1. Introduction

The topographic survey by sounding the water bottom is becoming an ever increasing part of the work of engineers and scientists on the German North Sea coast.

In order to fulfil the various tasks in the estuaries of tidal rivers in the foreshore areas, charts are needed with depth data of the greatest possible accuracy. The problem now is this: There exists a big accuracy gap between, on the one hand, the values obtained by the levelling method or by means of the water line method, often applied in the tidelands, yielding accuracies from 1/4 to 1 dm and, on the other hand, the soundings carried out from vessels in the adjacent sea area.

According to the investigations into the accuracy of nearshore hydrographic surveying, a tenfold of inaccuracy - compared with the above given values - must be assumed. Scientific studies of the natural processes in the foreshore area (the major programme of the German Research Society (DFG) "Sandbewegung im Deutschen Küstenraum" / The shift of sand in the German coastal area / may be cited here as an example) do not yield a reliable picture so long as the accuracy standard of tideland surveys is not approximately reached.

Moreover, the Offices of the Federal Republic of Germany and of her coastal provinces that have been entrusted with sea surveying, each use their own methods which have often evolved differently in the course of history. So, additional difficulties by frequent overlappings of surveyed areas are inevitable.
Comparative studies, e.g. calculating balances of sandy material from maps showing variations in depth will inevitably entail criticism so long as no standard methods are used.

The problem of hydrographic surveying is to precisely determine the water level at the moment of sounding. Due to the tides with constantly changing water levels, the reduction of each measured water level to a standard reference level is of special importance.

2. General Aspects

2.1. Reduction methods in the Outer Elbe Estuary

For the calculation of water levels which are needed for the charting of depths, different methods are used that have been dealt with several times in the literature. GÖHREN, 1968, has given a summary of the methods that are especially used in the Outer Elbe. He has studied and inter-compared the following four reduction methods:

a) Reduction according to the propagation velocity of the tidal wave;

b) Reduction according to the propagation velocity as under a) and height reduction according to the high water gradient between two stations;

c) Reduction according to mean tidal curves;

d) Reduction according to the propagation velocity of the tidal wave as under a) and height reduction according to changes in tidal range.

In the area of the Outer Elbe, the method b) is used by FORSCHUNGSGRUPPE NEUWERK, the method c) by WASSER- UND SCHIFFFAHRTSAMT CUXHAVEN and the method d) by DEUTSCHES HYDROGRAPHISCHES INSTITUT.
The result of GÖHREN's investigation shows that the method b) yields the best values just about the time of high water, and that with the method c) the best correlation is obtained between the reduced and observed water levels during the remaining times of the tides. The method d) is mainly used with good results in sea areas for the compilation of charts that are to serve for the calculation of water levels; however, in the estuaries it is inferior to the methods b) and c).

On account of the special tasks of the WASSER- SCHIFFFAHRTSAMT CUXHAVEN, the sounding of shipping routes cannot be limited to the times of high water, but has to be carried out during the whole working day with different tide phases.

Furthermore, the WASSER- UND SCHIFFFAHRTSAMT CUXHAVEN, in contrast to the FORSCHUNGSGRUPPE NEUWERK which uses several subsidiary gauges for the reduction, uses only the tide gauge of 1st order at Cuxhaven-Steubenhöft.

For about 50 years, the method c) - though improved in the course of time - has been used quite successfully in the Elbe Estuary. Owing to the network of subsidiary gauges in the Elbe Estuary that has been densified in the meantime, there was now the opportunity for a more critical examination of the method.

Comprehensive preliminary examinations of many details have shown that reduction errors in the order of 3-4 dm occurred with the method c) applied so far by WASSER- UND SCHIFFFAHRTSAMT CUXHAVEN; in exceptional cases they amounted to between 5 and 6 dm and on average they lay between 1 and 2 dm. These errors were found by reducing tides (that had actually occurred and had been observed at the reference gauge) to the subsidiary gauges established in the Outer Elbe and by comparing them with the water levels registered there.
2.2 Description of the method c) that has been applied so far.

The reduction method which is still being used by the WASSER- UND SCHIFFFAHRTSAMT CUXHAVEN refers the sounded water depths to the mean tidal conditions in the area of observation. The Outer Elbe area has been subdivided into individual zones for which, according to the tidal phase, water level correction values against the reference gauge at Cuxhaven-Steubenhöft can be taken from Tables. Thus the water level at the place of sounding is obtained from the simultaneously observed water level at the reference gauge, corrected by the above mentioned value.

For soundings in the Outer Elbe, Tables have been established showing the necessary reduction values for certain zones of the area to be sounded. These Tables are based on recordings of subsidiary gauges or other gauges of 3rd order, from which mean tidal curves have been constructed.

If during sounding operations mean conditions occur, a good correlation is obtained between reduced and measured water levels. With increasing deviations from the mean conditions the reduction error will also increase.

