CHAPTER 133

TIDE-INDUCED MASS TRANSPORT IN LACOON-INLET SYSTEMS[1]

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ABSTRACT

This paper examines the tide-induced net discharge in lagoon-inlet systems. In particular, attention is given to the role inlets play in inducing a steady current.

The flow in the lagoon is described by the one-dimensional long wave equations, the flow in the inlets is described by a semi-empirical equation. Both numerical and analytical techniques are employed to solve for the net discharge.

The results of the study indicate that 1) the net discharge can be significant provided the tidal amplitude to depth ratio is not small 2) the net discharge can be considerably increased by the proper selection of the inlet dimensions.

INTRODUCTION

In an earlier publication van de Kreeke[8]has pointed out the possibility of increasing the tide-induced flushing of lagoons and coastal channels by proper selection of the inlet dimensions. This concept of using inlets for environmental controls was discussed in a wider context by Lockwood and Carothers [3] who in their paper also incorporated the possibility of salinity control. In a recent paper by Mei, Liu and Ippen [5], an example of increasing the tidal flushing by introducing two narrow asymetric constrictions near each entrance of a coastal channel with opposite orientation is presented.

The material presented here constitutes an elaboration of the concepts set forth in the previously quoted papers. The practical significance of the tide-induced net discharge is discussed and the role of the inlets in inducing a steady current in coastal channels and lagoons is examined.

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EQUATIONS

A definition sketch of the lagoon-inlet system is presented in Figure 1.

The equations used to describe the flow in the lagoon are

Continuity $$b \frac{\partial \eta}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{1}$$

Momentum $$\frac{\partial Q}{\partial t} + gA \frac{\partial \eta}{\partial x} + \frac{1}{A} \frac{\partial Q^2}{\partial x} = \frac{-F\, Q|Q|b}{A^2} \tag{2}$$

b = the width of the lagoon (assumed constant)

η = water surface elevation with respect to still water

t = time

Q = discharge

x = horizontal Cartesian coordinate

g = gravitational acceleration

A = $b(h+\eta)$ = cross-sectional area

h = depth with respect to still water

F = resistance coefficient

In deriving Equations (1) and (2) terms of $O(\frac{a}{h}^2)$ have been retained.

The equation used to describe the flow in the inlets is the semi-empirical relation

$$gA \frac{\partial \eta}{\partial x} = \frac{-F_i Q|Q|b}{A^2} \tag{3}$$

in which F_i is an inlet coefficient accounting for friction and exit and entrance losses and b and A refer to the width and cross-sectional area of the inlet.

NUMERICAL SOLUTION

A feeling for the order of magnitude and the effects of the inlets on the net discharge is obtained by solving Equations (1) - (3) for Q and η numerically and integrating Q over a tidal cycle. The computations are carried out for various combinations of inlet and lagoon dimensions, phase angles and amplitudes of the ocean tides. The lagoon and inlet dimensions

