CHAPTER 22

WAVE HEIGHT DISTRIBUTION OF WIND WAVES OVER LONG WAVES

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ABSTRACT

Influence of long wave on the wave height distribution of wind waves was studied through the laboratory experiment. Experiments were conducted in a wind-wave tank where the wind waves were generated by a wind blower and the long waves were developed by an oscillating pendulum type wave generator.

The wave height distribution of the wind waves over long wave is slightly different from the Rayleigh distribution in the small steepness of long wave. The ratios between the average of highest 1/n-th waves vary with the steepness of long waves. The magnitude and the location of the spectral peak of wind waves are altered. The amount of the attenuation of wind wave energy is larger than the results of Mitsuyasu (1966).

1. INTRODUCTION

The design of marine structures have often been done with the concept of representative wave. Offshore structures are mostly designed against the maximum probable wave at the structure site. Coastal structures are usually designed against the significant wave at the locality. However, marine structures are subjected to the action of irregular waves. The analysis of irregular wave action on marine structurees requires the information of statistical distribution of wave heights. Ito, et.al.(1966) employed the wave height distribution function to estimate the possible sliding distance of the breakwater. This method has been adopted in the practical breakwater design (The Japan port and Harbour Association, 1970). Goda(1970) calculated the rate of wave overtopping of sea walls from the wave height distribution.

In this kind of analysis, the detailed distribution function of wave heights is needed. Theoretical distribution of wave heights, defined by the crest-to-trough method, was developed by Longuet-Higgins(1952) as the Rayleigh distribution, under the assumptions of narrow spectrum and random phases. There are lots of investigators confirm this wave height distribution. However, it is uncertain in case where the wind waves are propagated over the surface of long waves such as ocean swell or tidal currents.

The interaction of short wave and long wave has been studied by Longuet-Higgins and Stewart(1960) for the case of same wave type and by Mitsuyasu(1966) for the case of different wave type. In this paper, the influence of long wave on the wave height statistics of wind waves is presented.

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2. EXPERIMENTAL EQUIPMENT AND PROCEDURE

The experiment was conducted in a wind - wave tank of 75 m long, 1m wide and 1.2 m deep as shown in Fig.1. The long waves were generated by an oscillating pendulum type wave generator at one end of the tank. Wind waves are generated in the same direction by a 10 Hp motor, located at 10 m down the pendulum wave generator. The cross section of the outlet of wind blower is 1m in width and 0.5 m in height. The bottom of the duct is 0.2 m above the still water level such that the regular waves may pass through the gap. The wind velocity at the center of the wind passage was measured by a pitot - static tube in conjunction with a micromanometer. The micromanometer was allocated in a slope of $\sin\theta = 1/4$, and filled with two insoluble liquids of ink solution and colorless benzene, each has the spacific gravity of 0.998 and 0.875 at the temperature of 20°c respectively. The pressure difference bearing at the pitot - static tube can be read out easily and hence the wind velocity was obtained. Waves were measured by the resistance type wave gauge and recorded on the visigraph.

Wind waves were generated by three free stream wind velocities, i.e. 8.10m/s, 10.73m/s and 12.45m/s. The regular long waves were produced by three periods and five wave heights. The properties of regular long waves used in the present experiment were listed in the table 1. The steepnesses of the long waves are all below 0.01. Each experimental condition includes the case of three wind velocities, that makes altogether 45 experimental runs.

Table 1. Regular long waves used in the experiment

Mean period (sec)	Mean wave height (cm)	Steepness
3.06	1.49	0.0010
3.06	2.14	0.0015
3.13	3.62	0.0024
3.07	6.34	0.0043
3.09	7.44	0.0050
2.82	6.94	0.0056
2.81	5.01	0.0041
2.83	4.78	0.0038
2.84	3.48	0.0028
2.80	1.98	0.0016
2.51	2.47	0.0025
2.51	3.64	0.0037
2.48	4.99	0.0052
2.48	5.91	0.0062
2.48	8.10	0.0084

In the experiments the regular long wave was generated first, and then wind was sent over the surface of long wave. Each run lasts for five minutes, and the co - existent system was recorded for last two minutes. The wave data at fetch 16.4 m were presented here.

3. EXPERIMENTAL RESULTS

3.1 General features of the combined wave field

Figure 2 shows the examples of the wave record. In the figure, (a) is the record of wind wave unaffected by the long wave; (b) is the record of mechanically generated long wave: (c) is the record of co- existent system in almost steady state. Figure 3 shows the examples of co - existing system under different wave steepness of long waves. The wave heights are taken as crest-to-trough wave height through out this analysis. The wave statistics of wind waves and long waves are calculated separately, and the combined wave field of wind waves and long wave was also computed as given in Table 2, where the valves in backet denoted the wave statistics for wind waves only.