The reduction results given under paragr. 2.1 do no longer meet the requirements concerning the accuracy of depth charts as given in the Introduction. This induced us to develop a method the maximum errors of which were not to exceed 2 dm and with mean errors not greater than 0.5 - 1 dm. The following results have been obtained when studying the new method which differs fundamentally from all methods known so far.
3. Reduction method according to the variation in travel time of the tidal wave and height reduction according to the M.H.W./M.L.W. gradient.

3.1 Theoretical principles

M.H.W./M.L.W. gradient

If the tidal curve of an outer gauge is compared with the pertaining tidal curve of a reference gauge, differing vertex values of H.W./L.W. are obtained as a rule. From the distance of the outer gauge to the reference gauge and from the above mentioned vertex values the H.W./L.W. gradient is determined.

The difference of the pertaining vertex values will in the following be denoted as $\Delta h$-value.

When inter-comparing many tides, a good correlation of $\Delta h$-values is in general found, both in themselves and between each other.

From the statistical mean value the M.H.W. resp. the M.L.W. gradient can be determined for each outer gauge according to the conditions that will be laid down below, under paragr. 3.2

Variation in travel time

Travel time means the temporal motion of the tidal vertices between the outer gauge and the reference gauge. The travel times of the H.W. vertices of several tides are approximately equal, and so are the travel times of the L.W. vertices.

When intercomparing the travel times of both vertices, greater differences dependent on distances, are obtained. The variation in travel times between the vertices that is not directly measurable is an essential part of this paper.
In order to make the variation in travel times measurable, the $\Delta h$-value had to be assumed as constant. This was possible because for a number of studied tides the $\Delta h$-values showed only minor fluctuations (the deviations of the $\Delta h$-values from the statistical mean values amounted to a maximum of only 5 cm).

When using the averaged $\Delta h$-value the travel time can be determined in the required intervals (according to fig. 1 as an example for every hour, for the establishment of Reduction Tables in intervals of 10 minutes).

Apart from the natural astronomical influences, stronger meteorological influences may essentially affect both the travel time and the $\Delta h$-values. Therefore, as far as possible, only undisturbed tides, the so-called "fair weather tides" have been used for the investigations. As soundings of the Outer Elbe that are carried out from vessels are only possible up to wind forces of 5 Bft., only tides up to this limit were evaluated.

It must be noted that, for the determination of the mean $\Delta h$-values, the statistical mean tidal curves of the respective outer gauge could not be used; mean tidal curves in temporal relation to the reference gauge had to be calculated (cf. fig. 2). Moreover, as an additional limitation of the sources of errors, the tide was divided (as with the method used so far) into L.W. and H.W. areas (fig. 2).

Through the introduction of the variation in travel time, a reduction according to equal times its inherent errors became unnecessary.

From fig. 8 it can be seen that the characteristic form of the tidal curve obtained through reduction is maintained in spite of changes due to morphological and meteorological conditions. This has, to the authors' knowledge, so far not been the case with the known methods throughout the entire tide.
For the better understanding of the different effects of both reduction methods, two tides that have actually occurred have been reduced according to both methods (fig. 7). The resulting differences in water levels have been compared with the values read from the outer gauge. It can be seen that with the reduction according to the old method, only a constant line of water level differences, in dependence of the respective tide of the reference gauge can be obtained. Here, the new method has advantages, as the water level differences remain variable and adapt to a great extent to the measured water level differences.

According to the afore described general knowledge, the following principles can be set up for the establishment of a reduction method by means of Tables:

a) From statistical data the rise of the M.H.W. and M.L.W. lines referred to the reference gauge can be determined and calculated as a constant mean value, dependent on the reduction place. This is the \( \Delta h \)-value.

If the \( \Delta h \)-value is not equal to the difference value that has actually occurred at the time of sounding, this value is entered directly into the reduction as an error. According to an examination it will amount to just a few cm (cf. fig. 6).

b) By introducing the constant difference value \( \Delta h \), the variation in travel time can be determined throughout a tide in any phase.

If the variation in travel time that has thus been determined does not agree with the values that have actually occurred during the time of sounding, cm-differences will be obtained resulting from the minute-deviation (fig. 6). Depending on the rise of the tidal curve von the day of measurements, these differences are additionally entered as errors in the reduction. They may, however, reduce the error in the final result or eliminate it, as well.
From the above given theoretical considerations the reduction according to fig. 3 can be formulated as follows:

The water level at the place of soundings at the time $t'$ is equal to the water level at the reference gauge in the time interval $T'$ (theoretical time interval between H.W. resp. L.W. vertices of the reference gauge and the sounding time) $+ \Delta T$ (travel time in the time interval $T$) $+ \Delta h$.

The reduction method developed from the described criteria can be named:

"Beschickung nach der Laufzeitveränderung der Tidewelle und Höhenreduzierung nach dem MThw/MThw-Gefälle"

(Reduction according to the variation in travel time of the tidal wave and height reduction according to the M.H.W./M.L.W. gradient).

