CHAPTER 132

EXCHANGE CHARACTERISTICS OF TIDAL INLETS

by

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ABSTRACT

Measurements of the exchange characteristics at tidal inlets are presented and interpreted in the framework of an idealized conceptual model. The conceptual model considers the primary cause of exchange to be the result of the differences in flow patterns away from and toward an inlet. The efflux from an inlet is considered to occur as a separated flow whereas a sink-type attached pattern is assumed for flow toward the inlet. The combined results of these two patterns is an effective lateral mixing. Field measurements were conducted from an anchored boat and a dye injection and monitoring approach were utilized. The measured results, expressed as "Basin Mixing Coefficients" are presented for three inlets and are interpreted in terms of the geometric and flow characteristics of the inlet and adjacent waters.

INTRODUCTION

One of the most difficult problems that must be addressed by investigators working with computer simulation models of long wave dispersive hydromechanics of bay systems and estuaries is the mixing and flushing mechanisms associated with the various inlets connecting a basin to the open ocean and at the interfaces to the various connected sub-basin regions. Unfortunately little information is available to obtain a predictive capability in this area. This paper describes a field measurement program designed to: (1) obtain specific information regarding the exchange characteristics of the tidal inlets of the Lower Biscayne Bay System for use in a computer simulation model of the bay, and (2) provide insight into the physical parameters governing tidal inlet exchange characteristics. In addition to a discussion of the field measurement program and the techniques used for data analysis, the mixing mechanism of an idealized tidal exchange cycle through an inlet is presented which provides some insight into the effects of inlet geometry and basin bathymetry on the exchange characteristics of the system.

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The work described herein is part of a study of the circulation, mixing, and flushing characteristics of the Lower Biscayne Bay System being conducted by the Coastal and Oceanographic Engineering Laboratory, University of Florida, for the Florida Power and Light Company, see Figure 1. Results of the field measurement program are being incorporated in the University of Florida's two dimensional numerical modeling system, described in References 1, 2, and 3, to provide adequate treatment of the exchange characteristics across the boundaries of the Card Sound sub-basin within the bay system. The Card Sound Basin is of particular interest to the overall study because at one time, consideration was given to the discharge of cooling water from the Florida Power and Light Company's Turkey Point facility into this basin.

A total of three inlets were selected for the injection of Rhodamine-WT fluorescent dye used as the tracer material for the experiment. The locations of these inlets are shown in Figure 1. The data were collected over a three month period during the Summer of 1972.

Discussion of the Problem

The flushing and exchange characteristics of a bay system or sub-basin are normally considered on a time scale of one or more complete tidal cycles. The mechanism for this net mass transport across an interfacial region, such as an inlet, from one mixing basin, such as a bay, to another mixing basin, such as the ocean, is the integrated effect of the mixing which occurs between the inflow volume of water crossing the interface during one-half of a tidal cycle and the water within the receiving mixing basin. The mixing characteristics of each basin can be expressed in terms of a single parameter, the Basin Mixing Coefficient, $M$, defined as

$$M = \frac{V_{mb}}{\Omega}$$  \hspace{1cm} (1)

where,

$V_{mb}$ = volume of water in the receiving basin that mixes with the inflow tidal prism

$\Omega$ = tidal prism

The net mass transport of a substance to or from a mixing basin over a period of time encompassing one or more tidal cycles depends on the Basin Mixing Coefficients for the two basins and the concentration of substance within the basin mixing volume and the inflow tidal exchange volume. Therefore, if a Transport Coefficient, $R$, is defined as

$$R = \frac{\Omega_0}{\Omega_d}$$  \hspace{1cm} (2)
FIGURE 1 SCHEMATIC OF LOWER BISCAYNE BAY SYSTEM FOR EXCHANGE COEFFICIENT MEASUREMENTS TEST
where,

\( \Omega_{dI} \) = volume of substance transported across interface during one half of a tidal cycle of inflow to mixing basin

\( \Omega_{do} \) = volume of substance transported across interface during one-half of a tidal cycle of outflow from mixing basin immediately following inflow.

then R must be expressible as a function of M, and the concentration of substance in the basin mixing volume, and the inflow tidal exchange volume. This will be shown to be true later in the paper. It should be noted from Eq. (2) that values of R>1 signify a net transport of substance out of the mixing basin during the tidal cycle whereas values of R<1 signify a net transport of substance into the mixing basin during the tidal cycle.