3.2 Wave height statistics

The theoretical wave height distribution derived by Longuet - Higgins for a narrow band spectrum is given as

$$P(H)dH = \frac{1}{4} \frac{H}{\sigma^2} \exp\left(-\frac{H^2}{8\sigma^2}\right) dH \quad \ldots\ldots\ldots\ldots\ldots (1)$$

where σ^2 is the variance of surface elevation. The ratios of average heights of the highest 1/n-th waves have been calculated as

$$\frac{H_{1/3}}{H_a} = 1.598, \quad \frac{H_{1/10}}{H_{1/3}} = 1.270 \text{ and } \frac{H_{max}}{H_{1/3}} = 1.601 \text{ (for N=160)} \ldots \quad (2)$$

Although several reasons could affect the wave height distribution, such as the nonlinear wave - wave interaction, the breaking effect, the instrument of wave recorder, and the use of a finite number of component wave heights, etc., the wave height distribution observed in the field (Grodnight, 1963; Collins, 1967; Koele, 1960: Goda, 1974; etc.) has been shown generally in good agreement with the Rayleigh distubution.

In the present analysis, the distributions of crest - to - trough wave heights of wind waves over the long wave are computed. Figure 4 is the wave height distribution of wind waves over the long wave as given in Figure 3. It is seem that the wave height distributions do not deviate too much from the Rayleigh distribution, except for the wind waves only and the low wave steepness. All the experimental data of wave beight distribution are summarized into four groups According to the steepness of long wave, i.e., H/L = 0, 0.001-0.003, 0.003-0.006, 0.006-0.009. The data in the same group are averaged and formed the representive wave height distribution as given in Figure 5. Seeing that the wave height distributions vary with the steepness of the long

Table 2. Properties of co-existent system

Run	Wind waves		Long waves			Wind waves plus long waves					
	Velocity (m/s)	$(E)_0$ (cm^2)	T (sec)	H (cm)	H/L	E (cm^2)	H_{max} (cm)	$H_{1/10}$ (cm)	$H_{1/3}$ (cm)	H_a (cm)	H_r (cm)
	8.10	0.73					(3.10)	(2.17)	(1.67)	(1.95)	(2.24)
1			3.06	1.49	0.0010		5.46	4.16	3.30	2.76	3.06
2			3.06	2.14	0.0015	2.07	6.38	4.23	3.35	2.90	3.23
3			3.13	3.62	0.0024	3.32	3.77	2.77	2.04	1.85	2.28
4			3.09	7.44	0.0050	13.56	7.55	5.30	3.64	2.72	3.45
5			3.07	6.34	0.0043	10.91	12.60	7.43	4.21	3.30	4.36
6			2.82	6.94	0.0056	4.84	6.72	5.19	4.04	3.09	3.43
7			2.83	4.78	0.0038	20.37	13.85	10.23	5.81	4.61	6.54
8			2.81	5.01	0.0041	13.10	8.81	5.87	3.99	3.07	3.86
9			2.84	3.48	0.0028	5.16	5.88	4.67	3.45	2.69	3.70
10			2.80	1.98	0.0016	1.96	5.88	4.40	3.40	2.62	2.94
11			2.51	2.47	0.0025	2.62	6.72	4.01	3.02	2.51	2.83
12			2.51	3.64	0.0037	1.79	6.30	4.19	3.13	2.55	2.87
13			2.48	4.99	0.0052	2.91	6.30	4.38	3.15	2.50	2.85
14			2.48	5.91	0.0062	4.71	6.72	4.51	3.28	2.57	3.05
15			2.48	8.10	0.0084	7.11	7.97	5.59	3.96	2.76	3.42
	10.73	3.26					(9.65)	(6.95)	(5.95)	(4.93)	(5.22)
16			2.48	8.10	0.0084	10.3	10.53	7.61	5.73	4.45	5.32
17			2.48	5.91	0.0062	7.36	10.50	7.08	5.39	4.00	4.66
18			2.48	4.99	0.0052	5.78	10.50	7.11	5.94	4.76	5.23
19			2.51	3.64	0.0037	5.04	9.65	7.61	6.37	4.95	5.46
20			2.51	2.47	0.0025	4.09	10.92	7.29	6.07	5.27	5.69
21			2.80	1.98	0.0016	4.01	9.54	7.54	6.77	5.31	5.66

Table 2. Properties of co-existent system(continued)

