CHAPTER 139

HORIZONTAL DIFFUSION IN A TIDAL MODEL

by

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Abstract

The effect of the tidal residual flow on the horizontal diffusion in a shallow tidal bay is investigated through a hydraulic model experiment, for which Mikawa Bay in central Japan was used as a prototype.

A hydraulic model of about 20 x 30 m including Mikawa Bay and neighboring sea area, with a horizontal and vertical scale of 1/2000 and 1/160 respectively, was used, and a semi-diurnal tide was provided for it. Experiments have shown that tide and tidal current are well reproduced in the model. The tidal locus does not close, that means the existence of the residual flow. The distribution of the concentration of the dye, which is discharged from the bay bottom, corresponds to the pattern of the residual flow.

The diffusion coefficient in the bay obtained through one dimensional analysis is the order of 10^5 cm^2/sec and that through two dimensional analysis is less by one order and the dispersion coefficient becomes 10^5 cm^2/sec. It is concluded that the dispersion due to the residual flow plays more important role on the distribution of the material in the shallow bay, as Mikawa Bay, than the diffusion due to the tidal current itself.

1. Introduction

There are many kinds of current depending on various motive forces in the nearshore sea area. Among these the most predominant current through all seasons in the bays and inlets in Japan is the tidal current. The so-called tidal current, which is directly generated by the tide generating force, generates the minor scale currents such as the compensating current and eddy current under the influence of the geographical configuration such as headland and island as well as the bottom topography and the smaller eddy currents due to the bottom stresses.

In the existing shallow sea area, although the water particle goes and returns during flood and ebb, the locus does not close after one tidal cycle. This displacement is called as "the tidal residue". When the vectors of such tidal residues are at random, it is considered that there exists the turbulence, and when the vectors are uniform, there is the constant flow. The so-called constant flow is what is defined as the mean value of the current during one tidal cycle at a fixed point, therefore, this is a phenomenon seen from Eulerian standpoint. On the other hand, the residual flow is that seen from Lagrangean standpoint. Although it seems that both flows are different from each other, it is considered to be the same phenomenon essentially. The constant flow is also mainly caused by the geographical configuration, and it forms the horizontal circulation often in a bay.

The pollutants discharged into the sea are diffused initially by small eddies, and then it begins to be affected by larger eddies gradually depending on the scale of the patch. Beside the diffusion due to such eddies, the distribution of pollutants is affected by the advection due to the current of larger scale such as the constant flow. The relationship between diffusion and advection is discussed on the base of the results of a hydraulic model experiment.

2. Prototype

Mikawa Bay is in the central part of Japan, which is surrounded by both Atsumi and Chita peninsula, and connected with the Pacific Ocean through the southern part of Ise Bay. (Fig.1.) The bay is about 30 km long east to west and 20 km north to south, 500 km^2 in area, and 12 m deep.

The tidal wave comes into the bay from the Pacific Ocean through Irako Straits. The spring tidal range in the bay is about 1.9 m, which is 1,43 times of the Pacific coast. The phase difference of M_2 constituent is 12 degree. The maximum flood current appears 3 hours before high water in the bay and maximum ebb current 3 hours after