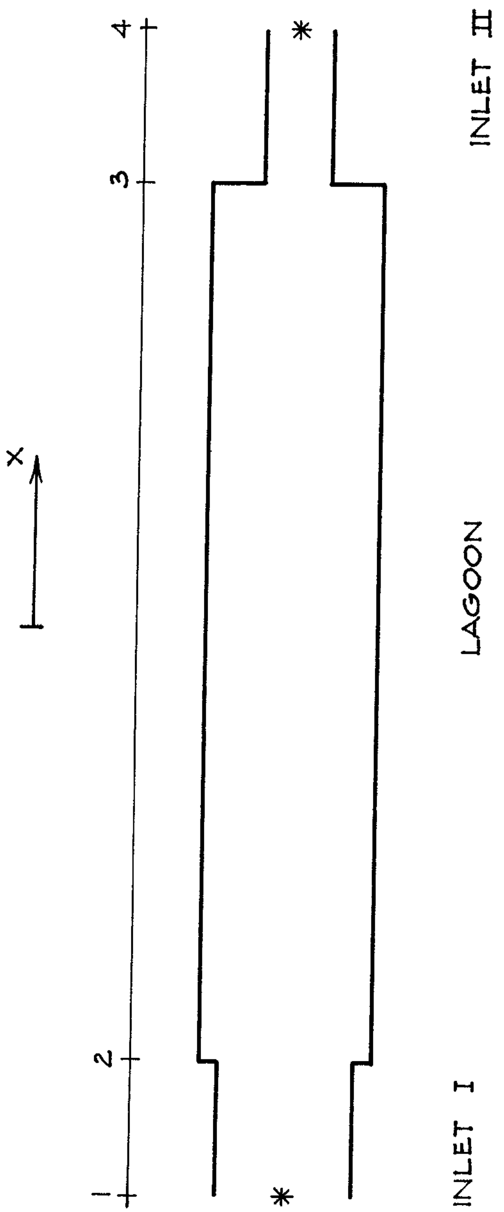

FIGURE 1 LAGOON — INLET SYSTEM ; PLAN VIEW.

used in the computations are typical for those found along the Florida Atlantic and Gulf of Mexico coastline, i.e., for the lagoons the length is 0(10,000 ft) the width is 0(1,000 ft) and the depth is 0(10 ft), for the inlets the length and width are 0(1,000 ft) and the depth is 0(10 ft).

As an example the results of the computations for the lagoon-inlet system indicated in Figure 1 with values of pertinent parameters listed in Table 1 are presented. An explicit finite difference scheme is used in the computations; see Reid and Bodine [6] and van de Kreeke [7]. The time and space step are respectively $\Delta t = 100$ sec and $\Delta x = 5,000$ ft.

TABLE 1
NUMERICAL VALUES USED IN COMPUTATIONS

BOUNDARY CONDITIONS

$$\eta_1 = a_1 \cos(\sigma t + \delta)$$
$$\eta_4 = a_4 \cos(\sigma t)$$
$$a_1 = a_4 = 1.3 \text{ ft} \quad \delta = 0^o$$

The subscripts refer to stations indicated in Figure 1.

GEOMETRY OF LAGOON

$h = 10$ ft, $b = 2,500$ ft, $L = 80,000$ ft

GEOMETRY OF INLET I

$h = 10$ ft, $b = 800$ ft, $L = 2,500$ ft

GEOMETRY OF INLET II

The following combinations of depth, width and length are used for Inlet II

$h = 2 \rightarrow 10$ ft	$b = 800$ ft	$L = 2,500$ ft
$h = 10$ ft	$b = 100 \rightarrow 2,500$ ft	$L = 2,500$ ft
$h = 10$ ft	$b = 800$ ft	$L = 0 \rightarrow 10,000$ ft

No computations were carried out for depths of Inlet II smaller than 2 feet and widths of Inlet II smaller than 100 feet. For small depth, Inlet II will be dry for part of the tidal cycle. For small width the depth to width ratio of Inlet II is no longer small, a condition for Equation (3) to be valid.

The net discharge as a function of depth, width and length of Inlet II is plotted in respectively Figures 2a, 2b and 2c. The results show a zero net discharge for Inlet II closed,(i.e. width = 0, depth = 0, length = ∞ (not indicated in Figure 2c)) and for Inlet II being identical to Inlet I (symmetrical lagoon-inlet system). The magnitude of the net discharge depends on the relative dimensions of the inlets and can be either positive or negative.

