

CHAPTER 118

ECONOMIC APPROACH TO OPTIMIZING DESIGN PARAMETERS

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ABSTRACT

For coastal engineering works, as for other structures, the designer must search for the economic optimum point. This point represents the minimum in the sum of direct cost and cost of possible future damage. By setting up functional representations of these costs the optimum can be obtained directly. This approach is illustrated by models developed independently in the Netherlands and in Canada. At this stage the output of the models may be denoted as the 'perfect knowledge' optimum, in the sense that parameters of the cost functions are assumed to be known with complete accuracy.

In the 'real world' case, however, the estimated values of the parameters will be subject to considerable uncertainty. It is shown that because of this uncertainty the 'real world' design optimum will generally be shifted to give a structure larger than that indicated by the 'perfect knowledge' assumption. The novel contribution of the present paper consists of analyzing this shift to obtain simple expressions for the apparent overdesign due to uncertainty and for the resulting cost increase. An illustrative example is presented.

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INTRODUCTION

Structures which must resist natural forces are generally designed to withstand a potential damage event which can be defined by a specific return period. For most coastal engineering works damage can be associated with wave action. Although a rigorous description of potentially damaging waves would involve spectral analysis, in common practice it is found that a single wave height parameter is sufficient to quantify a potential damage event, i.e. a given storm. The wave height can then be combined with the appropriate tide height to yield the height parameter h . This parameter is taken to be a random variable which has a probability density function $f(h)$ (Figure 1) and cumulative distribution function $F(h)$. Engineers commonly use the exceedance probability, $p = 1 - F(h)$, or its reciprocal, the return period T , as indices of the magnitude of the event. Selecting a specific value of the height parameter as the design height, H_d , is equivalent to fixing the return period, T , of the potential damage event which the structure is to withstand. Conversely, specification of a return period defines the design height.

In principle, the optimum design can be obtained by economic analysis. With small T (and H_d) the initial outlay for construction will be low, but the cost of probable future damage will be large. On the other hand with large T (and H_d) probable future damage will be small but construction cost will be greatly increased. The optimum return period, \hat{T} , which lies between these extremes, produces a minimum in the sum of construction cost plus cost of probable future damage. Figure 2 shows this optimum point.

The optimization process can be carried out by successive trial designs but this is tedious, and in many cases a valuable simplification can be made by approximating both the construction cost and the cost of probable future damage by mathematical functions. In particular, this approach provides a suitable method for dealing with groups of similar structures. Although some design studies may be required to determine the numerical values of coefficients included in the functions, the optimum point can now be obtained directly by differentiating the total cost for a minimum. This approach to the optimum design of coastal works was pioneered in studies made in the Netherlands. An analysis developed independently in Canada for flood-control works in inland waters is conceptually similar and provides a valuable alternate formulation for coastal structures. In both cases the expressions obtained for the optimum show the effects of the regional probability function and the type of structure. In addition, the importance of economic factors such as interest rate is taken into account.