Significant differences exist between the Basin Mixing Coefficient and the Transport Coefficient. The Basin Mixing Coefficient characterizes the mixing capacity of the basin for the existing conditions of tide, wind, basin geometry, etc. and should therefore remain reasonably unchanged as long as these conditions hold. The Transport Coefficient, however, should reflect the time-varying nature of the concentration of substance within the basin in addition to the mixing capacity of the basin. If the concentration of substance in the basin mixing volume changes significantly from one tidal cycle to the next then R should also be expected to change.

As an aid in understanding the differences between M and R and their physical significance, consider the time series of events shown in Figures 2a - 2g following the injection of tracer during the flood tide in a tidal inlet connecting a small bay with the ocean. In this example the ocean represents one mixing basin, the small bay the other and the tidal inlet the interfacial region connecting the two. As shown in Figure 2 the characteristic mixing volume, \( \Omega_{mb} \), of the bay is of limited extent and remains essentially constant with time. The corresponding volume in the ocean and the energy available for mixing, however, are much larger and therefore provide a greater dilution of the substance transported across the interface in the tidal prism, \( \Omega \). Thus, these figures illustrate qualitatively how the Basin Mixing Coefficient, M, represents the characteristic mixing capacity of a basin. The net mass transport of substance over a complete tidal cycle from bay to ocean or from ocean to bay is also qualitatively shown in Figure 2. This is seen to depend not only on the Basin Mixing Coefficient, M, but also on the concentration of substance existing in the inflow tidal prism, \( \rho \), and the basin mixing volume. Using the key for concentration level provided in Figure 2 the following is observed:

(1) Bay - For sequence 2a - 2c the transport coefficient, R, is <1 signifying a net transport of substance into the bay. For sequence 2e - 2g, R>1 signifying a net transport of substance out of the bay.
FIGURE 2 SEQUENCE OF MIXING AND EXCHANGE PROCESSES AFTER INITIAL INJECTION, IDEALIZED DESCRIPTION
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(2) Ocean - For sequence 2c - 2e and subsequent tidal cycles, the Transport Coefficient, R, is >1 signifying a net transport of substance into the ocean. This is the result of two factors: (1) The characteristic mixing volume of the ocean is large relative to the tidal exchange volume; and (2) The high energy characteristics of the open coast resulting from wind, wave, and tidal activity cause a relatively rapid decay of concentration levels with the mixing volume.

In the remaining portions of this paper these concepts are developed more fully and are applied in the analysis of data obtained from the three locations shown in Figure 1. However, while the Basin Mixing and Transport Coefficients are useful parameters for describing the exchange and flushing characteristics of a particular system, they do not provide a basis for understanding the exchange mechanism. Therefore, as previously mentioned, a conceptual model will be developed for idealized representation of Mixing Basin Coefficients.

FIELD MEASUREMENT PROGRAM

The field measurements were conducted from a 31 ft. houseboat anchored by a two point moor at each of the three locations described previously. The houseboat served as a mobile equipment platform, field station, and living quarters for the field party.

Rhodamine-WT fluorescent dye was chosen for the tracer material to be used in these experiments because of its ability to be detected in concentrations as low as 0.1 ppb and also because of its minimum biological uptake.

To commence a run of the experiment, dye was continuously injected from the anchored houseboat for one half of a tidal cycle during which the flow across the interface was into the Card Sound Basin (approximately 6.2 hours). The dye was injected by means of a hand siphon arrangement running from a dye container on deck to a point just below the surface of the water. The rate of injection was controlled by means of a screw clamp on the end of the siphon hose. Injection rates were periodically monitored and adjusted to a value of approximately 20 ml/min. Visual observations of the dye plume were made, both from above and below the water surface. These observations demonstrated the presence of large horizontal eddies and confirmed that the injected dye was quickly mixed over the cross section of the inlet and vertically over the water column.