Run	wind waves		Long waves			Wind waves plus long waves					
	Velocity (m/s)	(E)₀ (cm²)	T (sec)	H (cm)	H/L	E (cm²)	Hmax (cm)	H1/10 (cm)	H 1/3 (cm)	Ha (cm)	Hr (cm)
22			2.84	3.48	0.0028	4.74	8.71	6.14	5.24	4.16	4.73
23			2.81	5.01	0.0041	8.41	12.45	7.70	5.96	4.81	5.59
24			2.83	4.78	0.0038	11.30	9.13	5.39	4.43	4.00	5.16
25			2.82	6.94	0.0056	17.4	10.37	8.54	5.88	4.63	5.72
26			3.07	6.34	0.0043	11.35	10.79	7.52	5.37	4.01	4.78
27			3.09	7.44	0.0050	8.94	9.13	6.84	5.44	4.59	5.24
28			3.13	3.62	0.0024	5.91	9.13	6.66	5.52	4.32	4.87
29			3.06	2.14	0.0015	4.3	8.30	7.08	6.21	5.03	5.38
30			3.06	1.49	0.0010	4.5	8.71	6.99	5.67	5.12	5.60
	12.45	7.52					(18.26)	(10.25)	(8.63)	(7.41)	(7.79)
31			3.06	1.49	0.0010	6.95	12.45	10.01	8.77	7.00	7.39
32			3.06	2.14	0.0015	7.33	12.45	9.67	8.25	6.89	7.23
33			3.13	3.62	0.0024	7.00	10.37	8.37	6.89	5.94	6.35
34			3.09	7.44	0.0050	10.56	14.94	10.37	7.96	6.62	7.27
35			3.07	6.34	0.0043	9.01	15.35	10.24	7.46	5.29	6.13
36			2.82	6.94	0.0056	17.36	15.35	12.27	7.81	5.75	7.00
37			2.83	4.78	0.0038	13.69	14.11	9.77	6.92	5.39	6.51
38			2.81	5.01	0.0041	8.64	12.45	9.10	7.29	5.66	6.22
39			2.84	3.48	0.0028	7.56	10.79	8.90	7.27	6.10	6.59
40			2.80	1.98	0.0016	6.78	14.94	10.01	8.22	6.87	7.28
41			2.51	2.47	0.0025	6.94	12.86	9.70	8.08	6.80	7.18
42			2.51	3.64	0.0037	6.05	10.79	8.76	7.14	5.94	6.43
43			2.48	4.99	0.0052	6.60	11.62	8.29	6.79	5.82	6.50
44			2.48	5.91	0.0062	9.50	14.52	9.82	7.58	5.80	6.55
45			2.48	8.10	0.0084	10.22	13.28	10.07	7.06	5.60	6.44

wave. The wave height distribution of wind waves in absence of long wave does not follow the Rayleigh distribution. As the steepness of long wave increases, the densities in the lower part of the distribution fuuction begin to grow, and the wave height distribution of the co - existent system of wind waves and long wave show slightly close to Rayleigh than before, as in the case of H/L = 0.001-0.003 and H/L = 0.003-0.006. However, when the steepness of long wave continue to increase as in the case of H/L = 0.006-0.009, the distribution shows deviate from Rayleigh again.

Figure 6 shows the values of ratios among various wave height in relation to the steepness of long waves. It is noted that the values of $H_{\frac{1}{3}}$ /Ha and $H_{1/10}$ / $H_{1/3}$ are smaller than the theoretical values given by Equation (2), but the ratio of H max/$H_{1/3}$ is larger, comparing the theoretical values. The ratios of $H_{1/3}$ / Ha, $H_{1/10}$/ $H_{1/3}$, H max/$H_{1/3}$ are increased in proportion to the steepness of long wave.

Figure 7 shows the relations between $H_{1/10}$ / Ha - U/Co and $H_{1/10}$ / $H_{1/3}$ - ε^2, where U is the free stream velocity, Co is the phase velocity of long wave, ε is the bandwidth of the co-existent system computed from power spectrum. It is seen that $H_{1/10}$ / $H_{1/3}$ increases with ε^2, and $H_{1/3}$ / Ha seems has nothing to do with U/Co.

3.3 The spectrum and the attenuation of wind wave energy

Spectra of wind waves alone and the co-existent system of wind waves and long waves are analysed in order for the better understanding of the detailed properties of wind waves on long waves. Each record of two minutes in length is digitized with 0.1 sec interval. The computation of wave spectrum was done by the IBM 1130 digital computer. Each run consumes 7 minutes of computer time. The adopted program was based on Blackman and Tukey method with sample size N=1024, maximum lag M=102, Niquist frequency f_N = 5.0 c/s, resolution f=0.049 c/s.