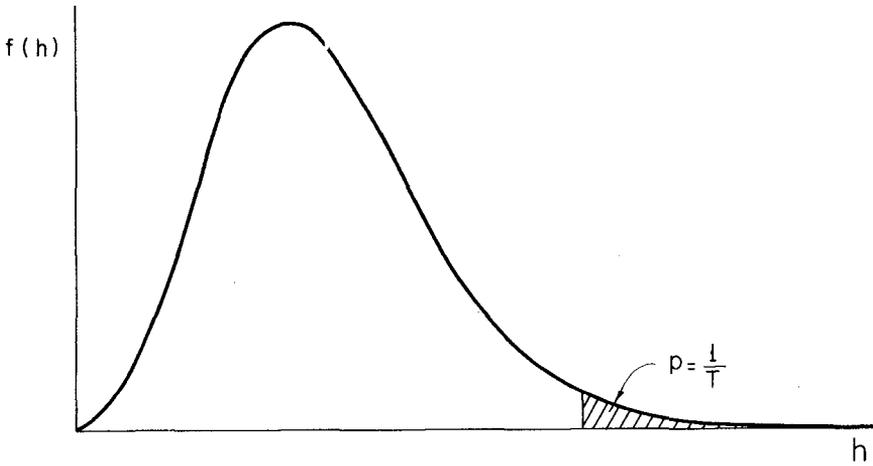


FIGURE 1 PROBABILITY DENSITY FUNCTION FOR h

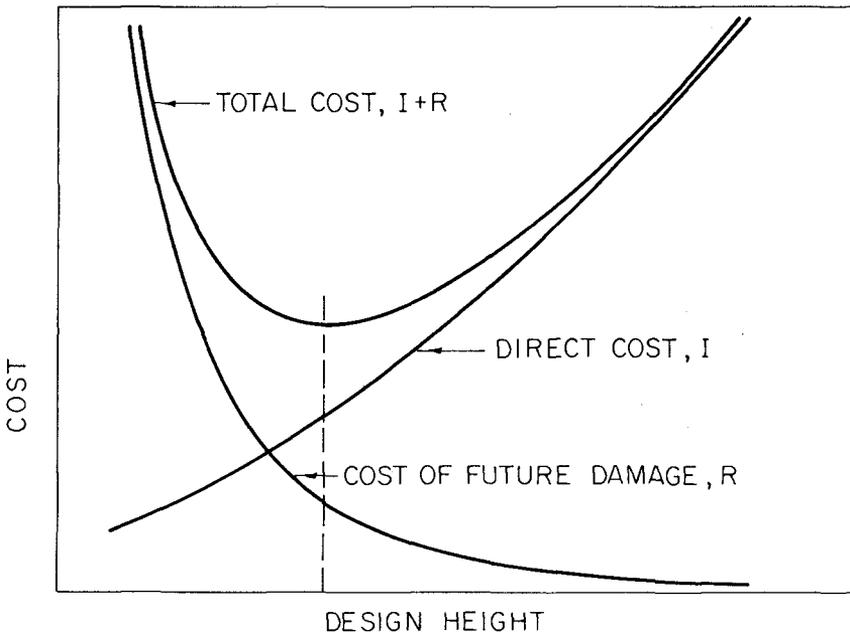


FIGURE 2 COST RELATIONS FOR COASTAL STRUCTURES

'PERFECT KNOWLEDGE' OPTIMIZATION

Overall Concept

The development of the analytical model is carried out in two stages. In the first stage the relations leading to the economic optimum design for a typical structure are set up to yield the common return period for a regional aggregate. In dealing with these relations it is assumed that all the required parameters, including the characteristics of the regional probability function, are known with complete accuracy. This may be termed the 'perfect knowledge' case, on the understanding that the knowledge of extreme events extends only to the statistics of their distribution and not to their particular values in specific future years.

This first stage -- the 'perfect knowledge' model -- is outlined below. In the second stage, which will be dealt with in subsequent sections, the analysis is modified to take account of the effects of uncertainties in the input data, including uncertainty in the assumed regional probability function.

Cost Functions

As mentioned previously, economic analysis could be carried out by determining direct cost and damage cost for a series of trial designs. However, greater insight can be obtained by using continuous cost functions rather than discrete trial values. The functions for direct cost and damage cost are expressed in terms of an appropriate variable, denoted as X , which is known as the scope of the trial design. In coastal engineering problems X would represent the height of a dike or breakwater above some datum level. Note that here the scope X is quite distinct from the height parameter h . The lower the value of X which is selected, the greater is the probability of overtopping, and hence of damage.

The specific relations linking direct cost and damage cost to the scope X are somewhat different in the Netherlands and the Canadian models. These relations are set out in Table 1. However, in the text below these relations are given in general functional form. Whenever possible, the nomenclature used in the Netherlands study has been adopted.