To establish background levels of fluorescence due to natural causes existing in the Bay System and offshore waters, a series of readings was made for the two mixing basins connected by the inlet into which dye was to be injected. These readings were made prior to injection at each test site and the values recorded for future use.

The monitoring phase of each experimental run was begun at the end of the initial injection phase, i.e. when slack water was observed.
following the flow of water into Card Sound. During this phase, readings of dye concentration, \( c \), and water particle velocity, \( u \), were taken every 30 minutes at the same location (houseboat anchorage) for periods up to 63 hours (5 tidal cycles) following dye injection. Sample plots of the measured \( u \) and \( c \) values versus time are shown in Figures 3 and 4.

Water particle velocity readings were obtained from a sensor suspended from the houseboat at a fixed depth with a remote direct readout located inside the houseboat. Water samples for monitoring dye concentrations were obtained by pumping water through a continuous flow sampling door on a G. K. Turner Model 111 Fluorometer. Lengths of garden hose were used to route the water from the intake, located approximately four feet below the water surface, inside the houseboat to the pump and fluorometer, and finally outside again and over the side. This arrangement proved to be very efficient allowing all equipment to be centralized at one location protected from the weather. This is particularly critical for the proper operation of the Turner fluorometer which is sensitive to ambient lighting conditions and improper ventilation.

Weather conditions during the conduct of the measurements at Broad, Angelfish and Caesar Creeks were generally fair with light prevailing winds from the northeast.

**IDEALIZED CONSIDERATIONS AND DATA ANALYSIS**

**Idealized Considerations of Exchange Across Tidal Inlets**

The renewal of waters in a bay system occurs due to flow into the bay from upland sources, direct precipitation, and through exchange at inlets connecting the bay to the adjacent ocean. The general case of exchange at an inlet includes effects of stratification and possibly a net flow through the inlet.

In discussing the mechanics of exchange across an inlet, the primary problem of concern here will be that pertinent to the Biscayne Bay - Card Sound system in which there is little apparent net flow and in which the stratification characteristics are believed to be minimal. Two components of the mixing responsible for the exchange are: (1) the lateral effects in which flow toward the inlet occurs as a sink-type flow, however flow away from the inlet occurs similar to a separated jet, and (2) the vertical effects due to the velocity shear which is similar to the transport that would occur in an oscillating flow in a straight and uniform channel. In the following paragraphs, the contributions to mixing from lateral effects will be discussed.

**Mixing Due to Lateral Effects** - Consider the situation shown in Figure 5(a) in which separated flow is occurring away from the inlet into the bay. The flow separation is due primarily to the direction of the momentum of the exiting water and represents a jet-type flow with the jet expanding away from the inlet due to a decrease in velocity.
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Coosar Creek (8/19/72 - 8/21/72)

Plot of measured $u$ and $c$ values for run numbers 10, 10a, 10b, 10c of exchange coefficient measurements test.
Figure 4: Plot of measured $u$ and $c$ values for run numbers 12, 12a, 12b of exchange coefficient measurements test.
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FIGURE 5 IDEALIZED MECHANICS OF INLET EXCHANGE DUE TO DIFFERING OUTFLOW AND INFLOW PATTERNS
through a lateral momentum transfer to the adjacent water by shear and through a vertical transfer of momentum to the bottom through friction. For flow from the bay toward the inlet; however, the flow features occur in a sink-like manner as presented in Figure 5(b). Denoting the outflow volume as $\Omega_0$, the inflow volume as $\Omega_I$ and the volume common to the two as $\Omega_{OI}$, considerations of the proportional dilution of $\Omega_{OI}$ by bay water yields the following expression for $M_B$

$$M_B = \frac{\Omega_0 - \Omega_{OI}}{\Omega_{OI}} = \frac{\Omega_0}{\Omega_{OI}} - 1$$

(3)

where it is assumed that $\Omega_0 = \Omega_I$. This provides a qualitative description of one component of the exchange mechanism at an inlet. It is noted that the flow patterns considered away from and toward an inlet are in reasonable qualitative accord with some observations. A similar definition and qualitative description would apply for the ocean mixing coefficient, $M_o$.

