CHAPTER 107

INTERACTION OF PLANE WAVES WITH

VERTICAL CYLINDERS

by

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ABSTRACT

This study deals with the interaction of linear, plane water waves with stationary groups of rigid, vertical, circular cylinders under conditions in which the inertial forces on the cylinders dominate over the drag forces. A direct matrix solution as well as multiple scattering as suggested by Twersky (1952) are used to obtain the velocity potential in the vicinity of the cylinders. The groups may consist of a number of cylinders having any geometric arrangement, may have Dirichlet, Neumann, or mixed boundary conditions, and need not have identical diameters. The study represents an extension of the single-cylinder case presented by MacCamy and Fuchs in 1954.

Basic scattering coefficients for 192 different arrangements of two cylinders are obtained with the aid of a Bessel coordinate transformation and a matrix inversion procedure. The resulting potential function is then applied to calculate force components in the direction of wave advance and orthogonal to it. For the cases considered the former departs as much as 65% from the force on a single cylinder and the mass coefficient is found to range from 1.19 to 3.38 - a not insignificant departure from the often used value of 2.0. Furthermore the orthogonal force may be as large as 67% of the single-cylinder force.

INTRODUCTION

As off-shore construction continues to expand around the world, the need for an improved understanding of the effects of water waves on various structures in the sea becomes increasingly evident. Basic to many such problems is the fundamental
one involving plane, periodic waves and vertical, circular cylinders, since many such structures include one or more cylindrical legs.

In the present study a general approach to the problem of describing the interaction of linear, plane waves with stationary groups of rigid, vertical circular cylinders is examined. In particular cylinders located in intermediate or deep water and having relatively large diameters (when compared with wave height) are considered in as much as they are representative of a type of off-shore construction that has received much attention recently.

Dean and Harleman (see Ippen (1966)) demonstrate that as the ratio of wave height to cylinder diameter, \( H/D \), diminishes and the ratio of water depth to wave length, \( h/L \), grows, the ratio of inertial force to drag force as described by the Morison Equation (Morison, et al (1950)) increases. For example, if \( H/D = 1.00 \) and \( h/L = 0.40 \), the inertial force will be ten times as large as the drag force. Many off-shore structures, and in particular those considered in the present study, are therefore subject primarily to inertial forces with drag effect considered negligible.

Under these circumstances classical diffraction theory, which presupposes a frictionless fluid and therefore neglects drag seems to be ideally suited to the solution of problems involving the interaction of plane waves with large cylinders in deep or intermediate-depth water. MacCamy and Fuchs (1954) were the first investigators to apply diffraction theory to this interaction problem. Their study of the diffraction of periodic plane waves about a single circular cylinder led to a new approach to the problem of predicting wave forces on structures.

The present study is an attempt to extend the work of MacCamy and Fuchs to a consideration of wave interaction with more than one vertical cylinder. One approach to the solution of such problems is multiple scattering as suggested by Twersky (1952). A direct matrix method appears to offer more rapid and reliable solutions and is therefore emphasized herein.

THEORETICAL ANALYSIS

Problem Statement. The problem under consideration is the interaction between incoming plane water waves and an arbitrary collection of vertical circular cylinders located in the path of the waves. The following conditions are assumed to prevail:

1. The waves are linear (small amplitude theory), and are not breaking.
2. The bottom is horizontal and impermeable with a depth sufficient for deep water or intermediate depth wave conditions.
3. The cylinders are circular, rigid, vertical, stationary, impermeable and have a relatively large diameter with respect to the wave height.

4. Drag effects are negligible (i.e. the water behaves as an ideal fluid).

A general procedure for determining the velocity potential for any number of cylinders is outlined first and then the specific case of two cylinders, Fig. 1, is analyzed more completely. Details of the analytical procedure are omitted herein, but are described by Spring (1973).