Direct Cost: The direct cost, I , comprises construction cost plus capitalized cost of annual maintenance over the lifetime of the structure. Therefore, I is an increasing function of the scope of the trial design, which may be written as

$$I = \phi_1(X) \quad (1)$$

TABLE 1
SUMMARY OF PERFECT KNOWLEDGE OPTIMIZATION

	<u>Relation</u>	<u>Netherlands Model</u>	<u>Canadian Model</u>
Scope	X	$X = H - H_0$	$X = Q - Q_0$
Direct Cost	$I = \phi_1(X)$	$I = I_0 + I^*X$	$I = K_1 X^m$ *
Exceedance Probability	$p = \frac{1}{T} = \phi_2(X)$	$p = p_0 e^{-\alpha X}$	$p = (X/A)^{-q}$
Damage Cost per Event	$\phi_3(X)$	W	$K_3 X^r$ **
Present Value Function	$\psi(\delta, L)$	$1/\delta$	$f = ((1 + \delta)^L - 1) / \delta$
Cost of Probable Future Damage	$R = \psi(\delta, L) \phi_2(X) \phi_3(X)$	$R = R_0 e^{-\alpha X}$ $(R_0 = p_0 W / \delta)$	$R = K_2 X^{-n}$ $\left[\begin{array}{l} n = q - r \\ K_2 = f K_3 A^q \end{array} \right]$
Optimum Scope	\hat{X}	$\hat{X} = \frac{1}{\alpha} \ln \frac{R_0 \alpha}{I^*}$	$\hat{X} = \left[\frac{n K_2}{m K_1} \right]^{1/(m+n)}$
Optimum Return Period	$\hat{T} = 1/\phi_2(\hat{X})$	$\hat{T} = \frac{1}{p_0} e^{\alpha \hat{X}}$	$\hat{T} = (\hat{X}/A)^q$
Relative Loss	$D = \phi_4(y)$	$D = e^{-y} - 1 + y$	$D = \frac{n}{m} (e^{my/q-1}) + (e^{-ny/q-1})$
	$\left[\begin{array}{l} D = \frac{(I+R) - (\hat{I}+\hat{R})}{\hat{R}} \\ y = \ln(T/\hat{T}) \end{array} \right]$		

* In the Canadian model the direct cost can also be expressed in the more general form $I = I_0 + K_1 X^m$. This change does not affect the other equations given above.

** The expression given here for 'damage cost per event' is one of the possible formulations studied in the Canadian model. For further information see reference 4. Note that when $r = 0$, $X^r = 1$ and K_3 corresponds to W of the Netherlands model.

Cost of Probable Future Damage: The cost of probable future damage or future losses, R , is the total present worth of possible damage events during the service life. Hence, R equals the product of two factors; the first is the present worth function of the interest rate and the service life, $\psi(\delta, L)$, and the second is the probable cost of damage in any individual year. The second factor, in turn, is given by the product of the probability of a damage event, $\phi_2(X)$, and the ensuing damage cost, $\phi_3(X)$.

$$R = \psi(\delta, L) \cdot \phi_2(X) \cdot \phi_3(X) \quad (2)$$

Optimum Scope and Optimum Return Period

The optimum scope, \hat{X} , which is associated with the minimum in the total cost can be determined by differentiating equations 1 and 2 with respect to X and equating the sum of the derivatives to zero. The corresponding optimum return period is obtained from the reciprocal of the exceedance probability.

$$\hat{T} = 1 / \phi_2(\hat{X}) \quad (3)$$

Relative Loss

The difference between the total cost, $1 + R$, for any value of X and the total cost for \hat{X} can be defined as a loss due to lack of optimization. This loss, divided by some typical cost (say \hat{R}), is denoted as the relative loss, D . In this work D is expressed not in terms of $(X - \hat{X})$ but in terms of the relative return period T/\hat{T} or, to be more specific, its natural logarithm, denoted here by y .

$$D = \phi_4(y) \quad (4)$$

Figure 3 shows the form of this function.

CONSEQUENCES OF UNCERTAINTY

The optimization process dealt with in the previous section was based on the assumption that all the required parameters were known exactly. In the 'real world' situation, however, the values employed necessarily represent estimates made from available data. These estimates are, of course, subject to significant uncertainty.

On initial inspection it might be thought that such uncertainty would not influence the design return period, which could still be taken as the 'perfect knowledge' optimum, \hat{T} . However, further consideration

will show that this is not generally the case. The total cost function is not symmetric about its minimum point, and for structures of the type considered here the left hand limb of the curve rises much more rapidly than the right hand limb, as illustrated by Figure 2. It follows that the penalty for underdesign is greater than that for an equivalent overdesign. If the 'perfect knowledge' optimum were adopted for design, then cases of both underdesign and overdesign would occur, but the total penalty, or financial loss, for the cases of underdesign would exceed that for the cases of overdesign. In these circumstances the total loss can be reduced by the strategy of adopting a design return period larger than the 'perfect knowledge' optimum. The analysis of the 'real world' case which is given below relates this apparent overdesign to the degree of uncertainty and other parameters.