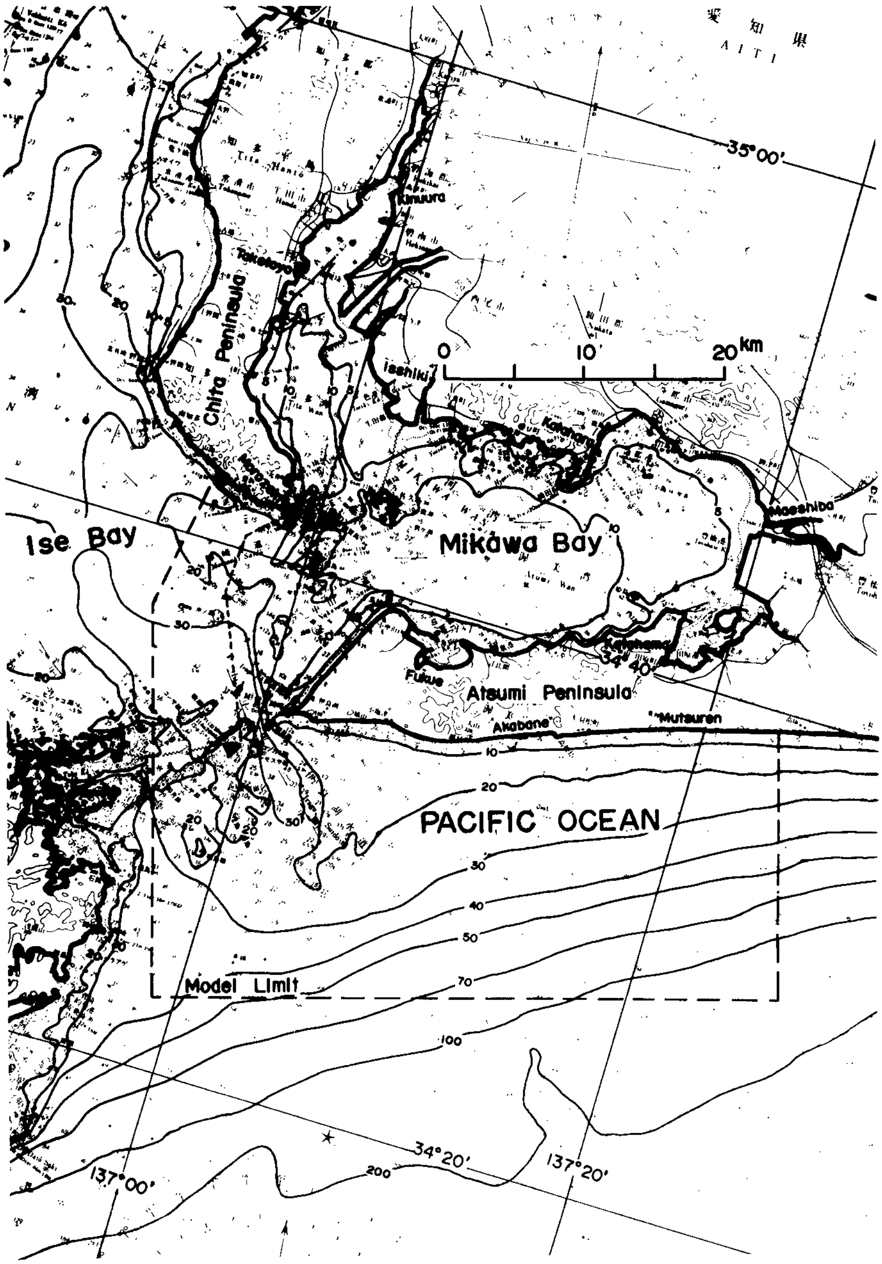

Fig.1 Bathygraphic chart of Mikawa Bay

that. The mean value of the maximum current is about 30 cm/sec in the bay.

3. Similitude[1)]

In the hydraulic model experiment for the tidal current, it is necessary for the following equations to be valid in order to hold a dynamical similitude between the prototype and the model,

$$t_r = x_r h_r^{-1/2}, \tag{1}$$

$$C_r = x_r^{-1} h_r, \tag{2}$$

where x is the horizontal length, h the vertical length, t the time, C the friction factor, and suffix r shows the ratio of the quantity in the prototype and the model.