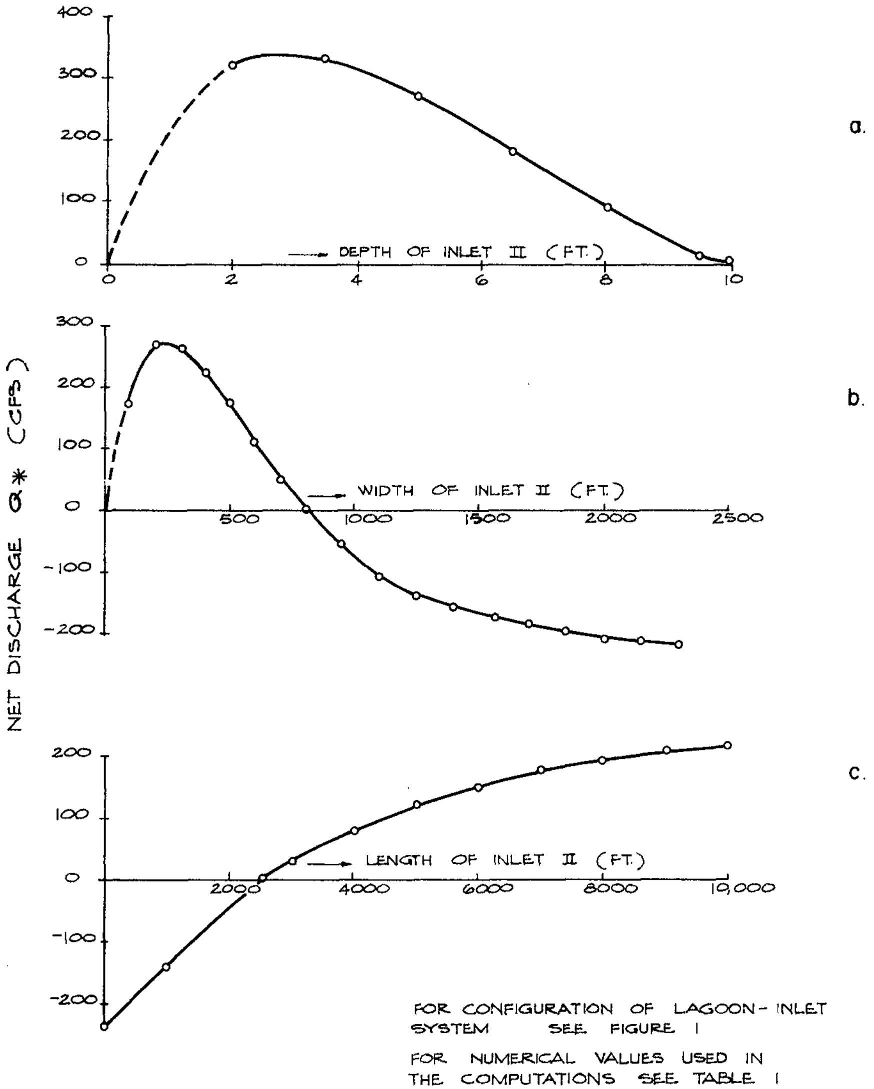

FIGURE 2 VARIATION IN NET DISCHARGE WITH DEPTH, WIDTH AND LENGTH OF INLET II

The results of the numerical computations provided enough incentive to attack the more complicated task of finding an analytic solution for the purpose of

1) obtaining a better understanding of the physics of the phenomenon of the tide-induced net discharge

2) to delineate the effects of the inlets on the net discharge

In particular the latter would be helpful when contemplating possible man-made modifications to existing lagoon-inlet systems.

ANALYTIC SOLUTION; PHYSICAL INTERPRETATION

Because the nonlinear friction term is of the same order as the linear terms, the long wave equations do not lend itself very well to an analytic solution. Therefore in the following treatment the friction term is linearized, realizing that in doing so the resulting solution can at best be an approximation. Linearization is accomplished by assuming a linear relation hetween bottom stress and mean velocity. The mechanics of the linearization process are described in detail in van de Kreeke [8] and Dronkers [2].

Upon introduction of the linearized friction term Equations(1) - (3) become

$$b \frac{\partial \eta}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{4}$$

$$\frac{\partial Q}{\partial t} + gA \frac{\partial \eta}{\partial x} + \frac{1}{A} \frac{\partial Q^2}{\partial x} = - \frac{F_{\ell} Qb}{A} \tag{5}$$

$$gA \frac{\partial \eta}{\partial x} = \frac{-F_{i_{\ell}} Qb}{A} \tag{6}$$

in which F_{ℓ} = "linear" resistance coefficient

$F_{i_{\ell}}$ = "linear" inlet coefficient

Substituting $A = b(h+\eta)$ in Equations (4) - (6) and neglecting terms of $0(\frac{a}{h}^3)$