It may be remarked that the analysis of such an apparent overdesign in terms of degree of uncertainty represents the formal expression of a concept which is implicit in the traditional factor of safety. The apparent or 'perfect knowledge' optimum design for most simple structures represents the boundary between underdesign, which results in failure, and overdesign, which adds excess material. Here again, the penalty for underdesign exceeds that for equivalent overdesign, and this is taken into account in practice by apparent overdesign inherent in the use of the factor of safety. It is an old truism that the factor of safety might well be called the 'factor of ignorance'. This is demonstrated by the increased factor of safety used in situations of increased uncertainty. For example, a large factor of safety will be applied for a natural material with highly variable properties, but a smaller factor will be applied for a manufactured material of controlled quality.

OPTIMIZATION UNDER UNCERTAINTY

Probability Distribution for Optimum Return Period

As noted above, in the 'real world' case, the various quantities included in the expression for \hat{T} are not known with absolute certainty, but must be considered as probabilistic in the sense that during the design process their values can only be estimated within some margin of variation or uncertainty. Therefore, if the best present estimates of the various quantities are substituted into equation 3, the result will be affected by these uncertainties. It follows that the estimated optimum return period \hat{T} (based on the 'perfect knowledge' formulation) can be expected to differ from the true optimum \hat{T}_0 by some amount which represents the combined effect of the uncertainties of all the components.

Therefore, the value of the true optimum, \hat{T}_0 , can only be expressed by means of a probability statement. This statement would

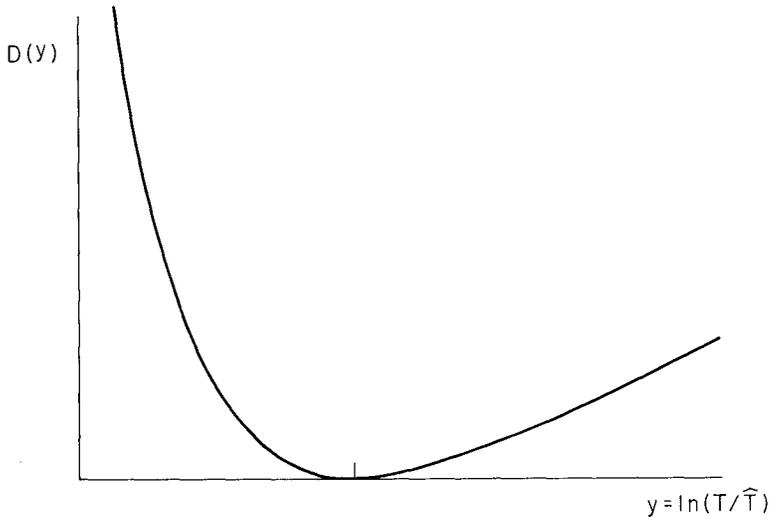


FIGURE 3 RELATIVE LOSS FUNCTION

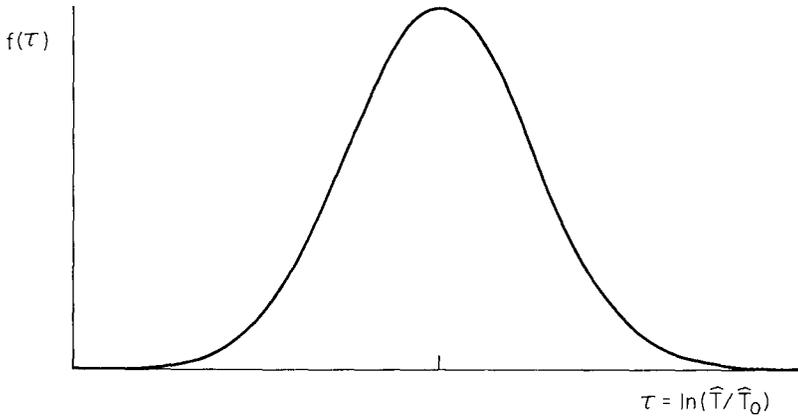


FIGURE 4 PDF FOR OPTIMUM RETURN PERIOD

normally take the form of a cumulative distribution function or its derivative, the probability density function (pdf). The pdf for \hat{T}_0 could be determined by a Monte Carlo type analysis, using Equation 3, if information were available on the pdf for each of the variables.