Figure 1. Definition Sketch for Cylinder "o" and Cylinder "s"

The approaching plane waves are conveniently expressed in terms of cylindrical coordinates since circular cylinders are under study. Thus in terms of the coordinate center "s" the
incoming wave may be written (see Twersky (1952) and MacCamy and Fuchs (1954))

$$\varphi_{in}(r_s,\theta_s) = -\Re \left[ f(z,t) \exp(ikr_s \cos(\theta_s - \sigma)) \right.$$ 

$$\sum_{n=-\infty}^{\infty} J_n(kr_s) \exp(in(\theta_s - \sigma + \pi/2)) \left. \right]$$ (1)

where the symbols are as defined in Fig. 1, $J_n(kr_s)$ is the Bessel function of the first kind, of order "n" and

$$f(z,t) = \frac{gH}{20} \frac{\cosh[k(h+z)]}{\cosh kh} e^{-i\sigma t}$$ (2)

where $g$ = the acceleration due to gravity
$H$ = the wave height
$h$ = the depth of water from the still water level to the bottom
$k$ = the wave number, $2\pi/L$
$L$ = the wave length
$T$ = the period
$z$ = the vertical coordinate, measured positive upward from the still water level
$\sigma$ = the frequency, $2\pi/T$

The waves scattered by the cylinders have as yet undetermined amplitude but must vanish at large distances from the cylinders due to circular dispersion. Also the waves must be outgoing rather than incoming. Thirdly, the scattered wave expression must be rather general to provide enough flexibility to account for the non-symmetrical scattering of the waves from the cylinder or cylinders. With negative sign on the time exponential, the Hankel function of the first kind (see MacCamy and Fuchs (1954)),

$$H_p^1(kr) = J_p(kr) + iY_p(kr)$$ (3)

will adequately express the radial dependence of the scattered waves. Since there will be no need for any Hankel function of the second kind (which describes incoming circular waves), the superscript on $H_p^1(kr)$ will be dropped and it will be understood that $H_p(kr) = Hankel$ function of the first kind with argument kr.

The scattered wave from the "sth" cylinder will then be
\[ \phi_{sc}(r_s, \theta_s) = \text{Re} \left[ f(z,t) \sum_{n=-\infty}^{\infty} A_n^s H_n (kr_s) e^{in\theta_s} \right] \] (4)

and for the cylinder located at "o"

\[ \phi_{sc}(r_o, \theta_o) = \text{Re} \left[ f(z,t) \sum_{n=-\infty}^{\infty} A_n^o H_n (kr_o) e^{in\theta_o} \right] \] (5)

where the \( A_n \)'s are complex constants (as yet unknown) of the form \( a + ib \), with appropriate superscript.

Since linear waves (small amplitude wave theory) are being considered, the velocity potential at any field point "p" may now be represented by superposing the various wave components to give

\[ \phi_p = \text{Re} \left[ f(z,t) \exp(ikr_{os} \cos(\theta_{os} - \alpha)) \right. \\
\left. \cdot \sum_{n=-\infty}^{\infty} -J_n (kr_s) \exp(in(\theta_s - \alpha + \pi/2)) \right. \\
\left. + \sum_{j=1}^{Q} \sum_{n=-\infty}^{\infty} A_n^j H_n (kr_j) e^{in\theta_j} \right] \] (6)

where the summation term on \( j \) accounts for the potential of the scattered waves from all cylinders present (\( Q \) being the number of such cylinders).

The potential function described by Eq. 6 is difficult to handle since each scattered wave is expressed in terms of a different coordinate center. There is however, a Bessel "addition theorem" or coordinate transformation available to express all wave components in terms of one selected coordinate center (see Watson (1966) pp. 359-61, or Abramowitz and Stegun (1965) p.363).

Referring to Fig. 2, the Bessel coordinate transformation of \( C_v(\tilde{\omega}) \) (which may be any one of the cylinder functions \(-J, Y, H^1, H^2\) or a derivative or linear combination thereof) from coordinate center 1 to a new coordinate center 2 is given by
\[ C_{\nu}(\hat{w}) e^{i\psi} = \sum_{m=-\infty}^{\infty} C_{\nu+m}(z) J_m(z) e^{im\theta} \quad (7) \]

with the restriction that \(|z e^{i\theta}| < |z|\).

It should be noted that the restriction is lifted if \(\nu\) is an integer or zero and the only functions involved are of the first kind.

\[ \frac{\partial \phi}{\partial r_s} = 0 \quad \phi \quad r_s = a \quad (8) \]

for each cylinder and must be applied to evaluate the coefficients \(A_k\) in Eq. (6). In the instance of water waves scattered by impermeable vertical cylinders, the Neumann condition is applied on each cylinder. It should be noted however, that the method does not require all boundary conditions to be of the same type.

