave height variability suggested in the analysis of individual waves. Comparison of the probability distributions at the middle line of the array are inconclusive. Although the trend of the individual distributions is consistent with the inverse process argument posed earlier.

Analysis of variance was carried out for each of the three lines of the array. The results of this analysis supported the hypothesis that significant differences exist between individual probability distributions at the outer and inner lines of the array, $F\ (2,600) = 5.27$, $p < 0.01$ and $F\ (2,600) = 10.54$, $p < 0.01$ respectively. The results of the analysis failed to support the hypothesis that significant differences exist between individual probability distributions at the middle line of the array, $F\ (2,600) = 4.09$ rejected at $p < 0.01$.

Spectrum analysis of the nine stations individual time series indicated two major spectral peaks common to all of the series. A 6.0 second peak corresponded to the incident swell period and a 3.3 second peak corresponded to the breaker period. A long period component ($T > 50$ seconds) was present in all of the time series from the monitoring stations seaward of the breaker zone. This component also appears in all but one of the six breaker zone time series.

DISCUSSION

Longshore variation in wave crest height is neither an unexpected nor unrecognized physical occurrence in the surf zone. Bowen (1969) showed that periodic longshore variation in wave height caused by either bottom geometry or mean water level fluctuations was responsible for the generation and spacing of rip cells. Bowen and Inman (1969) suggested that edge waves could account for longshore variations in wave height on a planer beach. Inman, Tait, and Nordstorm (1971) assert that "the interaction of edge waves with incoming waves of the same frequency produces systematic fluctuations in breaker height along the beach." Guza and Davis (1974) showed that waves incident on a planer beach can in fact generate these edge waves, but of a mode different from that of Bowen and Inman (1969). Finally, Dalrymple (1975) has shown that longshore variation in wave height can be anticipated for conditions where two wave trains of the same frequency arrive at the outer surf zone from two distinct directions.

Subsequently there are four mechanisms which can be postulated for the causation of longshore variation in wave crest height: significant longshore variation in bottom geometry $\beta/\varepsilon \rightarrow 1$; interaction of edge waves with the incident wave field; interaction of long-crested swell arriving from two distinct directions; and arrival of a "short-crested" wave field generated by local storm winds. Experimental conditions during the observations from this study tend to control for the first and third postulated causations. There were, of course, two distinct frequencies present in the wave spectrum at 6.0 and 3.3 seconds which clearly indicates the presence of two wave groups. However, the direction of arrival of these wave groups appeared to be coincident and their frequency separation was quite large for significant interaction to occur.

The possibility of edge wave interaction with the incident wave field is likely. Fluctuations in the total longshore velocity field with a periodicity close to that of the zero mode edge wave have been observed under similar field conditions (Wood and Meadows, 1975, Meadows, 1976). The wave length of the longshore wave height fluctuations, from this experiment, are equivalent to the incident wave length of the breaking waves. Thus the periodicity of these fluctuations would seem to be much shorter than those calculated from edge wave theory for the observed conditions. Likewise the observed wave height fluctuations are in excess of those anticipated by edge wave theory.

The final postulated mechanism, of an incident wave field arriving with a "short-crested" structure independent of the first three mechanisms, is theoretically possible but, within the limitations of this experiment difficult to substantiate. It has long been recognized that turbulence in a wind field can result in the generation of a short-crested wave field (Jefferies, 1924). Likewise McClenan and Harris (1975) analyzed 40,000 aerial photographs of waves in shallow-water and concluded that the dominant pattern is one of short-crested waves and randomness. Notably missing between these studies is the concurrent observation of wind and wave field structure in shallow-water. The crest height wave length correspondence to incident breaking wave length and the large fluctuations in wave height along the crest do, however, present intriguing possibilities for short-crested wave structure. Clearly, the question left to be answered is whether energy is being transmitted along the crests of these waves.

CONCLUSIONS

Significant variations in wave height occur along the crest of waves incident at the outer surf zone. These variations are independent of bathymetric control and persist through the surf zone due to differential effects of shoaling and breaking. The wave length of these variations is the same as that of the incident breaking waves.

Of the four postulated mechanisms which might cause these variations only bathymetric variability seems an unlikely mechanism under these experimental conditions. Therefore, the assumption, accepted implicitly by Meyer and Turner (1967), that longshore variability in the incident wave field is negligible whenever $\beta/\varepsilon \rightarrow 0$ and the normal beach slope is moderate to steep, is incorrect. Consequently, treatment of surf zone dynamics utilizing two-dimensional shallow-water equations seems to be generally inappropriate. In fact, the full three-dimensional equations of surf (8, 9, and 10) appear to be necessary to properly represent natural surf zone dynamics.

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REFERENCES

Bowen, A.J., Rip Currents, 1, Theoretical Investigations, Jour. Geophys. Res., 74, 5467-5478, 1969.

Bowen, A.J. and D.L. Inman, Rip Currents, 2, Laboratory and Field Observations, Jour. Geophys. Res., 74, 5479-5490, 1969.

Dalrymple, R.A., A Mechanism for Rip Current Generation on an Open Coast, Jour. Geophys. Res., 80, 3485-3487, 1975.

Friedrichs, K.O., On the Derivation of the Shallow Water Theory, Commun. Pure Appl. Math., 1, 81-85, 1948.

Guza, R.T. and R.E. Davis, Excitation of Edge Waves by Waves Incident on a Beach, Jour. Geophys. Res., 79, 1285-1291, 1974.

Ho, D.V., R.E. Meyer, and M.C. Shen, Long Surf, Jour. Marine Res., 21, 219-232, 1963.

Inman, D.L., R.J. Tait, and C.E. Nordstrom, Mixing in the Surf Zone, Jour. Geophys. Res., 76, 3493-3514, 1971.

Jefferies, H., On Water Waves Near the Shore, Phil. Mag., Series 6, 48, 44-48, 1924.

McClenan, C.M. and D.L. Harris, The use of Aerial Photography in the Study of Wave Characteristics in the Coastal Zone, Technical Memorandum, No. 48, CERC, U.S. Corps of Engineers, 72 pp., 1975.

Meadows, G.A., Time Dependent Fluctuations in Longshore Currents, Proc. 18th Conf. Coast. Eng. (in press) 1976.

Meyer, R.E. and A.D. Taylor, On the Equations of Surf, Jour. Geophys. Res., 68, 6443-6445, 1963.

Meyer, R.E. and R.B. Turner, Some Three-Dimensional Effects in Surf, Jour. Geophys. Res., 72, 2513-2518, 1967.

Stoker, J.J., Surface Waves in Water of Variable Depth, Quart. Appl. Math., 5, 1-54, 1947.

Stoker, J.J., Water Waves, Interscience Publishers, New York, 1957.

Wood, W.L. and G.A. Meadows, Unsteadiness in Longshore Currents, Geophys. Res. Lett., 2, 503-505, 1975.