In Fig.8, the solid line shows the power spectrum of wind waves for wind velocity equeals to 12.45 m/s. As long waves appear, the spectrum of co-existent system shows a double peaks, which can clearly separated each other. The high frequency peak represents the wind wave spectrum, and the low frequency peak represents the long wave spectrum. Fig.8 shows that the wind wave spectrim is attenuated gradually wity increasing the steepness of long waves. The scale of ordinate in the figure is changed in the high frequency range so as to see more clearly the changes of wind wave spectrum.

The total energy in each run as listed in Table 2 is computed through the spectrum calculation. In Table 2, $(E)_0$ indicates the total energy of wind waves in the absence of long waves; E represents the total energy of the co-existent system of wind waves and long waves. The total energy of the co-existing system is increased in most cases due to the appearance of long waves. But that part of wind waves energy in the co-existent system is decreased, as has shown in Fig.8. Let E' be the energy contained in the wind waves of the co-existent system. E' can be obrained by the numerical integration of each spectrum. Fig.9 shows the attenuation of wind wave energy as relation to steepness of the long waves. The attenuation rate according to long wave steepness is larger than the result of Mitsuyasu (1966). The attenuation of wind waves energy might due to the breaking of wind waves near the crest of the long wave, as has pointed out by Longuet-Higgins and Stewart (1960). The other reason of attenuation is due to the change of wind field induced by the passing long waves. The peak of wind wave spectrum is also attered. Fig.9 shows the results.

4. CONCLUSION

The interaction between short wind-generated waves and the long mechanically-generated waves involves complicated energy transfer in the wind filld and wave field. The experimental results show the inflnence of long wave on the wave statistics of wind waves. The wave height distributions of wind waves over small steepness of long waves show not much deviate from the Rayleigh distribution, but the ratios of various wave heights are varied both with the steepness of long waves and the band width of the co-existent spectrum. The attenuation of wind wave energy is directly proportion to the steepness of long wave. The peak of wave wave spectrum is shifted.

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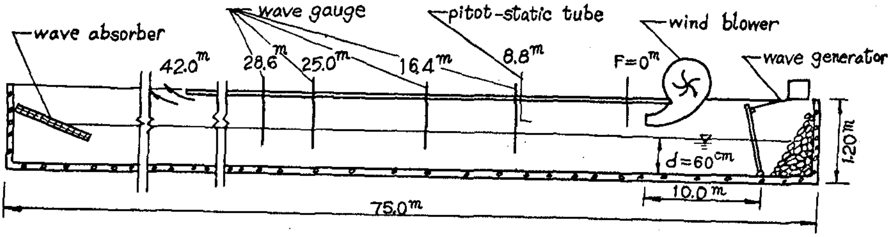


Fig. 1 EXPERIMENTAL WIND WAVE TANK.

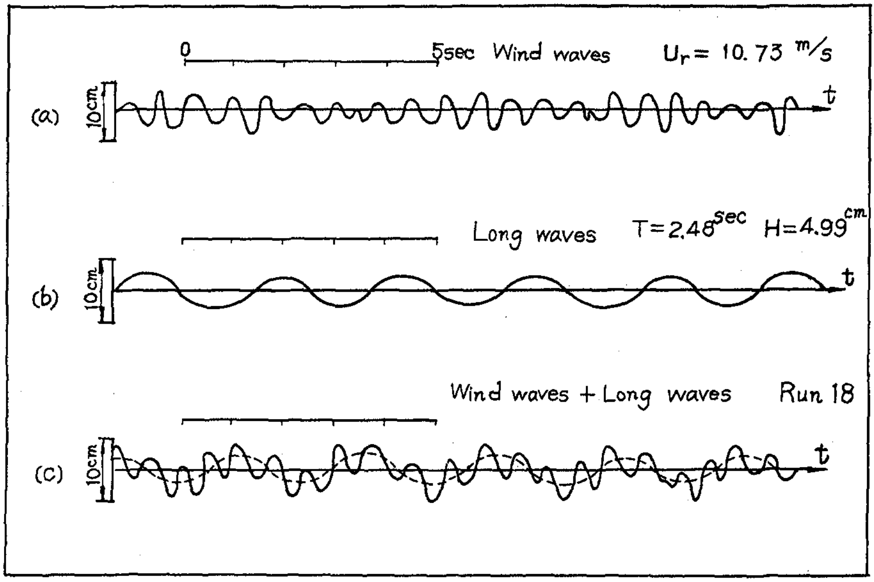


Fig. 2 SAMPLES OF WAVE RECORDS (AT 16.40M)