This description presented can be extended somewhat further to obtain a crude formulation of the exchange coefficient. Consider the flow away from the inlet to be governed by the following vertically integrated equation of motion

$$\frac{\partial q_x}{\partial x} + \frac{\partial (q_x q_y)}{\partial y} = f q_y q_x + \frac{gh}{\rho} \frac{\partial v}{\partial y} + \frac{h}{\rho} \frac{\partial \tau}{\partial y}$$

(4)

in which the motion has been considered to be steady, $f = \text{Darcy Weisbach friction coefficient}$, $h$ and $n$ are the water depth and tidal elevation and $\tau$ is the average lateral shear stress acting on the water column to retard its flow. This shear stress represents a lateral (y-direction) flux of momentum from the jet to the adjacent waters. The quantities $q_x$ and $q_y$ represent the transport components per unit width in the x and y directions and q is the transport magnitude per unit width. In order to proceed further with this formulation, the following simplifications will be made:

$$\frac{\partial (q_x q_y)}{\partial y}, \frac{\partial n}{\partial x}, \frac{\partial \tau}{\partial y} \text{ are assumed negligible}$$

$$q_x = q$$

The resulting equation is

$$\frac{\partial q_x^2}{\partial x} = - \frac{fq_y^2}{8h}$$

(5)
and the solution is
\[ q(x) = q_0 e^{-\left(\frac{x}{\frac{16}{h}}\right)} \] (6)

which, through continuity and the assumption of no entrainment, yields an equation for the width, \( w(x) \), of the jet
\[ w(x) = w_0 e^{\frac{x}{\frac{16}{h}}} \] (7)

The flow toward an inlet is considered to be governed by sink flow in which
\[ \Omega_1 = \frac{\pi r e^2 h}{2} \] (8)

and an approximate expression for the Basin Mixing Coefficient, \( M \), is determined from Eq. (3) as
\[ M = \frac{\pi r e^{2f}}{f r e^{\frac{x}{\frac{16}{h}}} - 1} \] (9)

Unfortunately, the form above is not readily interpreted due to the many factors involved. Therefore computations were carried out to demonstrate the effect of depth, \( h \), Darcy-Weisbach friction factor, \( f \) and tidal prism, \( \Omega \). The computations required values of \( \Omega \), and \( w_0 \), etc. Values were selected which are reasonably representative for the three inlets studied. The approximate values of \( \Omega \) and \( w_0 \) and the values selected for computation are presented in Table I.

<table>
<thead>
<tr>
<th>Inlet</th>
<th>( h_0 ) (ft.)</th>
<th>( w_0 ) (ft.)</th>
<th>( m^* ) (ft.)^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broad Creek</td>
<td>7</td>
<td>2180</td>
<td>0.9 x 10^9</td>
</tr>
<tr>
<td>Caeser Creek</td>
<td>10</td>
<td>1800</td>
<td>1.3 x 10^9</td>
</tr>
<tr>
<td>Angelfish Creek</td>
<td>12</td>
<td>750</td>
<td>2 x 10^8</td>
</tr>
<tr>
<td>Base Value Used in Computations</td>
<td>Not Required</td>
<td>2000</td>
<td>1 x 10^9</td>
</tr>
</tbody>
</table>

*Based on O'Brien's Equilibrium Cross-Section Tidal Prism Relationship (Reference 5)
Effect of water depth on mixing coefficient - The effect of water depth was investigated by fixing all other variables equal to values which are considered realistic for the measurements and calculating $M$ for $h = 5, 8,$ and $10$ ft. The values for the other variables used are: $f = 0.02, \alpha = 1 \times 10^9 \text{ ft.}^3,$ and $w_o = 2000 \text{ ft.},$ thereby reducing Eqs.(8) and (9) to

$$M = \frac{1}{1.6 \times 10^{-3} h^2 \left(e^{31/h^3/2} - 1\right)} - 1 \quad (10)$$

The effect of varying $h$ is shown in Table II where it is seen that smaller water depths tend to inhibit mixing.