From the comparison of both equations of diffusion in the prototype and the model, the following equation is obtained,

$$K_r = x_r^2 t_r^{-1}, \tag{3}$$

where K is the horizontal diffusivity. On the other hand, the horizontal diffusivity is expressed as follows,

$$K = \varepsilon^{1/3} l^{4/3}, \tag{4}$$

where ε is the rate of energy dissipation and l the scale of the phenomenon. Therefore, the ratio of the diffusivity is written as follows,

$$K_r = \varepsilon_r^{1/3} l_r^{4/3}. \tag{5}$$

Assuming that the rate of energy dissipation in the model is equal to that in the prototype, that is, $\varepsilon_r = 1$, we obtain

$$K_r = x_r^{4/3}, \tag{6}$$

since $l_r = x_r$. From the equation (3) and (6), we get

$$t_r = x_r^{2/3}. \tag{7}$$

From the equation (1), (2), and (7), we obtain

$$h_r = x_r^{2/3}, \tag{8}$$

$$C_r = x_r^{-1/3}. \tag{9}$$

Consequently, the scale ratios for the time, vertical length, and friction factor are determined only by that for horizontal length. When the friction factor determined by the equation (9) is reasonal value, these scale ratios are acceptable.

4. Experiment

A hydraulic model of about 20 x 30 m including Mikawa Bay and neiboring sea area, of which the model limit is shown by broken lines in Fig.1, was constructed. The horizontal scale is 1/2000, the vertical and time scales are 1/160, which are determined by the equations (7) and (8). A semi-diurnal tide of the period of 280 sec was provided for it by an automatically controlled pneumatic tide generator.

The distribution of the tidal range in the model agrees well with that in the prototype. The flow patterns in the flood and ebb also agree with those in the prototype. From these two results, it is considered that the dynamical similitude for the tidal current holds well.

The loci of floats are shown in Fig.2. A float moves along the solid line during the flood and then it does along the broken line during the ebb. Generally, as shown in this figure, a float does not come back to the initial location after one tidal cycle, that is, the tidal locus does not close. This suggests the existence of the constant flow caused by the geographical configuration. This residue is called as "the tidal residue" and the current causing such a residue is called "the residual flow". Such a residual flow forms several horizontal circulation in the bay. An example is shown in Fig.3. The velocity of the residual flow is several percent of the tidal current.

The horizontal distribution of the concentration of the dye, which was continuously released from the coast, at 90 tidal cycle after the commencement of release is shown in Fig.4. The numeral shows the concentration in per mill (o/oo) of that of released dyed water. The concentration is the mean value of the water column from the surface to the bottom. In comparing Fig.4 with Fig.3, the distribution of the concentration seems to highly depend on the residual flow.