In the present analysis, \hat{T}_0 was assumed to be lognormally distributed about its estimated value \hat{T} (i.e. $\ln(\hat{T}/\hat{T}_0) = \tau = N(0, \sigma_\tau)$).

This is shown on Figure 4. The standard deviation, σ_τ , of this distribution is then an index of the uncertainty which results from the combined effect of the uncertainties in the various economic and physical factors included in the expression for the optimum return period.

Minimization of Expected Loss

The uncertainties in estimating the optimum return period which have been dealt with above imply that for a given installation, \hat{T} is either greater or less than the true value \hat{T}_0 . Hence a design based on \hat{T} ($T_d = \hat{T}$) will result in total cost greater than the minimum, and this excess cost can be considered as a penalty or loss. Because of the uncertainties inherent in the 'real world' case, this loss cannot be completely eliminated. The expected loss is defined as the summation of the product of loss and probability density over the full range of relative return period.

$$EL = \int_{-\infty}^{\infty} D(y) \cdot \hat{R} \cdot f(\tau) \cdot d\tau \quad (5)$$

As a result of the asymmetry of the total cost function, and hence $D(y)$, the penalty for underdesign is greater than that for an equivalent overdesign, and it follows that the expected loss can be minimized by selecting a design return period greater than the 'perfect knowledge' optimum. As shown on Figure 5, this can be visualized as a shift of the pdf, $f(\tau)$, relative to the loss curve, $D(y)$. The amount of this shift is denoted by a (i.e. $a = y - \tau$ where $a = \ln T_d/\hat{T}$, $y = \ln T_d/\hat{T}_0$ and $\tau = \ln \hat{T}/\hat{T}_0$). The value of a required to minimize the expected loss (see Figure 6) can be written functionally as