<table>
<thead>
<tr>
<th>Water Depth, $h$ (ft.)</th>
<th>Basin Mixing Coefficient, $M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.67</td>
</tr>
<tr>
<td>8</td>
<td>2.33</td>
</tr>
<tr>
<td>10</td>
<td>2.75</td>
</tr>
</tbody>
</table>

Effect of friction coefficient on mixing coefficient - The same procedure was followed to investigate the effect of variations in the friction coefficient as was described previously for the water depth. The fixed values were: $h = 6$ ft., $\alpha = 1 \times 10^9 \text{ ft.}^3,$ and $w_o = 2000$ ft. resulting in the approximate equation

$$M = \frac{f}{1.2 \times 10^{-3} \left(e^{107f} - 1\right)} - 1 \quad (11)$$

The effects of varying friction coefficient are presented in Table III where it is seen that a large friction coefficient causes the ebb jet to be retarded close to the inlet and to widen. Reference to Figure 5 will demonstrate that this results in a reduced mixing.
TABLE III
EFFECT OF FRICTION COEFFICIENT, f
ON MIXING COEFFICIENT, M.

<table>
<thead>
<tr>
<th>Darcy - Weisbach Friction Coefficient, f</th>
<th>Basin Mixing Coefficient, M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>3.35</td>
</tr>
<tr>
<td>0.02</td>
<td>1.22</td>
</tr>
<tr>
<td>0.03</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Effect of tidal prism of mixing coefficient - Repeating the described procedure for the tidal prism, \( \Omega \), as the variable and \( f = 0.02 \), \( w_o = 2000 \) ft., and \( h = 6 \) ft. as the fixed variables

\[
M = \frac{\Omega}{60 \times 10^6 (e^{0.7 \times 10^{-4} \sqrt{\frac{\Omega}{h}}} - 1)}
\]  

and, as shown in Table IV, the Basin Mixing Coefficient is a maximum for \( \Omega = 5 \times 10^8 \) ft.\(^3\). It should be recalled that although the Basin Mixing Coefficient decreases with increasing \( \Omega \) (\( > 5 \times 10^8 \) ft.\(^3\)), the volume of mixing water increases due to the definition of \( M \) being normalized by \( \Omega \).

TABLE IV
EFFECT OF TIDAL PRISM, \( \Omega \),
ON MIXING COEFFICIENT, M.

<table>
<thead>
<tr>
<th>Tidal Prism ( \Omega ) (ft.(^3))</th>
<th>Basin Mixing Coefficient, M</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 1 \times 10^8 )</td>
<td>0.64</td>
</tr>
<tr>
<td>( 5 \times 10^8 )</td>
<td>1.20</td>
</tr>
<tr>
<td>( 1 \times 10^9 )</td>
<td>1.05</td>
</tr>
</tbody>
</table>

It is noted that the equations based on the conceptual model are very approximate. Use of some ranges of values in these equations can result in negative M values. For example, small \( h \) values will result in negative M values when used in Eq. \((10)\).
Data Analysis

As discussed previously, the data consisted of water velocity, \( u \), and dye concentration readings, \( c \), which, under ideal circumstances, were taken every one-half hour.

To analyze the recorded data, values of \( u \) and the product of \( u \) and \( c \) for each half-hourly reading were multiplied by the time interval between readings and the resulting values summed for each half tidal cycle. These sums represent approximations of the tidal prism and volume of dye respectively crossing the interface in each half tidal cycle. This is shown to be the case if it is assumed that the total instantaneous discharge, \( Q(\text{ft}^3/\text{sec}) \), across the interface at any time, \( t \), is proportional to the velocity, \( u(\text{ft./sec}) \), measured at a point, i.e.

\[
Q = Ku
\]

where,

\[
K = \text{constant, (ft.}^2\text{)}
\]