The fundamental difference between the direct approach used herein and the procedure suggested by Twersky (1952) is in the method of application of the cylinder boundary conditions. In the "direct" approach the boundary conditions on all cylinders are applied simultaneously and all unknowns are obtained by means of a single matrix inversion of a set of simultaneous equations. The number of equations increases with the number of cylinders.
involved in the problem (except for such special cases as an
infinite row of cylinders of the same size and spacing); and
with the number of terms taken in the summations. The practical
limit on the method is therefore the limit of the matrix size
which can reasonably be handled by the digital computer available.

In contrast, the multiple scattering approach takes one cylin-
der at a time and sequentially solves for the scattering coef-
ficients. The sum of the multiple scattering coefficients for a
particular cylinder approaches the direct solution if enough
orders of scattering are considered.

The Two-Cylinder Problem. The general method described above
will be applied in this section to the case of two cylinders as
shown on Fig. 1. The two-cylinder case is used as an illustra-
tion of the techniques involved. Three, four, six, or more cylin-
ders could likewise be considered. On the other hand it
should be noted that the size, orientation and spacing of the
cylinders and the direction of wave approach are all arbitrary
and can be selected later to make computations for a particular
situation.

The velocity potential at any field point \(p\) in terms of
cylinder "s" may be formed by superposing the velocity poten-
tials for the incoming plane wave, the scattered wave from cy-
linder "s" and the scattered wave from cylinder "o" as suggested
by Eq. (6). The Bessel coordinate transformation from "o" to
"s" and the cylinder boundary on "s" are then applied and the
resulting complex expressions are separated into real and imag-
inary parts, while noting that \(A = a + ib, H = J + iY\) and \(H' =
J' + iY'\). This results in two sets of equations in the four
sets of unknowns \(a^p, b^p, a^o, b^o\), as follows:

\[
\begin{align*}
-J'_m(k_s a_s) \cos\left[(k r_{os} \cos(\theta_{os} - \alpha)) - m(-\alpha + \pi/2)\right] \\
+ (a^o_m - b^o_m)(ka_s) \\
+ \sum_{n=-\infty}^{\infty} J'_m(k_s a_s) \left[(a_n J_{n+m}(k l_{os}) - b_n Y_{n+m}(k l_{os}) \cos(n\theta_{os} + m\theta_s)ight. \\
-b_n J_{n+m}(k l_{os}) + a_n Y_{n+m}(k l_{os}) \sin(n\theta_{os} + m\theta_s) \right] = 0 . \quad (9a)
\end{align*}
\]
where $m = 0, \pm 1, \pm 2, \ldots$.

In similar fashion the Bessel coordinate transformation from "s" to "o" and boundary condition on cylinder "o" are applied, producing two more equation sets in the same four sets of unknowns $a_0^S, b_0^S, a_0^o, b_0^o$, as follows

$$
-J_m^s(k a_s) \sin\left[ (k r_o \cos (\theta_o - \alpha)) - m(-\alpha + \pi/2) \right] + (b_m^S J_m^s(k a_s) + a_m^S Y_m^s(k a_s))
$$

$$
- \sum_{n=-\infty}^{\infty} J_m^o(k a_o) \sin\left[ (k r_o \cos (\theta_o)) - m(-\alpha + \pi/2) \right] + (b_m^o J_m^o(k a_o) - a_m^o Y_m^o(k a_o))
\right) \cos(n\theta_o + m\theta_s) \right] = 0
$$

where $m = 0, \pm 1, \pm 2, \ldots$.

In order to reduce the coefficients and the equations to a finite number, $m$ and $n$ are given a range of $-M$ to $+M$ where the value of $M$ required to maintain a specified precision increases with increasing $k a_s$ and $k a_o$ and decreasing $k l_o$. For the two-cylinder problem this will therefore result in $8M^2 + 4$ equations and unknowns.