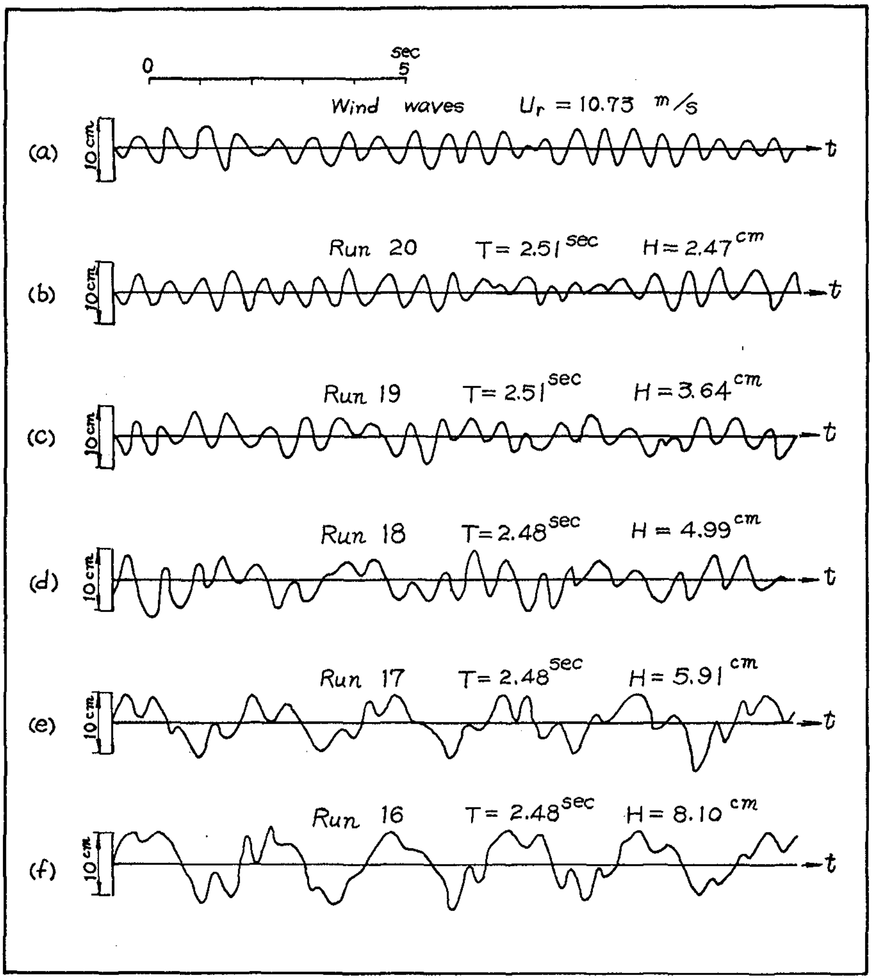


Fig. 3 SAMPLES OF CO-EXISTENT SYSTEM OF WIND WAVES AND LONG WAVES (AT 16.4 M).