$$a = \phi_5(D(y), f(\tau)) \quad (6)$$

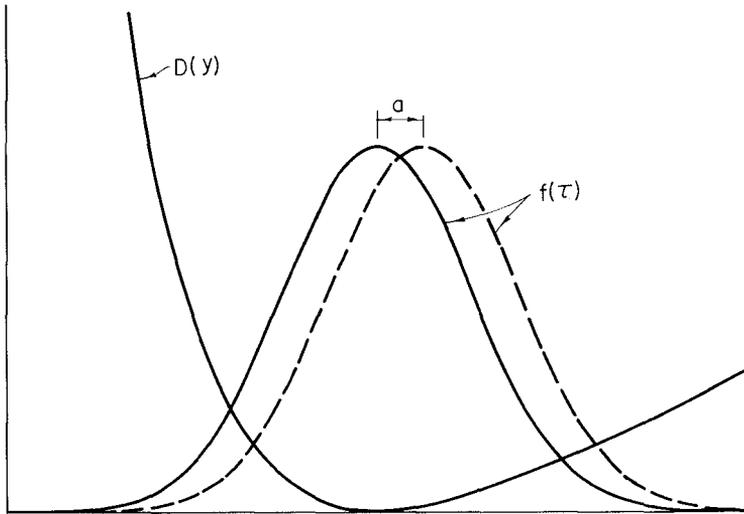


FIGURE 5 SHIFT TO MINIMIZE EXPECTED LOSS

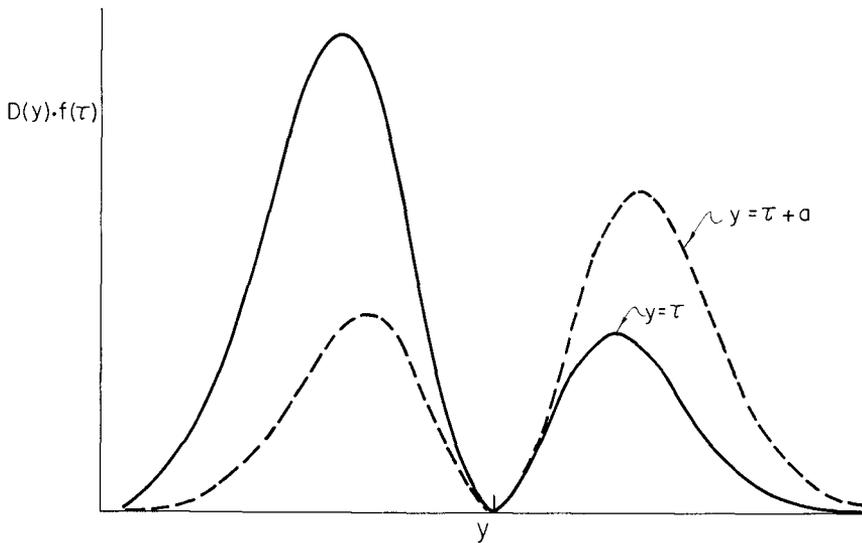


FIGURE 6 EFFECT OF SHIFT ON EXPECTED LOSS

Overdesign Factor

On substituting the particular functions for $D(y)$ given in Table 1 into equation 5 and differentiating for a minimum, one obtains the relations shown below for a .

For the Netherlands Model:

$$a = \sigma_{\tau}^2 / 2 \quad (7a)$$

For the Canadian Model:

$$a = (1 - (r + m) / q) \cdot \sigma_{\tau}^2 / 2 \quad (7b)$$

Here r , m , and q are exponents used in R - X , I - X and p - X relations respectively.

As an alternative formulation, it may be useful to express this shift in terms of the design scope X (i.e. dike height above some datum level) and its uncertainty as given by σ_X or $\sigma_{\ln X}$. In this case, using the scope-return period relation (equation 3) the following expressions are found for the Netherlands and Canadian models respectively.

$$X_d = \hat{X} + \frac{\alpha}{2} \sigma_X^2 \quad (8a)$$

$$\ln X_d = \ln \hat{X} + \frac{q-r-m}{2} \sigma_{\ln X}^2 \quad (8b)$$

Minimum Expected Loss

Substitution of the values for a (equation 7) into equation 5 yields the minimum value for the expected loss. As with D , a dimensionless form (L_{\min}) is obtained by dividing this value by \hat{R} .

The resulting expressions for L_{\min} are as follows.

For the Netherlands Model:

$$L_{\min} = \sigma_{\tau}^2 / 2 \quad (9a)$$

For the Canadian Model (ignoring certain higher-order terms):

$$L_{\min} = \frac{n(n+m)}{2q} \sigma_{\tau}^2 / 2 \quad (9b)$$

SIGNIFICANCE OF FUNCTIONAL MODELLING

The principal outputs of the two functional models of economic optimization have been presented above. Although the 'perfect knowledge' sections of the two models were developed completely independently, they show considerable basic similarity. The Netherlands model specifically assumes a straight line relation for construction cost and because of this linearization and the shape assumed for the regional probability distribution, it is able to use a minimum number of parameters. The Canadian model is not constrained in this fashion, and thus is more versatile. At the same time, however, it requires more parameters.

Under the conditions for which the Netherlands model was developed, it would appear that the results of the two models are virtually equivalent. This is illustrated in the appendix, using the data to which the Netherlands model was originally applied. It is seen that the calculated 'perfect knowledge' dike height is 5.97 m for the Netherlands Model and 5.92 m for the Canadian Model.

The more recent developments of functional modelling in Canada have been directed to the effect of uncertainty, as noted in a previous section. This work comprises the extension of both the original Netherlands model and the Canadian model to allow for uncertain data, and it has been shown that in this 'real world' case the design scope should be larger than the 'perfect knowledge' optimum. The fact that the functional representation of optimum design can provide simple quantitative expressions for this 'apparent overdesign' or 'factor of ignorance' demonstrates the advantages of this technique over the conventional trial design method.

The figures shown in the appendix again indicate that the two models give similar results, with the 'real world' optimum design heights being 6.14 m and 6.09 m for the Netherlands and Canadian formulations, respectively. For the data used in this particular example the difference in dike height may not be large, but nevertheless the increase in construction cost over the 'perfect knowledge' optimum amounts to about 7 000 000 guilders for both models. By including the effect of data uncertainty in the analysis, the 'real world' modelling has shown that this extra construction cost is justified by a reduction in the probable cost of future damage.