Once the coefficients have been computed the final velocity potential is obtained. In functional form, therefore,
\[ \phi = \sum_m (f_m \cos(m\theta + \omega t) + g_m \sin(m\theta + \omega t)) \]  

(11)

where \( f_m \) and \( g_m \) are functions of \( r \), and

\[ f(z) = \frac{gH \cosh[k(h+z)]}{2 \cosh kh} \]

Pressure on the Cylinder Face. The pressure field may be determined with the aid of the generalized Bernoulli Equation

\[ \frac{\partial \phi}{\partial t} + gz + \frac{P}{\rho} + \frac{1}{2} (v\phi)^2 = 0 \]

(12)

Neglecting the kinetic energy and considering only the dynamic pressure, \( P_d \), one obtains

\[ P_d = -\frac{\partial \phi}{\partial t} = \rho \sigma f(z) \sum_m (f_m \sin(m\theta + \omega t) - g_m \cos(m\theta + \omega t)) \]

(13)

For the pressure on a cylinder face, \( f_m \) and \( g_m \) must be evaluated at the appropriate radial distance.

Horizontal Force Components on the Cylinder Face. By integrating the dynamic pressure over the entire cylinder face the horizontal force is obtained. In general functional form the force components may be expressed as follows:

\[ F_x = \frac{\pi H g \sigma}{2} f(z) R_x \cos(\omega t + \delta_x) \]

(14a)

and

\[ F_y = \frac{\pi H g \sigma}{2} f(z) R_y \cos(\omega t + \delta_y) \]

(14b)

where \( R_x \), \( R_y \), \( \delta_x \), and \( \delta_y \) are amplitudes and phase angles which depend on \( k\alpha_x \), \( k\alpha_y \), \( k\beta_x \), \( k\beta_y \), and \( \theta_{OS} \).

ANALYSIS OF THE DATA

Computer Procedure. It is readily apparent that the solution of Eqs. (9) and (10) by matrix inversion requires the services of high speed computers. In this case the UNIVAC 1108 of the Madison Academic Computing Center and the Datacraft 6024 of the Engineering Computing Laboratory, both at the University of Wisconsin-Madison, were employed. The flow diagram describing the program for the solution of the two-cylinder problem is presented in Fig. 3.
Specify cylinder size parameters, $k_{a1}$ and $k_{a2}$, and cylinder separation parameter $k_{los}$.

Calculate Bessel functions of the 1st and 2nd kinds and their derivatives on $k_{a1}$, $k_{a2}$, and $k_{los}$.

Specify horizontal angle $\theta_{os}$.

Calculate matrix elements (coefficients of the unknowns $a_n$ and $b_n$). Matrix subroutine solves for the scattering coefficients $a_n$ and $b_n$.

Calculate multiple scattering coefficients up to 9th order. Compare summation of the first nine orders with the $a_n$ and $b_n$ obtained above.

Use scattering coefficients to verify cylinder boundary conditions on the cylinder faces, $r_1 = a_1$, and $r_2 = a_2$.

Calculate the pressure modulus and phase on both cylinder faces at 20° increments and the ratio to single cylinder pressure.

Calculate horizontal force components (modulus and phase), ratio to single cylinder force, and $C_n$.

Specify new angle $\theta_{os}$.

Specify new cylinder separation $k_{los}$.

Figure 3. Computer Flow Diagram: Two Cylinders
Calculations were aimed at the determination of the pressure distribution on the faces of the cylinders and the horizontal force components per unit depth. Pressures and forces were also calculated for the single isolated cylinder case of MacCamy and Fuchs (1954). Thus the pressures and forces which are presented herein are expressed as ratios to the single cylinder pressures and forces. These ratios clearly reveal the effect that the presence of the auxiliary cylinder has on the cylinder of interest.

All calculations were accomplished in non-dimensional form in terms of the parameters describing wave length and cylinder size, spacing, and orientation: $k_1$, $k_2$, $k_{0s}$, and $\theta_{0s}$ (where $a_1$ and $a_2$ are the radii of cylinders 1 and 2 respectively).

The Two-Cylinder Solution. The primary objectives in studying the two-cylinder case are to illustrate the technique involved, to compare the direct method with the multiple scattering approach, and to lay the groundwork for future extensions. Thus the full range of cylinder sizes and spacings is not considered here.

