CHAPTER 125

AMPLIFICATION OF LONG WAVES IN MARINA DEL REY

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ABSTRACT

Water surface oscillations in a man-made marina--Marina del Rey, California--have been studied through prototype experiments and theoretical analysis. The prototype experiments are conducted by using both the floating gauge and pressure type gauge. The field data show that a distinct mode of oscillation with wave periods between 45 minutes and 50 minutes are naturally present almost all the time. Such resonant wave periods also agree well with the theoretical results.

INTRODUCTION

Waves which propagate from the open-sea across the continental shelf into bays and harbors usually experience changes in propagating direction, wave profile due to local bathymetry and horizontal geometry of the embayment. The geographical configuration of a harbor can cause significant amplification or attenuation of the incident wave system especially for the long period waves. The phenomenon of large amplifications of long period waves in harbors or bays is well documented, e.g., Raichlen (1966), Wilson (1972), and Miles (1974).

The present work contains some prototype long period wave measurements in a man-made marina--Marina del Rey, California. In order to understand and explain the measured result, the theoretical results calculated by using the theory developed by Lee and Raichlen (1972), are compared.

Marina del Rey is the largest man-made small craft harbor in the United States. It is located approximately 15 miles west of central Los Angeles. At the present time there are approximately 6000 small boats which are moored inside the marina.

In the following, a brief outline of the theoretical analysis
will be presented followed by a description of the experimental equipment. The data obtained from the field measurements will be compared with the theoretical results.

THEORETICAL ANALYSIS

For the theoretical analysis, the flow is assumed irrotational and the fluid incompressible, thus one can define a velocity potential $\Phi$ such that the fluid particle velocity vector can be expressed as the gradient of the potential. In order to simplify the problem, the horizontal layout of Marina del Rey (as shown in Fig. 1) is approximated by only the main channel with the entrance open to the open-sea region. The water depth in the marina is considered to be constant. This assumption plus the linearized free surface condition at the water surface and the no-flow condition at the bottom will enable one to find the velocity potential in the form of:

$$
\Phi(x, y, z, t) = \frac{1}{i \sigma} \frac{A g \cosh k (h + z)}{\cosh kh} f(x, y) e^{-i \sigma t}
$$

where $i = \sqrt{-1}$, $\sigma$ is $2\pi$/period, $k$ is $2\pi$/wave length, $h$ is the water depth, $z$ is the vertical coordinates which originate from the water surface, $x$ and $y$ are the horizontal coordinates. The function $f(x, y)$ must satisfy the Helmholtz's equation:

$$
\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0,
$$

and the boundary conditions that $\frac{\partial f}{\partial n} = 0$ along all solid boundaries and the radiation condition.

To solve for $f(x, y)$ in the harbor, the total domain of consideration is divided into three regions: the open-sea region, the region bounded by MNOP, and the region bounded by OPQR (see Fig. 1).

The function $f$ in any position $(x, y)$ in Region I is expressed as:

$$
f(x, y) = -\frac{i}{4} \int_s \left[ f(x_o, y_o) \frac{\partial}{\partial n} \left[ H_o^{(1)} (kr) \right] \right] ds
$$

$H_o^{(1)} (kr) \frac{\partial}{\partial n} f(x_o', y_o')$
Fig. 1 Layout of the Marina del Rey Model

S: Location of Harbor Patrol Station
Fig. 2. Surface Gauge for Long Wave Measurement
where \( x, y \) is the position of the boundary point, \( r \) is the distance between \( (x, y) \) and \( (x', y') \), \( n \) is directional normal of the boundary. If \( (x, y) \) is allowed to approach the boundary at \( (x', y') \), then an integral equation can be formed from Eq. (3). The integral equation so formed can become matrix equation by approximating the line integral with a discrete summation from divided boundary segments:

\[
F = G_n \cdot F - G^F_{-n}
\]

where \( F \) represents a vector of the value of \( f \) at each of the boundary segments, \( F_n \) is a vector representing the value of \( \partial f / \partial n \) at each boundary point. \( G_n \) and \( G \) are both matrixes, the size of the matrix is the number of segments into which the boundary is divided. Eq. (4) can be further reduced to a matrix equation with \( F \) expressed only in terms of the unknown normal derivative of \( \partial f / \partial n \) at the harbor entrance (MN) and the common boundary OP.

Following the same procedure, the solution of \( f \) in Region II can be expressed in terms of \( \partial f / \partial n \) at the common boundary OP. These unknown values of \( \partial f / \partial n \) at the common boundaries (MN and OP) can be solved by using the continuity condition, i.e., water surface elevation as well as the horizontal velocity must be continuous at these boundaries.

**EXPERIMENTAL EQUIPMENT AND PROCEDURES**

The field data were obtained using two types of long-period wave gauges. The first type of gauge is called the floating gauge; a schematic diagram of the floating gauge is shown in Fig. 2. It is seen that the gauge consists of a stilling well which is constructed of 1 1/2 inches I.D. schedule 40 PVC pipe of 14 feet long. Inside the stilling well a water surface float equipped with a permanent magnet is placed. A tube (constructed of 1/4 inch I.D. schedule 80 PVC pipe of 12 feet long) which contained a resistive voltage divider and 80 reed switches evenly spaced at 4 centimeter centers, was attached to the outside of the stilling well. The permanent magnet in the float which moves with the changes in water surface elevation closed the magnetic reed switch nearest to it and taped off a voltage which was proportional to the water surface elevation.