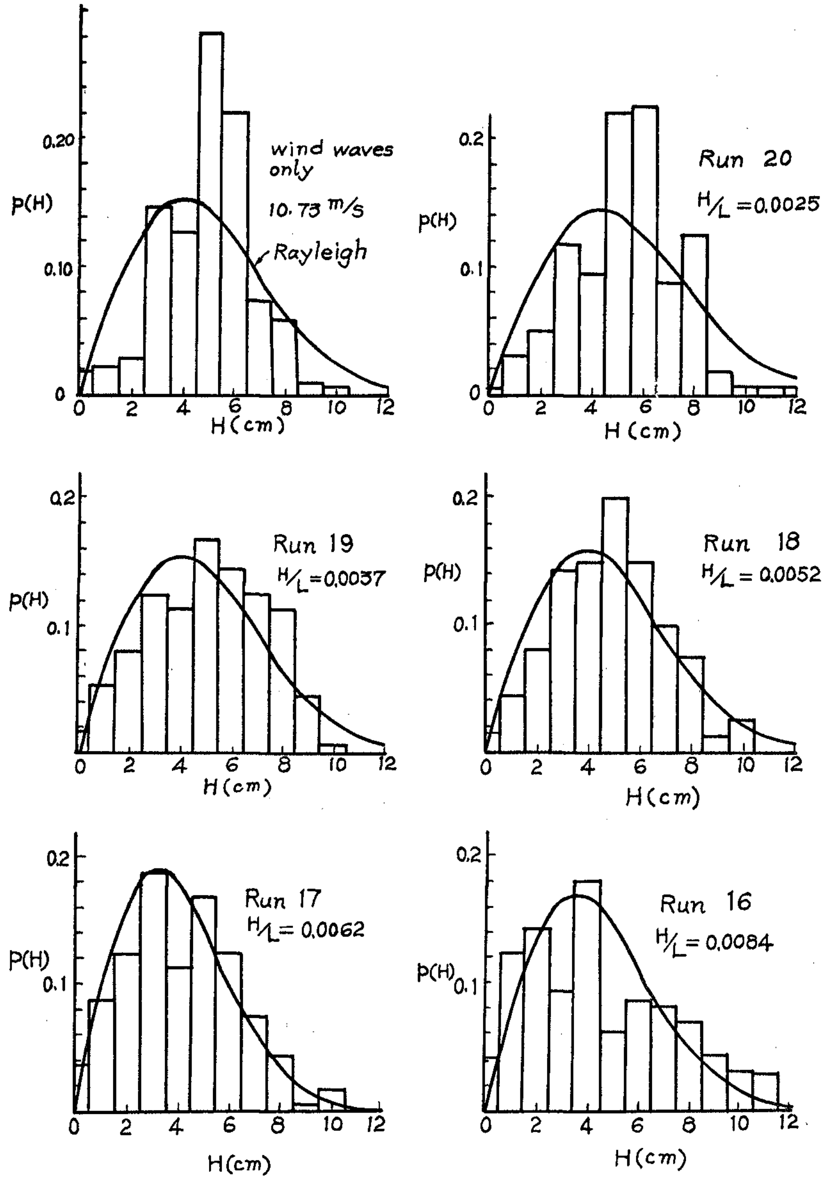
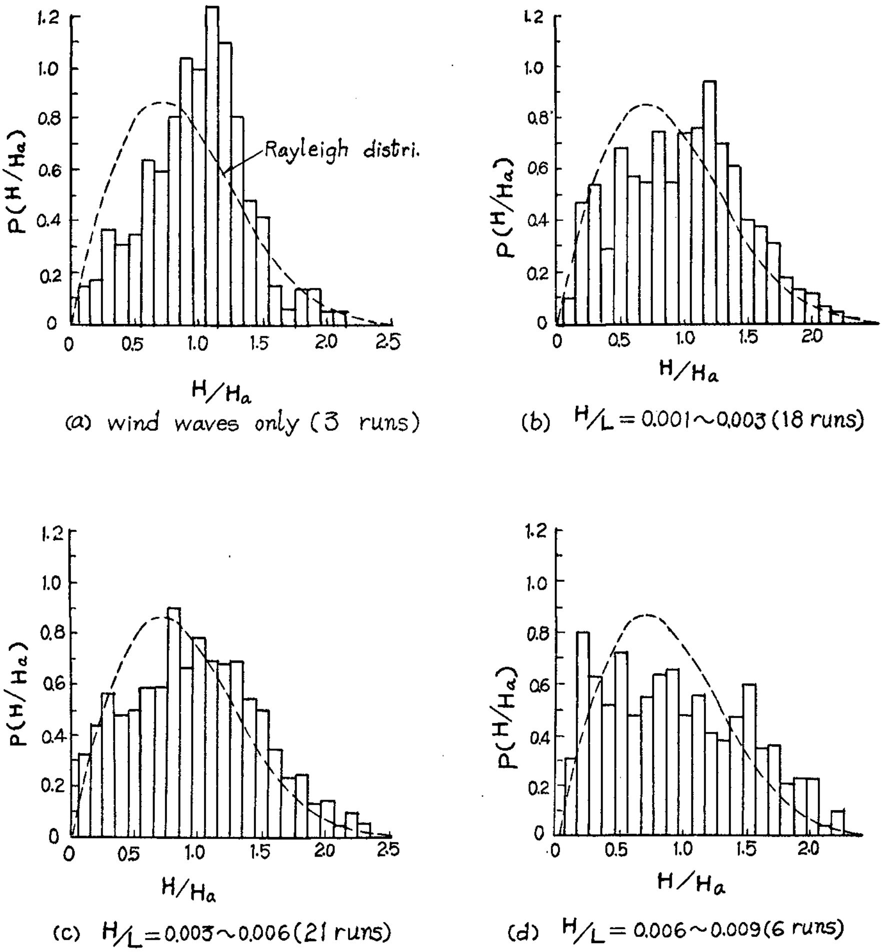


Fig. 4 COMPARISON OF WAVE HEIGHT DISTRIBUTION TO RAYLEIGH DISTRIBUTION (AT 16.4M).