5. Consideration

a) One dimensional analysis

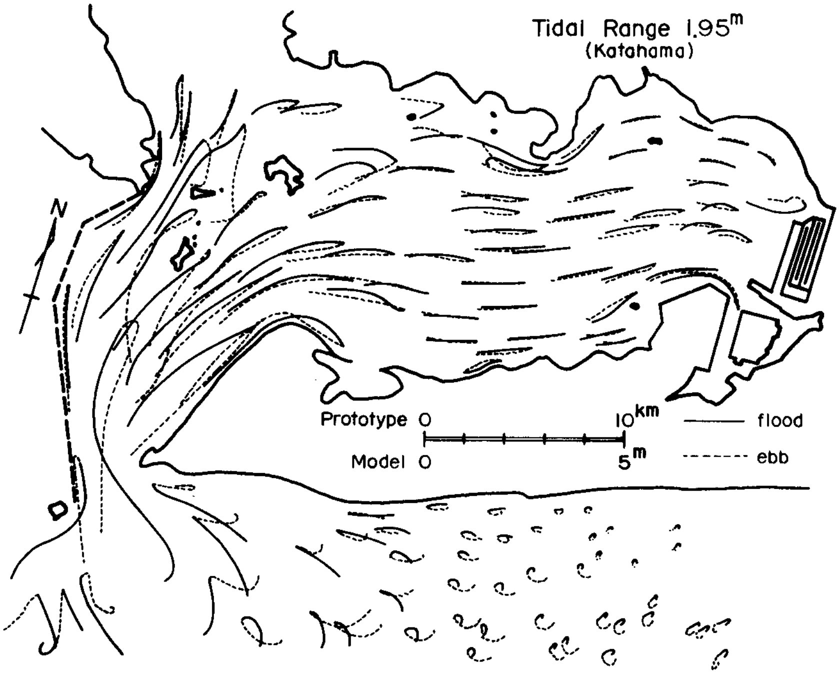

Fig.2 Tidal locus

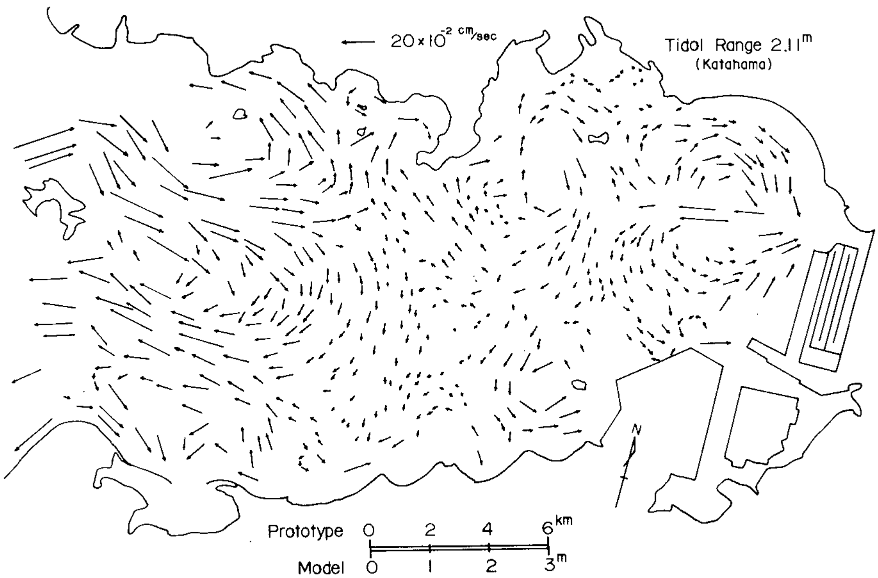

Fig.3 Residual flow

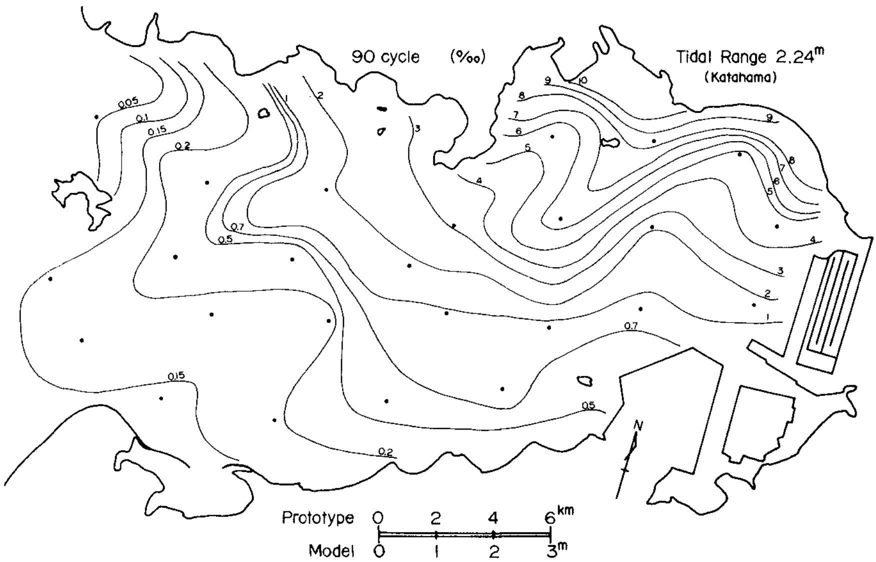

Fig.4 Horizontal distribution of dye concentration

The bay is divided into several sections along the center line as shown in Fig.5, where the point A is the releasing point and the dot means the sampling point.