If the assumption stated by Eq. (13) is valid, then the tidal prism, \( \Omega \), is given by

\[
\Omega = \int_0^{T/2} Q \, dt
\]

\[
= \lim_{\Delta t \to 0} \sum_{i=1}^{N} \frac{Q_i \Delta t_i}{\Delta t}
\]

\[
= \lim_{\Delta t \to 0} \sum_{i=1}^{N} Ku_i \Delta t_i
\]

and since \( K \) is assumed to be constant

\[
\Omega = K \sum_{i=1}^{N} u_i \Delta t_i
\]

where,

\( T = \text{tidal period, 12.4 hrs.} \)

\( N = \text{number of measurement intervals in one half of a tidal cycle (T/2)} \)

In a similar manner it may be shown that the volume of dye, \( \Omega_d \),
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Crossing the interface in one half tidal cycle is given by

\[ N_{i=1} = K \sum_{i=1}^{N} u_i c_i \Delta t_i \]  

Upon examination of the values of \( u \), it was found that in most cases the measured water particle velocities were significantly greater during one half of the tidal cycle as compared to the other half. This fact was dramatically displayed by the values of \( n \) computed using Eq. (14) which indicated that the tidal exchange volumes passing across the interfacial regions varied by as much as a factor of 4. Based upon results obtained from the calibrated numerical model (Reference 1), which indicate that the ebb and flood tidal exchange volumes are with 0.5% of being equal, it was concluded that the assumption given by Eq. (13) was not valid, and therefore the measured values of \( u \) could not be used. As an alternative approach to obtaining values for \( u \) suitable for use in Eqs. (14) and (15), sinusoidal velocities were generated by adjusting the amplitude of the velocity such that

\[ \left\{ \sum_{i=1}^{N} u_i \Delta t_i \right\}_{\text{ebb}} = \left\{ \sum_{i=1}^{N} u_i \Delta t_i \right\}_{\text{flood}} \]  

for each tidal cycle and having the same times of slack water as the observed velocity. In effect, this approach loses whatever difference there may be between the ebb and flood tidal exchange volumes while preserving the phase relationship between \( u \) and \( c \).

Values for \( n \) and \( n_{d} \) obtained using generated velocities, \( u \), and measured dye concentration, \( c \), were then applied to a mass balance of the dye during one tidal period. The mass balance considers the volume of dye crossing the interface during one half of a tidal cycle mixing with a volume of water containing a given concentration of fluorescence, and the resulting mixture crossing the interface in the opposite direction. From the mass balance, a "Basin Mixing Coefficient" is calculated which represents the mixing and flushing characteristics of the interfacial region, i.e. the Basin Mixing Coefficient, \( M \), is expressed as

\[ M = \frac{\bar{c}_I - \bar{c}_o}{\bar{c}_o - \bar{c}_mb} \]  

where

\[ c_I = \frac{n_{dI}}{\Omega_I} \]
It is noted from Eq. (17) that calculation of a basin mixing coefficient requires knowledge of, $c_{mb}^e$, the effective concentration of the basin waters which mix with the waters flowing into that basin. With the exception of initial background fluorescent levels, no attempt was made to determine $c_{mb}^e$. In fact, its determination would prove to be a difficult task because it is an effective value that is required. To rationally utilize Eq. (17), the following procedure was developed to estimate the time-varying value of $c_{mb}^e$ from the field measurements.

The expression utilized to describe $c_{mb}(t)$ is

$$\frac{dc_{mb}^e}{dt} = \lambda_I^e (c_I^e - c_{mb}^e) - \lambda_{mb}^e c_{mb}^e$$  \hspace{1cm} (19)

rate of increase of $c_{mb}^e$ due to mixing with inflow tidal exchange volume

rate of decrease of $c_{mb}^e$ due to mixing of high localized concentrations with adjacent bay waters

where

$\lambda_I^e =$ decay constant for the mixing of basin water with inflow tidal exchange volume ($t^{-1}$)

$c_I^e =$ effective constituent concentration in water crossing interface into mixing basin

$\lambda_{mb}^e =$ constant representing decrease rate due to mixing with adjacent bay waters