$$b\frac{\partial\eta}{\partial t} + \frac{\partial Q}{\partial x} = 0 \tag{7}$$

$$\frac{\partial Q}{\partial t} + gbh\frac{\partial\eta}{\partial x} = \frac{-F_\ell Q}{h} + \frac{F_\ell Q\eta}{h^2} - gb\eta\frac{\partial\eta}{\partial x} - \frac{1}{bh}\frac{\partial Q^2}{\partial x} \tag{8}$$

$$gbh\frac{\partial\eta}{\partial x} = \frac{-F_{i_\ell} Q}{h} + \frac{F_{i_\ell} Q\eta}{h^2} - gb\eta\frac{\partial\eta}{\partial x} \tag{9}$$

Because the nonlinear terms in Equations (7) - (9) are of $0(\frac{a}{h}^2)$, a first order solution for the water levels and discharges is found by considering only the linear terms in the equations. The resulting first order expressions for the water levels and discharges, designated respectively η_ℓ and Q_ℓ are periodic in time and space and have the same frequency as the ocean tide. An expression for the net discharge, Q_*, in the lagoon-inlet system may be found by substituting in the complete Equations (7) - (9), including the nonlinear terms, a trial solution of the form

$$\eta = \eta_*(x) + \eta_\ell(x,t)$$

$$Q = Q_*(x) + Q_\ell(x,t)$$

in which η_* = mean water surface elevation.

Averaging over a tidal period and neglecting terms of $0(\frac{a}{h}^3)$ and higher, this results in

$$Q_* = \text{constant}$$

$$\frac{\partial}{\partial x}\left[\frac{\overline{Q_\ell^2}}{bh} + gbh\eta_* + \frac{gb}{2}\overline{\eta_\ell^2} + \int\frac{F_\ell Q_*}{h}\,dx - \int\frac{F_\ell\overline{Q_\ell\eta_\ell}}{h^2}\,dx\right] = 0 \tag{11}$$

pertains to lagoon

$$\frac{\partial}{\partial x}\left[gbh\eta_* + \frac{gb}{2}\overline{\eta_\ell^2} + \int\frac{F_{i_\ell}Q*}{h}\,dx - \int\frac{F_{i_\ell}\overline{Q\eta}}{h^2}\,dx\right] = 0 \tag{12}$$

pertains to inlet

The bar denotes averaging over a tidal cycle.

Applying Equations (10)-(12) to the lagoon-inlet system in Figure 1, integrating with respect to x, setting $\eta_{*1} = \eta_{*4}$ and eliminating η_{*2} and η_{*3} (1, ...4 refer to station locations, see Figure 1) yields the following expression for Q_*

$$Q_* \left[\frac{F_{i_\ell} L}{gbh^2}_{I} + \frac{F_\ell L}{gbh^2_{LAG}} + \frac{F_{i_\ell} L}{gbh^2}_{II} \right] = -\int_1^2 f_I(x)\,dx$$

$$-\int_2^3 f_{LAG}(x)\,dx - \int_3^4 f_{II}(x)\,dx \qquad (13)$$

where $f_I(x) = \left[\frac{1}{h} \overline{\eta_\ell \frac{\partial \eta_\ell}{\partial x}} \right]_I$

$$f_{LAG}(x) = \left[\frac{1}{h} \overline{\eta_\ell \frac{\partial \eta_\ell}{\partial x}} + \frac{1}{gb^2h^2} \frac{\overline{\partial Q_\ell^2}}{\partial x} - \frac{\overline{F_\ell Q_\ell \eta_\ell}}{gbh^3} \right]_{LAG}$$

$$f_{II}(x) = \left[\frac{1}{h} \overline{\eta_\ell \frac{\partial \eta_\ell}{\partial x}} \right]_{II}$$

The subscripts I, LAG and II refer to respectively Inlet I, the Lagoon and Inlet II.

The right side of Equation (13) has been evaluated by Cotter [1]. The resulting expressions are very lengthy and complex and to a large extent defy the purpose of the analytic solution. Therefore, the complete results of Cotter's [1] analysis will not be presented here.