The second type of gauge was a strain gauge pressure transducer and signal conditioner mounted in a waterproof pressure protected case which was placed on the sea floor. The pressure variations at the sea floor were calibrated to corres-
pond to variations in water surface elevation. The signals were transmitted to the recording station located on shore through the undersea cable. This underwater instrument is primarily used in locations where surface obstruction must be avoided such as the location at the center of the marina entrance. The electronic signal is recorded on a Honeywell-Brown "Electronic" Servo-recorder.

**RESULTS AND DISCUSSION**

Numerically determined response of the simplified Marina-del Rey model to periodic incident waves is presented in Fig. 3. As mentioned earlier, the results are obtained by dividing the total region of consideration into three sub-regions: Open-sea Region, Region I, and Region II with the final results obtained by matching solution at all of the common boundaries. The ordinate in Fig. 3 represents the amplification factor which is defined as the maximum wave height within the marina divided by the standing wave height at the entrance if the marina is closed. The abscissa is the dimensionless wave number \((kl)\) where \(k\) is the wave number and \(l\) is the length of the main channel. In prototype dimension \(l\) is 11,000 ft. If one uses the mean depth of 12 ft., the wave period for the first resonant mode \((kl=1.49)\) is approximately 40 minutes. The wave period for the second resonant mode is 13 minutes \((kl=4.57)\), for the third mode the period is 7.5 minutes. Attention is also directed to the results of large amplification of wave amplitude for the first mode. The mode shape for each of the resonant mode is presented in Fig. 4. It is clear that for the first mode the water surface elevation is quite uniformly distributed except the region near the harbor entrance. This resonant mode is referred to as Helmholtz mode or pumping mode. For the second mode there exists one nodal line so that as part of the basin is in positive displacement the other part is in negative displacement. For the third mode of oscillation there are two nodal lines, and the shape of water surface elevation is relatively more complicated.

The observed long period wave amplitude in the marina varied from day to day, but were approximately 10 inches for the first mode, 2-4 inches for the second mode and perhaps 1 inch for the third mode. Here, only the first mode data will be compared with surface wave gages and the theoretical predictions. An example of large amplification of long period wave within the marina is shown in Fig. 5. The upper curve in Fig. 5 is the water surface history at the marina entrance taken by the pressure gage placed at the bottom. Other than the tide, long period waves are not obvious. The lower curve in Fig. 5 represents the water surface history
Fig. 3. Response Curve at Marina del Rey, Calif.
<table>
<thead>
<tr>
<th>Mode</th>
<th>Eigenvector Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Mode (kl = 1.49)</td>
<td>0.0 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0</td>
</tr>
<tr>
<td>2nd Mode (kl = 4.57)</td>
<td>0.0 2.0 4.0 5.0 5.5 5.0</td>
</tr>
<tr>
<td>3rd Mode (kl = 7.75)</td>
<td>-3.0 -3.25 -3.0 -2.0 -1.0 -1.0 -2.0 -3.0 -5.0 -5.5</td>
</tr>
</tbody>
</table>

**Fig. 4. Mode Shape of the First Three Modes**
Fig. 5. Water Surface History at Two Locations inside the Marina

2-12-73
12:00 NOON

Harbor Entrance

Station S.
Fig. 6. Water Surface History at Two Location inside the Marina
at the harbor patrol station (Station S shown in Fig. 1) taken by
the float gage. In addition to the tidal fluctuation it is clearly seen
that long period waves of approximately 45 minutes period also
exist. The wave height of this wave is about 0.8 ft. It is interesting
to note that the observed wave period is very close to that of the
first mode \((k_1=1.49)\) shown in Figs. 3 and 4. The amplification
factor based on the field results is not known because no estimate
of the incident wave amplitude at this period can be made. However
one may obtain the approximate ratio of the relative amplitude be-
 tween the lower curve and the upper curve for this period. This
ratio is about six, which agree reasonably well with that indicated
in the lower curve of Fig. 4. Thus, it appears that the field data
agree well with the theoretical prediction at least for the first mode.

Fig. 6 shows the wave records for the same respective loca-
tions as that shown in Fig. 5. These data were taken during a sea
storm on Feb. 28, 1973. A significant difference between Fig. 5 &
Fig. 6 is that the amplitude of higher frequencies waves is smaller
at Station S than that at the marina entrance. Thus the marina do
damp out waves with higher frequencies although they do amplify
certain long period waves.

In order to more fully demonstrate the nature of the first
mode of oscillation within the marina, records taken from three
measuring devices placed at three different locations within the
marina is shown in Fig. 7. These three locations are: Station S (harbor patrol station), Station X (the end of the main channel),
and Station Y (inside the Basin E). Records from the Station X
are taken by means of the hydrodynamically filtered pressure gauge
placed at the bottom, while at the other two locations the digital
type surface gages are used. It can be seen that the records are in
phase at these locations for the first mode. The amplitude of first
mode of oscillation at Stations X and Y are larger than that at
Station S as was predicted in the theoretical calculation.

CONCLUSIONS

Results from the field experiment conducted at Marina del
Rey, Los Angeles, California clearly showed the phenomenon of
long period oscillation within the marina. The wave period of the
first mode is approximately 45 minutes - 50 minutes. Such mode
is naturally present almost at all time. Theoretical results have
also been shown to agree reasonably well with the field experiments.
It will be of great interest to investigate further the source of
excitation of such long waves.
Fig. 7. Water Surface History at Three Locations Inside the Marina
ACKNOWLEDGEMENT

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LIST OF REFERENCES