For purposes of analysis; it is simpler and does not affect the results if the following equation is employed

$$\frac{dc_{mb}^e}{dt} = \lambda_I^e c_I^e - \lambda_{mb}^e c_{mb}^e$$  \hspace{1cm} (19)

where the definitions of $\lambda_I^e$ and $\lambda_{mb}^e$ are altered accordingly. Eq. (19) was solved using the following approximation for $c_I^e$

$$c_I^e(t) = c_{I_0} e^{-\lambda_I^e t}$$  \hspace{1cm} (20)

where $c_{I_0}$ represents the effective constituent concentration at the interface for $t = 0$. Inserting Eq. (20) into (19), yields as a solution
\[ c_{mb}(t) = c_{mbo} + \frac{c_{10} \lambda_1}{\lambda_{mb} - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_{mb} t}) \] 

(21)

The three unknowns \((\lambda_1, \lambda_{mb}, c_{10})\) in the above equation were determined by requiring that the basin mixing coefficient, \(M\), always be positive. Restating Eq. (17)

\[ M = \frac{\bar{c}_I - \bar{c}_o}{\bar{c}_o - \bar{c}_{mb}} \] 

(22)

it is seen that the sign of \(M\) can be positive or negative, depending on

**Situation 1:**

\[ M > 0 \] 

\( \{ \bar{c}_I > \bar{c}_o \) and \( \bar{c}_o > \bar{c}_{mb} \) or if \( \bar{c}_I < \bar{c}_o \) and \( \bar{c}_o < \bar{c}_{mb} \}\)

**Situation 2:**

\[ M < 0 \] 

\( \{ \bar{c}_I < \bar{c}_o \) and \( \bar{c}_o > \bar{c}_{mb} \) or if \( \bar{c}_I > \bar{c}_o \) and \( \bar{c}_o < \bar{c}_{mb} \}\)

The basis for utilizing the data collected to determine \(\lambda_1, \lambda_{mb}\) and \(c_{10}\), and hence \(c_{mb}\) and \(M\) is that \(M\) should be positive. The procedure then involves selecting trial values of the 3 unknowns along with the constraint \(M > 0\). This procedure is explained more fully in Reference 3. A graphical illustration of the constraints which the procedure places on \(c_{mb}(t)\) is presented in Figure 6. It is seen that the values of \(\lambda_1, \lambda_{mb}\) and \(c_{10}\) define the curve for \(c_{mb}(t)\) within a fairly limited range. Considering the Bay Mixing Coefficient, \(M_B\), during the initial period when \(\bar{c}_I > \bar{c}_o\), an upper limit is established for \(\bar{c}_{mb}\). In later phases of the testing program, after a significant buildup of dye in the bay has occurred, \(\bar{c}_o > \bar{c}_I\) and a lower limit on \(\bar{c}_{mb}\) is established. Using various trial values of the unknowns to meet these limits, a relationship for \(\bar{c}_{mb}\) is obtained which appears reasonable as shown in Figure 6.

**RESULTS**

For the three inlets, Broad Creek, Caeser Creek and Angelfish Creek (shown in Figure 1), measurements of basin mixing coefficients were carried out for a total of thirteen tidal cycles or portions of tidal cycles. As discussed previously these coefficients represent the number of tidal prisms of basin water mixing with the incoming water.
\[ C_{mb} = \frac{4.7}{C_{mb_0}} \]
\[ \lambda = 1.1 \]
\[ \lambda_{mb} = 0.021 \]
\[ C_{l_0} = 6.45 \]

Interval Used In Computations

FIGURE 6  PLOT OF \( C_{mb}(t) \) VS \( t \)  FOR BAY SIDE CAESAR CREEK
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prior to outflow through the inlet. As used in this context, the word "basin" can represent either the bay or the ocean.