As a matter of academic interest and to build faith in the numerical model, analytically and numerically computed net discharges for the lagoon-inlet system under consideration are compared in Figure 3. The net discharges computed with both methods show similar trends and orders of magnitude, however, the differences are too large to neglect. Since the process of linearizing the friction term is suspected of causing this discrepancy, a finite difference technique, similar to the one used in solving the equations including the quadratic friction term, is applied to the equations on which the analytic solution is based, i.e., Equations (4), (5) and (6). The time step and space step used in the computations are Δt = 100 seconds and Δx = 5,000 feet. The results of the finite difference computations for the linearized equations along with the analytic results presented earlier in Figure 3 are presented in Figure 4. From this figure it is seen that the two sets of results are

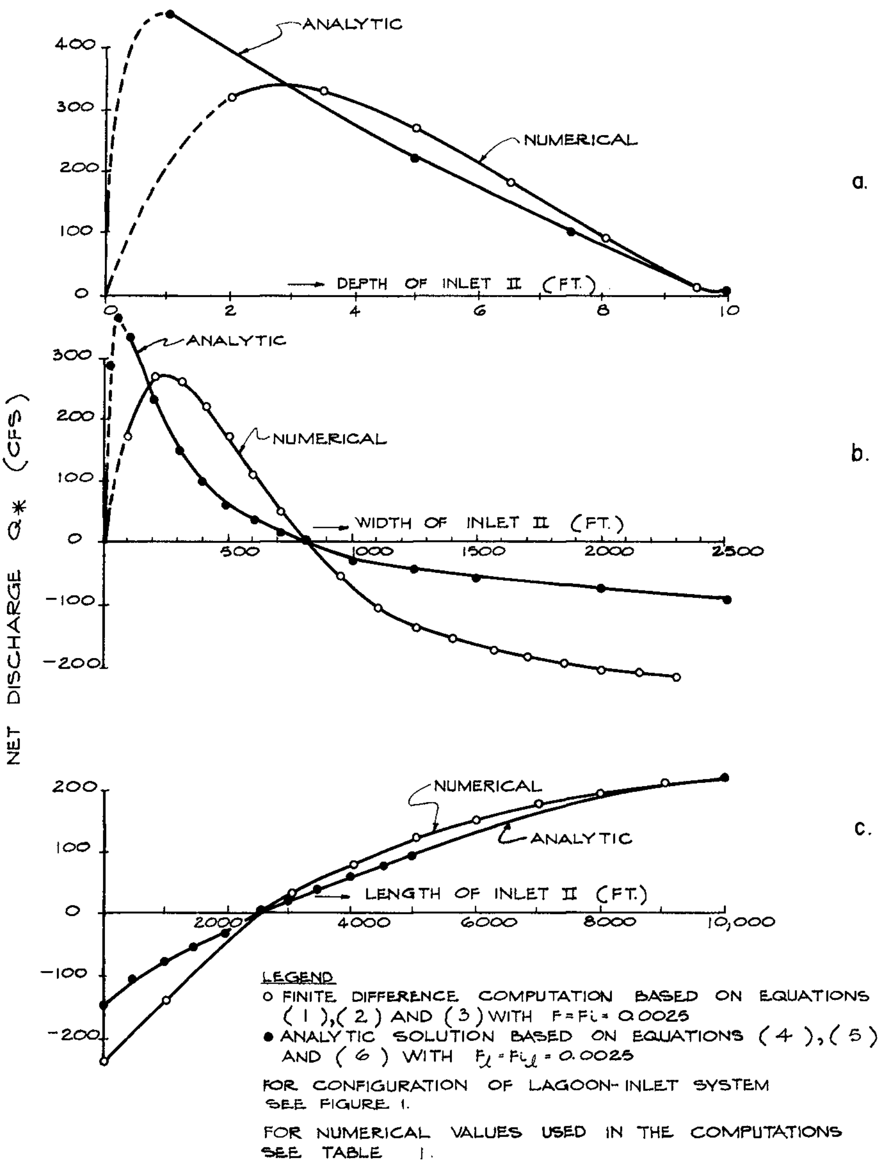

FIGURE 3 COMPARISON BETWEEN NUMERICALLY AND ANALYTICALLY COMPUTED NET DISCHARGE.