The one dimensional equation of diffusion is as follows,

$$A\frac{\partial C}{\partial t} + Q\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(A\cdot K\cdot\frac{\partial C}{\partial x}\right) \tag{10}$$

wher A is the cross-sectional area, C the concentration, Q the rate of flow of the dyed water, K the diffusion coefficient. The diffusion coefficient is expressed in the form of difference as follows,

$$K = \frac{A\cdot \Delta x\cdot \frac{\Delta C}{\Delta t} + Q\cdot \Delta C}{A\cdot \frac{\Delta C}{\Delta x}} \tag{11}$$

The one dimensional diffusion coefficient is shown in Table 1, which is calculated from the mean concentration in each section and the increment of the concentration from 80 to 100 cycles after the commencement of release of the dyed water. The average value of the diffusion coefficient in the whole bay is 2.7×10^5 cm^2/sec.

b) Two dimensional analysis

The two dimensional equation of diffusion is as follows,

$$\frac{\partial C}{\partial t} = -\nabla\cdot(uC) + \nabla\cdot(K\nabla C), \tag{12}$$

where u is the velocity of the constant flow. The diffusion coefficient is caluclated in a box model enclosed by vertical planes parallel with the major (x) and minor (y) axis of tidal ellipse (Fig.6). Equation (12) is rewritten in the form of the difference as follows,

$$V\frac{\Delta C_5}{\Delta t} = S_1u_1C_1 - S_2u_2C_2 + S_3u_3C_3 - S_4u_4C_4 + S_1K_x\frac{\Delta C_1}{\Delta x} - S_2K_x\frac{\Delta C_2}{\Delta x} + S_3K_y\frac{\Delta C_3}{\Delta y} - S_4K_y\frac{\Delta C_4}{\Delta y} \tag{13}$$

where V is the volume of the box, S the area of the side wall, u and v the velocity of the constant flow in x and y direction respectively. Assuming the same value of K_x and K_y respectively in the neighboring boxes, we obtain $K = 4.7 \times 10^4$ cm^2/sec and $K_y = 1.7 \times 10^4$ cm^2/sec in the central part of the bay. The length of the box is 4 km, the width 1.5 km. The mean water depth is 13 m and the maximum velocity of the tidal current is 20 cm/sec in the box. The two dimensional diffusion coeffi-

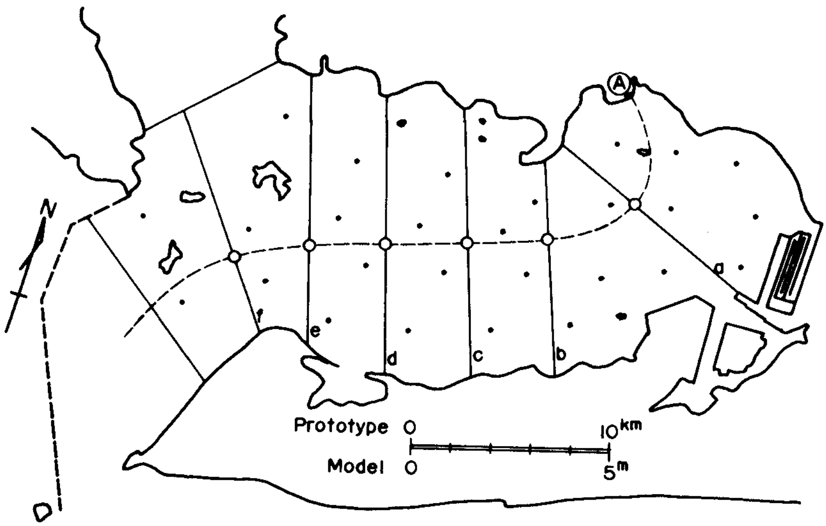

Fig.5 Section used for averaging

Table 1 Diffusion coefficient K in one dimensional model and dispersion coefficient D_1 in two dimensional model ($\times 10^5 \, cm^2/sec$)