Two series of calculations were made, both with the incident waves moving in the positive "x" direction ($\alpha = 0^\circ$). In the first series the two cylinders were of equal size, $k_1 = k_2 = 0.40$. The horizontal spacing between cylinders, $k_{0s}$, was varied from 0.80 (cylinders touching) to 9.5 (approximately 1.5 wavelengths apart) in twenty steps, and the angular position of the second cylinder, $\theta_{0s}$, was varied from 0° to 90° in steps of 30°. Thus eighty combinations of angle and distance were considered.

In the second series cylinders of unequal size ($k_1 = 0.40$, $k_2 = 0.60$) were considered. The parameter $k_{0s}$ was varied between 1.00 and 8.0 while the angular position of the second cylinder was varied between 0° (cylinder one leading) and 180° (cylinder two leading) in steps of 30°, providing 112 different locations of cylinder two in relation to cylinder one. For each of these situations the following items were calculated:

1. Basic scattering coefficients, by both the direct method and the multiple scattering approach.
2. Pressure amplitude and phase angles at 20° increments around the cylinder.
3. Ratio of pressure amplitude to the pressure amplitude on an isolated cylinder at 20° increments around the cylinder.
4. Force components in the two horizontal directions.
5. Ratio of force to that of a single cylinder (x-component only).
Horizontal Force Components. Amplitudes and phase angles of the horizontal force components \((R_x, R_y, \delta_x, \delta_y)\) as given in Eqs. (14) were calculated along with the ratio to the force on a single cylinder (x-component only). Typical results for two equal diameter cylinders and two unequal diameter cylinders are in Tables 1 and 2, respectively. Furthermore force ratios for the same two cases are shown graphically as follows:

- Figure 4: \(\theta_{os} = 0^\circ\) and \(180^\circ\)
- Figure 5: \(\theta_{os} = 30^\circ\) and \(150^\circ\)
- Figure 6: \(\theta_{os} = 60^\circ\) and \(120^\circ\)
- Figure 7: \(\theta_{os} = 90^\circ\)

The force ratios for both equal- and unequal-size cylinders appear in each figure for easy comparison. Moreover in Fig. 7 force ratios for an infinite row of cylinders \((ka = 0.400, \theta_{os} = \pm 90^\circ)\) are included.

The orthogonal or \(y\)-component of force was also calculated (see Tables 1 and 2), and in one case rose to 67 percent of the \(x\)-component force \((ka_1 = 0.400, ka_2 = 0.600, kl_{os} = 1.00, \theta_{os} = 30^\circ)\). Although the effect on the maximum resultant was generally less than 10 percent, partly due to the phase differences between the \(x\) and \(y\) components, in one case the maximum resultant was increased by 50 percent. From a design point of view, either static or dynamic, both force components may be significant.

The Mass Coefficient. The force ratio mentioned above and in Tables 1 and 2 can also be interpreted as the ratio of the mass coefficient for one of two cylinders to the mass coefficient of a single isolated cylinder. Therefore independent calculation of the latter using the equations derived by MacCamy and Puchs (1954), permits determination of the former. Such values of \(C_m\) are also included in Tables 1 and 2.

It is of interest to note that \(C_m\) may differ significantly from 2.0. In the limited range of this study \(C_m\) varies from 1.192 to 3.380 as shown in Table 2.

General Observations. The following general observations can be made about the cases studied:

1. The force ratios are periodic in the spacing parameter \(kl_{os}\), with attenuated amplitude, not unlike that of Bessel functions.
2. The effect of a neighboring cylinder on the force of the first cylinder is very significant with as much as a 59 percent increase for two cylinders of equal size at \(\theta_{os} = 90^\circ\) and in contact; and up to 42 percent decrease in the instance of two unequal cylinders at \(\theta_{os} = 0^\circ\) and in contact.
3. Cylinders lined up in the direction of wave advance generally exhibit greater variation in the \(x\)-component of force than do cylinders lined up orthogonal to the wave advance vector.
Table 1. THEORETICAL FORCE: TWO EQUAL-DIAMETER CYLINDERS  \( \text{ka}(1) = \text{ka}(2) = 0.400 \)