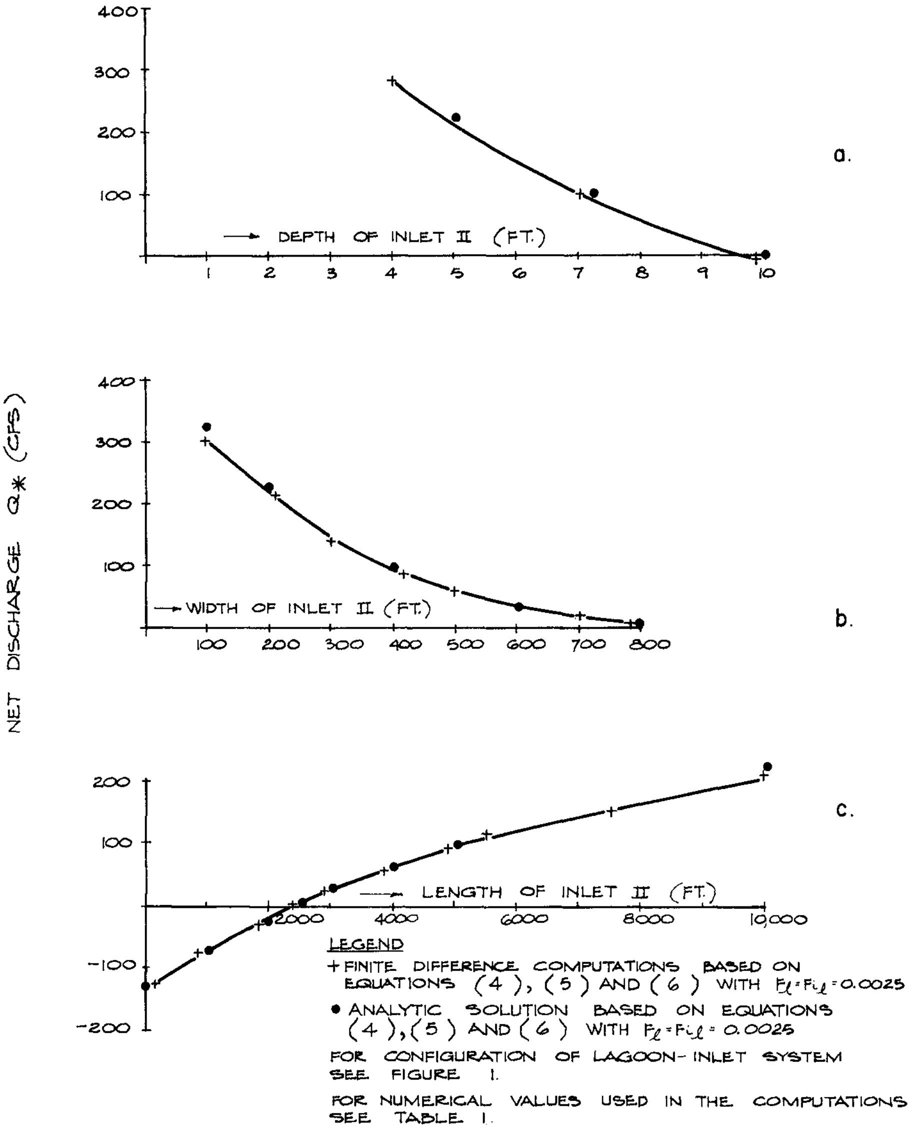

FIGURE 4 NUMERICALLY COMPUTED NET DISCHARGE USING "LINEARIZED" FRICTION.

similar in every respect. This leads to the conclusion that the differences between the numerical and analytic solution evident in Figure 3 arise from the linearization of the friction term employed in the analytic solution.