Area	a	b	c	d	e	f
K	1.49	5.03	1.28	1.26	4.03	3.13
D_1	1.03	2.93	0.28	—	3.38	

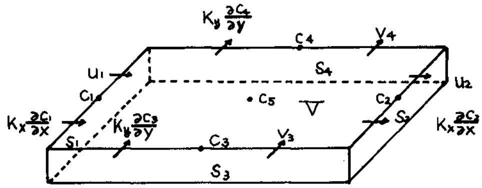

Fig.6 Box model

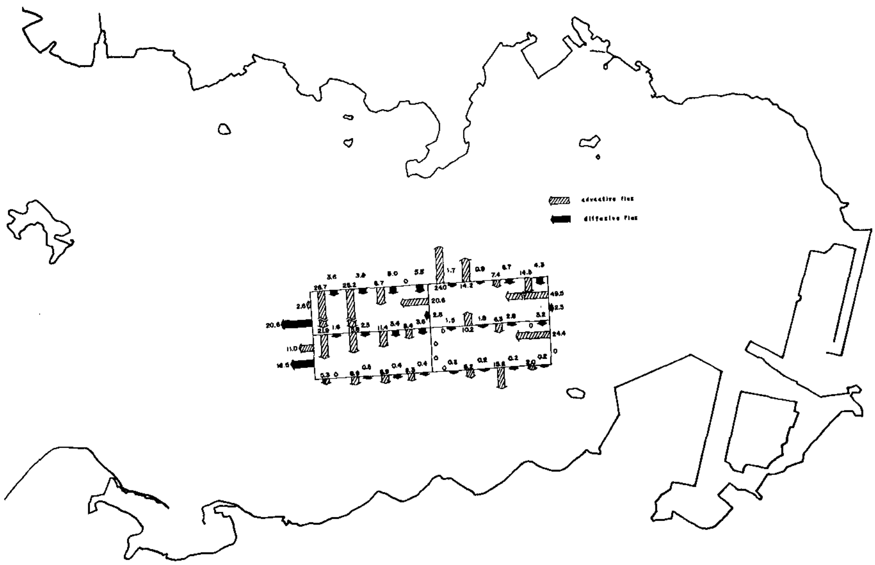

Fig.7 Advective flux and duffusive flux

cient is smaller by one order than one dimensional one.

The diffusive flux of the dye due to the tidal current and the advective flux due to the residual flow are shown in Fig.7, in which the black arrow shows the diffusive flux and dark one the advective flux. From this figure it becomes to be clear that the advective flux is much larger than the diffusive flux.

We devide the current velocity into 4 parts, after Fisher's model[2],

$$U(x,y,t) = u_o(x) + u_1(x,t) + u_s(x,y) + u'(x,y,t), \qquad (14)$$

where u is the velocity,

x the horizontal length from the bay bottom to the mouth,

y the lateral length,

u_o the cross-sectional mean velocity of the dyed water, $u_o = \langle \bar{u} \rangle$

$\bar{\ }$ shows the mean value in the cross-section.

$\langle\ \rangle$ the mean velocity of the tidal current in the cross-section,

$$u_1 = \bar{u} - u_o,$$

u_s the velocity of the residual current, $\bar{u}_s = 0$,

u' the deviation of the velocity, $\bar{u}' = \langle u' \rangle = 0$.