<table>
<thead>
<tr>
<th>Cylinder No. 1</th>
<th>Cylinder No. 2</th>
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<td></td>
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<tr>
<td>( k_{10} )</td>
<td>( R_x )</td>
</tr>
<tr>
<td>( \theta_{10} )</td>
<td>( \theta_{10} )</td>
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<td>0.5904 178.9 1.476 0.0000</td>
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Table 2. **THEORETICAL FORCE: TWO CYLINDERS OF UNEQUAL DIAMETER** \( ka(1) = 0.400, ka(2) = 0.600 \)

| Cylinder No. 1 | | Cylinder No. 2 | |
|----------------|----------------|----------------|
| **Separation Angle** | **Force Coeff** | **Phase** | **Force Ratio** | **Force Coeff** | **Phase** | **Force Ratio** |
| \( \theta_{st} \) | \( 90^\circ \) | \( R_x \) | \( S_X \) | \( C_m \) | \( R_y \) | \( S_Y \) | \( R_x \) | \( S_X \) | \( C_m \) | \( R_y \) | \( S_Y \) |
| 1.0000 | 0.4769 | -166.3 | 1.192 | 0.0000 | 1.5805 | 1.0569 | 109.1 | 1.761 | 0.0000 | 0.9184 |
| | 0.5629 | 166.6 | 1.407 | 0.3784 | 56.5 | 0.6852 | 1.1717 | 116.7 | 1.953 | 0.2551 | 79.4 | 0.1012 |
| | 1.0151 | 166.0 | 2.538 | 0.3651 | 35.6 | 1.2356 | 1.4176 | 132.2 | 2.363 | 0.3274 | 82.2 | 1.2318 |
| | 1.3521 | 150.7 | 3.380 | 0.1729 | 73.7 | 1.6439 | 1.5049 | 150.6 | 2.508 | 0.1529 | -110.1 | 1.3077 |
| | 1.2106 | 161.2 | 3.026 | 0.4638 | 139.3 | 1.4736 | 1.2571 | 175.8 | 2.095 | 0.2516 | 155.1 | 1.0923 |
| | 0.8065 | 166.2 | 2.016 | 0.4175 | 159.2 | 0.9818 | 0.9769 | -168.1 | 1.628 | 0.2951 | 149.4 | 0.5489 |
| | 0.6035 | 166.3 | 1.509 | 0.0000 | ----- | 0.7346 | 0.9421 | -127.7 | 1.570 | 0.0000 | ----- | 0.8187 |
| 1.3000 | 0.6560 | -159.9 | 1.640 | 0.0000 | 0.7985 | 1.1443 | 90.4 | 1.908 | 0.0000 | ----- | 0.9956 |
| | 0.5943 | -174.1 | 1.480 | 0.2264 | 70.5 | 0.7235 | 1.1702 | 101.8 | 1.950 | 0.0927 | 83.7 | 1.0163 |
| | 0.7279 | 158.7 | 1.820 | 0.2364 | 35.9 | 0.8860 | 1.2561 | 127.8 | 2.093 | 0.1569 | -100.8 | 1.0915 |
| | 0.9750 | 158.9 | 2.437 | 0.1592 | 56.9 | 1.1685 | 1.2560 | 158.1 | 2.093 | 0.1352 | -129.2 | 1.0974 |
| | 0.9539 | 168.4 | 2.385 | 0.2307 | 136.0 | 1.1612 | 1.0783 | -165.1 | 1.797 | 0.1383 | 170.3 | 0.9370 |
| | 1.0337 | 168.1 | 1.958 | 0.1887 | 176.6 | 0.9533 | 0.9983 | -124.2 | 1.664 | 0.1695 | 155.7 | 0.8765 |
| | 0.7557 | 162.3 | 1.795 | 0.0000 | 0.8739 | 1.0417 | 106.9 | 1.736 | 0.0000 | ----- | 0.9052 |
| 1.7000 | 0.9009 | -164.3 | 2.252 | 0.0000 | 1.0567 | 1.2088 | 65.3 | 2.015 | 0.0000 | ----- | 1.0504 |
| | 0.7717 | -168.5 | 1.929 | 0.1774 | -101.9 | 0.9394 | 1.1979 | 80.3 | 1.996 | 0.0403 | -59.3 | 1.6094 |
| | 0.6695 | 170.3 | 1.674 | 0.2015 | 56.9 | 0.8150 | 1.2216 | 116.8 | 2.036 | 0.0738 | -128.8 | 1.6015 |
| | 0.8581 | 162.8 | 2.145 | 0.2462 | 29.6 | 1.0445 | 1.1751 | 160.9 | 1.958 | 0.1209 | -156.6 | 1.0211 |
| | 0.8959 | 170.2 | 2.240 | 0.1347 | 128.4 | 1.0905 | 1.0372 | -147.2 | 1.729 | 0.1354 | 164.4 | 0.9013 |
| | 0.8109 | 167.4 | 2.027 | 0.1125 | -166.3 | 0.9871 | 1.1088 | 100.7 | 1.868 | 0.1314 | 147.7 | 0.9535 |
| | 0.8016 | 162.3 | 2.006 | 0.0000 | 0.9758 | 1.2083 | 84.6 | 2.014 | 0.0000 | ----- | 1.0499 |
| 2.1000 | 1.0662 | -175.0 | 2.666 | 0.0000 | 1.2979 | 1.2184 | 40.2 | 2.031 | 0.0000 | ----- | 1.0587 |
| | 0.9366 | -173.6 | 2.342 | 0.1525 | -139.0 | 1.1612 | 1.1961 | 58.6 | 1.994 | 0.0538 | -46.0 | 1.0394 |
| | 0.7121 | 178.1 | 1.780 | 0.1808 | 82.3 | 0.8669 | 1.2042 | 104.9 | 2.007 | 0.0270 | -167.4 | 1.0644 |
| | 0.8071 | 165.6 | 2.018 | 0.1318 | 3.5 | 0.9325 | 1.1416 | 162.6 | 1.903 | 0.1069 | 174.7 | 0.9920 |
| | 0.8708 | 170.4 | 2.177 | 0.0861 | 125.1 | 1.0600 | 1.0626 | -131.6 | 1.771 | 0.1229 | 149.2 | 0.9234 |
| | 0.6160 | 166.1 | 2.040 | 0.0950 | -149.5 | 0.9934 | 1.2251 | -82.5 | 2.042 | 0.1120 | 132.4 | 1.0645 |
| | 0.8294 | 160.9 | 2.073 | 0.0000 | ----- | 1.0096 | 1.3313 | 67.1 | 2.219 | 0.0000 | ----- | 1.1569 |
Figure 4. Force Ratio Versus Cylinder Separation, Two Cylinders, $\theta_{os} = 0^\circ$ and $180^\circ$
Figure 5. Force Ratio Versus Cylinder Separation, Two Cylinders, $\theta_{os} = 30^\circ$ and $150^\circ$
Figure 7. Force Ratio Versus Cylinder Separation, Two Cylinders, \( \theta_{os} = 90^\circ \)
Single Isolated Cylinder Pressure and Force. By way of verification, the pressure amplitudes for single, isolated cylinders with $ka = 0.5$ and $ka = 1.0$ were checked against those published by Wiener (1947) for sound diffraction by a single rigid cylinder. Agreement to the 4th decimal place was obtained.