For a physical explanation of the tide-induced net discharge it seems best to return to Equations (11) and (12). These equations are the time averaged conservation of momentum equations for respectively the lagoon and the inlets. In Equation (11), the first three terms constitute the momentum flux across a vertical plane in the lagoon (or equivalently the radiation stress, Longuet-Higgins and Stewart [4], the last two terms constitute the momentum flux across the bottom (or equivalently the bottom stress). A similar physical interpretation can be given for the terms in Equation (12). In both the lagoon and inlets the excess momentum flux associated with the wave motion is balanced by momentum flux associated with a net discharge Q_* and a momentum flux (or equivalently a pressure) associated with a change in mean water level η_*. Note also that the relation between the slope in mean sea level and the net discharge is not the simple relation known from steady state hydraulics.

SUMMARY AND CONCLUSIONS

The tide-induced net discharge in a schematized lagoon-inlet system is determined by solving the system of governing long wave equations and inlet equations numerically and integrating the discharge over a tidal cycle. The results of the computations show that 1) the net discharge can be of a magnitude that is important when considering the renewal of the waters of the lagoon and 2) the magnitude of the net discharge is strongly affected by the inlet dimensions. The last conclusion suggests that a considerable increase in net discharge can be obtained by properly designing the inlets.

The physics underlying the net discharge is generally obscured when applying numerical techniques to solve the governing equations and therefore an analytical solution is sought to delineate the causes and principles behind the net discharge. Unfortunately, the analytic expression for the net discharge turns out to be rather complicated and therefore provides little physical insight in the role of the inlets with respect to the magnitude of the net discharge. In addition, it appears from comparison with numerical results that because of the linearization process applied to the friction term, the analytic solution yields at best an approximate value of the net discharge. Therefore, for accurate results recourse should be taken to numerical techniques.

Physical insight can best be obtained from the time-averaged conservation of momentum equations for the lagoon and inlets. These equations express the balance between the excess momentum flux across a vertical plane and across the bottom associated with the long wave traveling in the lagoon-inlet system, and the momentum flux associated with the net discharge and the change in mean water level.

Because the net discharge results from the nonlinear (second order) terms in the governing equations, significant net discharge should only be expected for large values of $\frac{a}{h}$ (a = amplitude h = depth). Also, the study presented in this paper pertains to schematized lagoon-inlet systems with a homogeneous fluid and merely serves to illustrate a concept. When dealing with actual lagoon-inlet systems, it should be realized that second order effects such as the net discharge can be easily obscured by the geometric irregularities in the lagoons and inlets and by gravity currents, fresh water inflow and by wind-induced currents.

ACKNOWLEDGEMENT

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REFERENCES

1. Cotter, D.C. Tide-Induced Net Discharge in Lagoon-Inlet Systems. Masters Thesis, University of Miami, Department of Ocean Engineering June, 1974.

2. Dronkers, J.J., Tidal Computations in Rivers and Coastal Waters, North Holland Publishing Company, 1964.

3. Lockwood, M.G. and Carothers, H.P., Preservation of Estuaries by Tidal Inlets, Journal of the Waterways and Harbors Division, ASCE, Vol 93, No. WW4, November, 1967.

4. Longuet-Higgins, M.S. and R. W.Stewart, Radiation Stresses in Water Waves; a Physical Discussion with Applications. Deep Sea Research, Vol. II, pp. 529-562, 1964.

5. Mei, C.C., P.L.F. Liu and A.T. Ippen, Quadratic Loss and Scattering of Long Waves, Journal of the Waterways Harbors and Coastal Engineering Division, ASCE, Vol. 100 No. WW3, August, 1974.

6. Reid, R.O. and B.R.Bodine, Numerical Model for Storm Surges in Galveston Bay, Journal of the Waterways and Harbors Division, ASCE, Vol. 94, No. WW1, February, 1969.

7. van de Kreeke, J., A Numerical Model for the Hydromechanics of Lagoons. Proceedings, XIIIth Conference on Coastal Engineering, Vancouver, 1972.

8. van de Kreeke, J., Tide-Induced Mass Transport in Shallow Lagoons, Department of coastal and Oceanographic Engineering, University of Florida, Technical Report No. 8, October, 1971.