The concentration of the dye is expressed in the same way as the velocity.

$$C(x,y,t) = C_o(x) + C_1(x,t) + C_s(x,y) + C'(x,y,t) \qquad (15)$$

The flux of the dye passing through a cross-section is expressed as follows,

$$M = \frac{1}{T}\int_0^T\int_A (uC)\, dA\, dt, \qquad (16)$$

where T is the tidal period, A the cross-sectional area. Assuming that the change of the cross-sectional area due to the tide is small and $\langle u_1 C_1 \rangle \doteqdot 0$, equation (16) is rewritten as follows,

$$M = C_o Q_o + A_o(\overline{u_s C_s} + \langle \overline{u'C'} \rangle), \qquad (17)$$

where Q is the rate of flow of the dyed water, A_o the mean cross-sectional area during one tidal cycle. $C_o Q_o$ corresponds to the advective flux in one dimensional equation of diffusion. The flux of dye is shown by dark arrows in Fig.8, where the white arrow shows the rate of flow due to the residual flow.

The dispersion coefficient, after Fisher, is as follows,

$$D_1 = \frac{1}{(dC_o/dx)} \cdot \overline{u_s C_s}. \qquad (18)$$

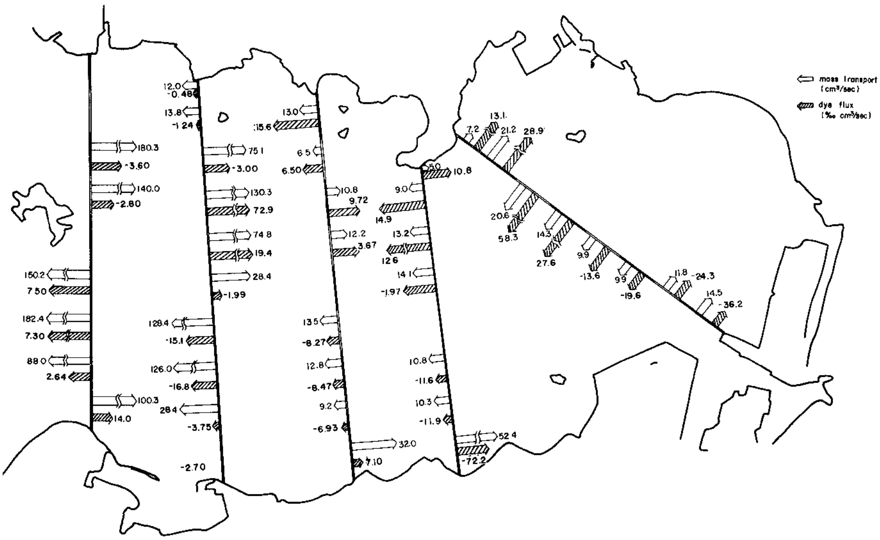

Fig.8 Rate of flow(mass transport) and dye flux

The value is shown in Table 1. From this table, it becomes to be clear that the one dimensional diffusion coefficient mainly consists of the dispersion due to the residual flow.

6. Conclusion

The mean value of the diffusion coefficient in Mikawa Bay is 2.7×10^5 cm^2/sec, which is obtained by one dimensional analysis from the distribution of the concentration of the dye released continuously from the bay bottom. The diffusion coefficient in a box model in the central part of the bay is smaller by one order than it, which is obtained by two dimensional analysis of the balance of the dye in the box. The mean value of the dispersion coefficient due to the constant flow is 1.9×10^5 cm^2/sec. The one dimensional diffusion is almost understood as the dispersion due to the constant flow.

In conclusion, the dispersion due to the constant flow plays more important role on the distribution of the material in the shallow tidal bay, as Mikawa Bay, than the diffusion due to the tidal current itself.

Referance

1) Higuchi,H. and Sugimoto, T. : Experimental study on the horizontal diffusion due to the tidal current, Paper No.29: Symposium on "The physical process responsible for the dispersal of pollutants in the sea with special reference to nearshore zone" Aarhus/University, Denmark, July 1972

2) Fisher,H.B.: Mass transport mechanisms in partially stratified estuaries, Journal of Fluid Mechanics vol.53, pp.671-687, 1972