The loss or extra cost due to uncertainty, which equals or exceeds the increase in construction cost noted above, can potentially be reduced by improved data-gathering systems and prediction techniques. In this connection the estimates of this loss provided by the 'real world' functional modelling can provide a rational basis for allocating resources to field measurement and analytic studies directed to reducing uncertainty.

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APPENDIX

EXAMPLE CALCULATIONS WITH NETHERLANDS AND CANADIAN MODELS

Data

The data given below have been abstracted from the Report of the Delta Committee cited in the list of references.

Annual exceedance probability of levels h metres above MSL given by

$$p = 2.63 e^{-2.97 (h-1.70)}$$

with estimated uncertainty (expressed in σ_τ units) of

$$\sigma_\tau = 0.23 (h-1.70)$$

Effective annual interest rate (interest minus inflation), 0.015

Costs, expressed in Mf (1 Mf = 10^6 guilders)

Washout Cost, $W = 24\,200$ Mf

Direct Cost, $I = 110$ Mf at $H_d = 5.00$ m
 $l = 150$ Mf at $H_d = 6.00$ m

'Perfect Knowledge' Optimization

- a) Netherlands Model In this case X has an arbitrary origin. It will be taken as $X = H_d - 5.00$. Also, from above $\alpha = 2.97$. The resulting expressions for I and R , in Mf, are

$$l = 110 + 40 X \quad \text{and} \quad R = 242 e^{-2.97 X}$$

$$\therefore X = \frac{1}{\alpha} \ln \frac{R_0 \alpha}{l} = 0.97 \text{ m}$$

$$\hat{H}_d = 5.00 + \hat{X} = 5.97 \text{ m}$$

$$\hat{I} = 148.8 \quad \text{and} \quad \hat{R} = 13.4$$

- b) Canadian Model In this case the origin of X is taken as the same as that of the height distribution, i.e.

$$X = H_d - 1.70$$

Equating direct and damage costs to the values at $H = 5.00$ m and 6.00 m ($X = 3.3$ and 4.3) gives $m = 1.17$. With $r = 0$, $q = n = 11.15$. Thus

$$I = 27.2 (X)^{1.17} \quad \text{and} \quad R = 1.46 \times 10^8 (X)^{-11.15}$$

$$\therefore \hat{X} = \left[\frac{n K_2}{m K_1} \right]^{1/(m+n)} = 4.22 \text{ m}$$

$$\hat{H}_d = 1.70 + \hat{X} = 5.92 \text{ m}$$

$$\hat{I} = 146.6 \quad \text{and} \quad \hat{R} = 15.4$$

Effect of 'Real World' Uncertainty

- a) Netherlands Model Since $H_d \approx 6 \text{ m}$, σ_τ should be evaluated at this point.

$$\text{i.e.} \quad \sigma_\tau = 0.23 (6 - 1.7) \approx 1.0$$

$$\sigma_X = \frac{\sigma_\tau}{\alpha} = .337$$

$$X_d = \hat{X} + \frac{\alpha}{2} \sigma_X^2 = 0.97 + 0.17 = 1.14 \text{ m}$$

$$H_d = 5.00 + X_d = 6.14 \text{ m}$$

$$I_d = 155.5$$

$$\text{Loss due to uncertainty} = L_{\min} \hat{R} = \frac{\sigma_\tau^2}{2} \hat{R} = 6.7$$

- b) Canadian Model Again, $\sigma_\tau = 1.0$, and, thus

$$\sigma_{\ln X} = \frac{\sigma_\tau}{q} = .0897$$

$$\ln X_d = \ln \hat{X} + \frac{q-r-m}{2} \sigma_{\ln X}^2 = 1.440 + .040 = 1.480$$

$$\therefore X_d = 4.39 \text{ m}$$

$$H_d = 1.70 + X_d = 6.09 \text{ m}$$

$$I_d = 153.6$$

$$\text{Loss due to uncertainty} = L_{\min} \hat{R}$$

$$= \frac{n(n+m)}{q^2} \frac{\sigma_\tau^2}{2} \hat{R} = 8.5$$