(a) wind waves only (3 runs)

(b) H/L = 0.001~0.003 (18 runs)

(c) H/L = 0.003~0.006 (21 runs)

(d) H/L = 0.006~0.009 (6 runs)

Fig. 5 COMPARISON OF OVERALL WAVE HEIGHT DISTRIBUTION TO RAYLEIGH DISTRIBUTION

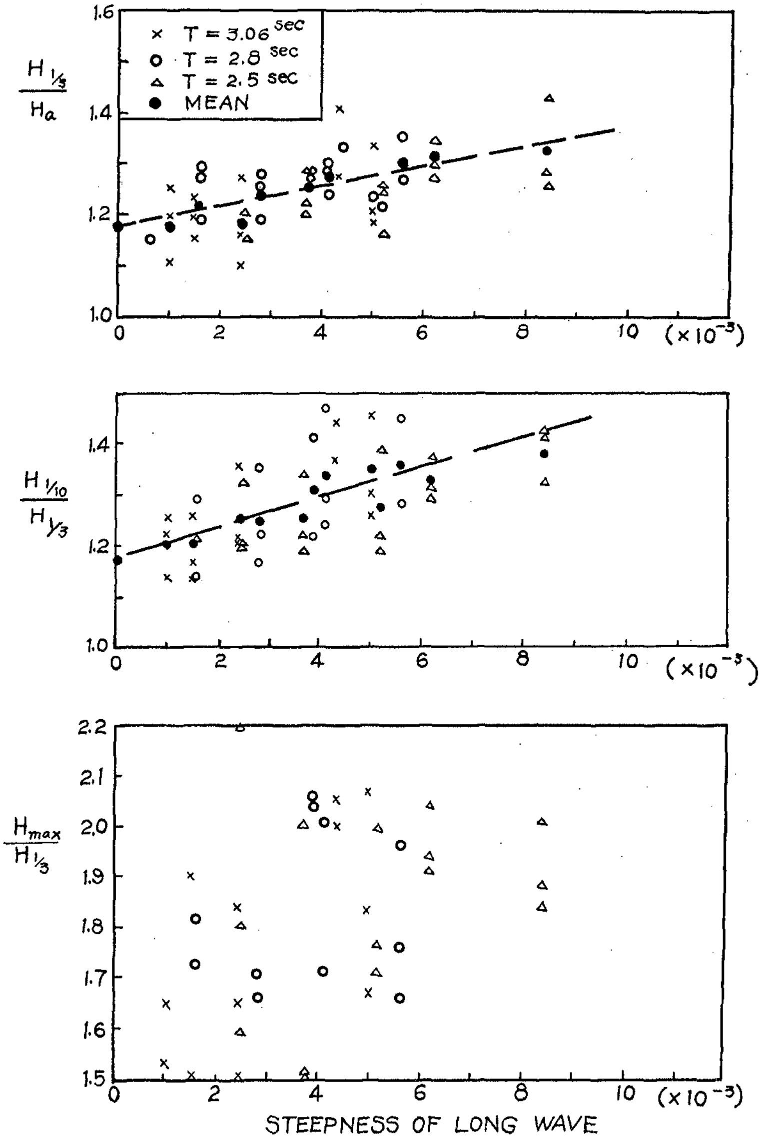
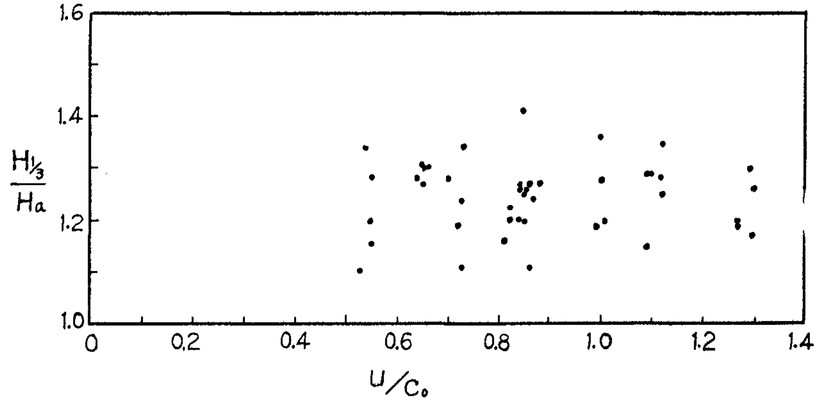
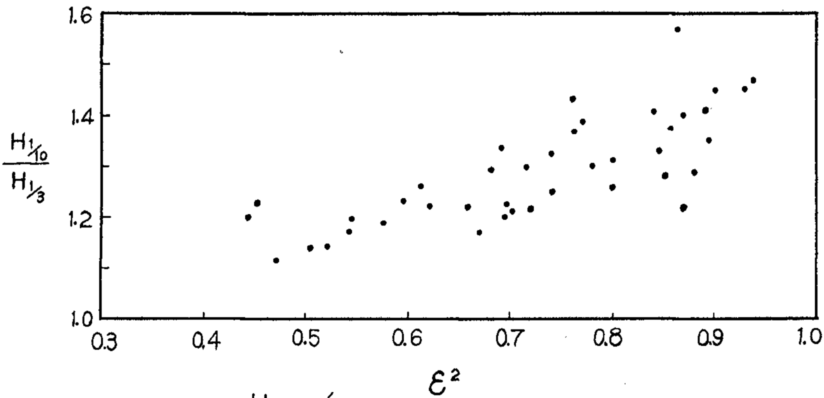