An example of measured dye concentration and inlet velocities is presented in Figure 3 for Caeser Creek and Runs 10, 10a, 10b, and 10c. The dye was injected on the flood cycle with monitoring commencing on the first following ebb cycle. Very briefly it is seen that there is considerable residual dye retained in the bay after injection as evidenced by the maximum dye concentration approximately coinciding with the end of each ebb cycle. Moreover, it is seen that the minimum dye concentration corresponds more or less with the completion of the flood cycle. The slow rate of decay is secondarily due to the low Bay and Ocean mixing coefficients, but is primarily due to the residual dye retained in the Bay following the injection and subsequently being partly released on each ebb cycle. Both the bay and ocean shoals of Caeser Creek are very substantial with the ocean shoals off Caeser Creek extending 3 nautical miles offshore and the bay shoals extending approximately one nautical mile bayward. These shoals tend to inhibit mixing of the tidal prism, thereby resulting in low values of the basin mixing coefficients.

The Basin Mixing Coefficients are presented in Table V for the three inlets studied. Only the Ocean Mixing Coefficients were obtained for Broad Creek because this was the first inlet studied and in those early phases of the measurement program, the measurements were limited to determination of the Ocean Mixing Coefficients. The asterisked data in Table V indicates that measured dye concentrations used to calculate these points were close to the ambient concentration. The denominator and numerator of Eq. (22) are therefore small and the accuracy of these results is questionable.

It is noted that during the measurements there was some variation in the reasonably constant light northeast wind conditions. Moreover any secondary longshore currents in the Bay or Ocean could result in "sweeping" the residual dye away from the influence of the inlet. This is believed to be of particular significance on the ocean side of the inlets where the proximity (5 miles) of the Gulf Stream is known to cause erratic nearshore currents parallel to the shoreline.

SUMMARY AND CONCLUSIONS

Summary

Basin Mixing Coefficients have been defined as a measure of the proportional mixing that occurs with the return portion of the tidal prism flowing into that basin. A conceptual model is proposed based on the effective lateral mixing resulting from the different flow patterns that occur on the inflow and outflow portions of the cycle. The flow away from the inlet is considered to occur as a separated flow whereas a sink-type pattern is assumed to prevail during the flow toward the inlet. Based on this simple concept a relationship is
### TABLE V

**SUMMARY OF BAY AND OCEAN MIXING COEFFICIENTS**

<table>
<thead>
<tr>
<th>Inlet</th>
<th>Run No.</th>
<th>Ocean Mixing Coefficient, ( M_o )</th>
<th>Bay Mixing Coefficient, ( M_b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Broad</td>
<td>6</td>
<td>1.53</td>
<td>-</td>
</tr>
<tr>
<td>Creek</td>
<td>7</td>
<td>4.40</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1.91 (Avg.: 2.61)</td>
<td>-</td>
</tr>
<tr>
<td>Caeser</td>
<td>10</td>
<td>0.86</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>10a</td>
<td>0.71</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>10b</td>
<td>0.73</td>
<td>1.97</td>
</tr>
<tr>
<td></td>
<td>10c</td>
<td>1.32</td>
<td>0.46</td>
</tr>
<tr>
<td></td>
<td>10d</td>
<td>0.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Avg.: 0.81)</td>
<td></td>
<td>(Avg.: 1.05)</td>
</tr>
<tr>
<td>Angelfish</td>
<td>11</td>
<td>2.10</td>
<td>10.02*</td>
</tr>
<tr>
<td></td>
<td>11a</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.53</td>
<td>4.59</td>
</tr>
<tr>
<td></td>
<td>12a</td>
<td>0.52</td>
<td>0.28*</td>
</tr>
<tr>
<td></td>
<td>12b</td>
<td>0.47*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Avg.: 0.91)</td>
<td></td>
<td>(Avg.: 4.96)</td>
</tr>
</tbody>
</table>

* Indicates value subject to considerable error
developed for the Basin Mixing Coefficient, M. Measurements were carried out at three inlets in South Florida and the resulting Bay and Ocean Mixing Coefficients are reported.

Conclusions

The field measurements suggest, and the conceptual model supports the importance of shoals in inhibiting mixing of the tidal prism in a basin. Additional measurements of the type reported here are needed to: (1) provide necessary exchange characteristics for numerical modeling of constituent transport in any particular area, and (2) extend the data base to different tidal prisms, bathymetries and climatic conditions.

ACKNOWLEDGEMENT

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REFERENCES