CONCLUSIONS

1. A means of calculating the pressures and forces on a cluster of vertical circular cylinders is developed. The method employs diffraction theory, but avoids multiple scattering techniques, in favor of a direct, matrix solution.

2. Theoretical calculations for the force in the direction of wave advance reveal as much as a 60% departure from the force on a single isolated cylinder in the instance of two equal-diameter cylinders and as much as a 65% departure for two cylinders of unequal diameter. The force on a given cylinder is thus significantly affected by the presence of neighboring cylinders. The mass coefficient, $C_M$, is found to range from 1.19 to 3.38, significant departures from the often assumed value of 2.0.

3. The component of force perpendicular to the direction of wave advance is found to be very significant when the cylinders are close together, rising in one case to 67% of the force component in the direction of wave advance. Although the effect on the maximum resultant is generally less than 10%, in one case a 50% increase is found. Both force components may be significant for design of cylinders used for offshore tower supports.

4. The method is not restricted to water waves, but can also be applied to other cases of scalar scattering in acoustics or electromagnetics.

ACKNOWLEDGEMENTS

This project was funded in part by the National Oceanic and Atmospheric Administration's Office of Sea Grant, Department of Commerce, through an institutional grant to the University of Wisconsin. Additional support was received from the Ford Foundation, the National Science Foundation, and the Wisconsin Alumni Research Foundation.

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