Fig.6 STATISTICAL RATIOS OF WIND WAVE HEIGHTS AND STEEPNESS OF LONG WAVE



(a) $H_{1/3}/H_a \sim U/C_0$



(b) $H_{1/10}/H_{1/3} \sim \varepsilon^2$

Fig. 7 CHANGES OF RATIOS OF WAVE HEIGHTS WITH ε^2 AND U/C_0

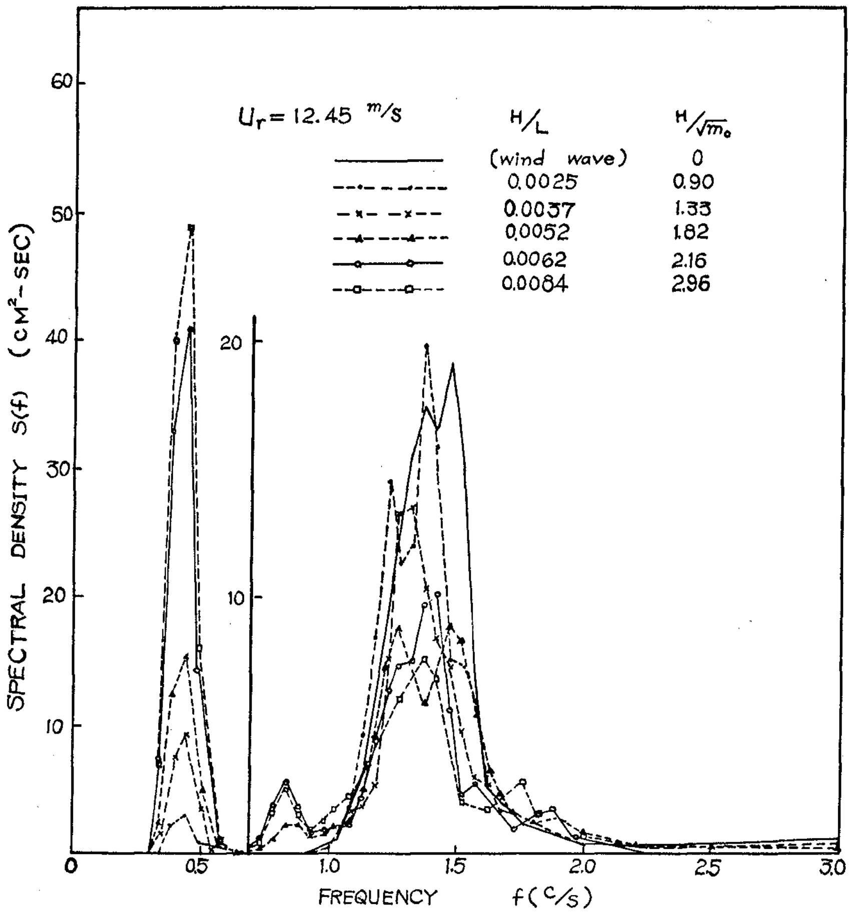


Fig.8 SPECTRA OF CO-EXISTING SYSTEMS OF WIND WAVE AND LONG WAVE.

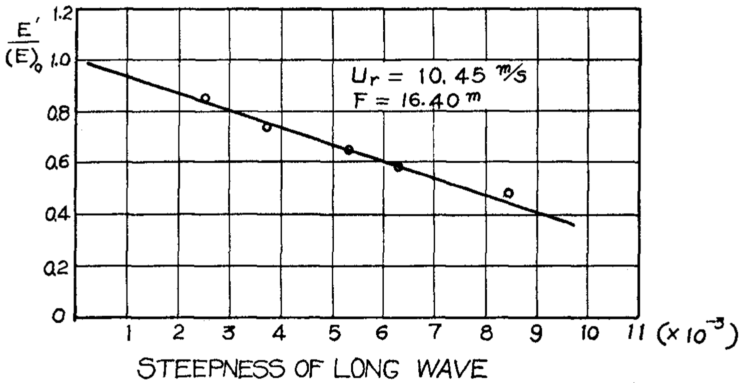


Fig. 9. ATTENUATION OF WIND WAVE ENERGY