### 3.2 Compilation of Reduction Tables

From these reduction formulae, tables can be compiled for practical use that will enable reductions in the area of investigation which - in comparison to the old method - require only little additional time.

For this purpose the work listed below was necessary;

a) From the outer gauges that were needed for the investigations, as well as from the reference gauge at Cuxhaven-Steubenhöft, 20 tidal curves correctly registered with regard to height and time were selected. Tides influenced by wind forces of more than 5 Bft. were not used for the further investigation.

b) For each of the 10 gauges that are available in the area of investigations the T.H.W. and T.L.W. gradients to the reference gauge were determined in cm as $\Delta h$-value. There it was found that the T.H.W. gradient nearly equals the T.N.W. gradient. Only in the "Neuwerker Fahrwasser"
that is separated from the main channel by the jetty "Kugelbake" did greater deviations from both these gradients occur. While in the main channel maximum differences of up to 5 cm were observed, differences of 15 cm could be stated in the "Neuwerker Fahrwasser". In order to avoid considerable reduction errors, a linear interpolation had to be carried out in the Tables for the "Neuwerker Fahrwasser".

c) For each gauge (outer gauge and reference gauge) the mean tidal curves (fig. 4) have been calculated, referred to the arrival time of T.L.W. resp. T.H.W. at the reference gauge. From the mean tidal curve thus obtained of each gauge the time intervals (in hours and minutes) referred to the mean arrival times of the T.L.W. resp. T.H.W. of the reference gauge have been calculated and compiled in Tables, under consideration of the previously determined height difference values \( h \) in 10 minutes intervals (referred to the reference gauge).

d) As in the area of studies only 10 subsidiary gauges were available, a division into a greater number of reduction squares had to be made in order to diminish the reduction errors. The side lengths of the squares should not be greater than 2-3 km.

As limitation of the individual squares the main hyperbolic lines according to the Hi-Fix method had been selected. This was necessary with regard to the treatment of sounding data and the manual and electronic data processing, especially in order to simplify the conversion into the GAUSS-KRÜGER coordinate system.

If the squares thus determined are not directly fitted with a subsidiary gauge - and this is the case with most of the squares - the Table values have to be determined by interpolation.

In contrast to the method used so far, with a division transverse to the river axis, the new division into squares takes into account the noticeable differential transverse gradient.
3.3 Instruction for the use of the Tables

By means of the Tables, the reduction can be made in the following steps:

a) To determine the gauge data of the reference gauge.
   For this purpose the arrival time of the vertex at the reference gauge prior to the sounding time has to be noted.

b) To determine the sounding place for the selection of the Reduction Table.

c) To determine, from the selected Table and the pertaining square number, the respective height difference $\Delta h$.

d) To calculate the time interval $T'$ by means of the sounding time $t'$.

e) To determine the theoretical time interval $T$ from the Table, by means of the time interval $T'$ and the pertaining square number.

f) To calculate the theoretical sounding time $t$ for the reference gauge and to read the water level above zero from the tidal graph.

g) To reduce the height of the water level at the reference gauge to the $\Delta h$-value.

Thus, the height of the water level above gauge zero at the sounding place is determined.

3.4 Reduction errors

The reduction errors keep within the limits required under paragraph 2.2.

4. Further improvements of the reduction

When thinking of a further limitation of the reduction errors, a much denser gauge network will have to be required. This requirement seems justified, as from gauges established in short
distances the water level at the sounding time can be obtained directly without conversion or through simple interpolation with decimetre accuracy.

This, however, requires an absolutely reliable operation of all gauges with regard to height and time; at least control readings at the respective gauges would have to be made before and after the sounding. Apart from the fact that thus uneconomical loss of time would have to be put up with, it cannot be hoped to realize this method, neither today nor in the foreseeable future. When taking into account maximum distances of 4-5 km between the gauges in the Outer Elbe area, about 25 gauges would have to be available.

As with the new reduction method errors exceeding 1.5-2 dm are not to be expected (on the average less than 1 dm) and as, on the other hand, with a view to future coastal research and coastal engineering, an accuracy of ± 1 dm will have to be required, a reliable permanent gauge of 1st order at the lower end of the sounding area, i.e. approximately off the position of the lightvessel ELBE I at the "Grossvogelsand-West" would have to be required.

This new gauge as a radio linked tide gauge which could permanently be interrogated would be of valuable help for further diminishing the error that remains even with the application of the new reduction method.

Improvements of the present method seem to be possible as here the travel time of the tide vertices from the "outer gauge" (required above) to the reference gauge, as well as the height difference value \( h \) become known. From the data of both gauges, the distance of which could approximately be 45 km, more important deviations from the \( h \)- and travel time values incorporated in Tables could be found and eliminated. Besides, it would be imaginable to use the outer gauge as reference gauge. Thus, waiting periods would not arise in the reduction, and a reduction on board with the appropriate additional electronic equipment would be possible. The effect of the location of the reference gauge is represented in fig. 5.
5. Final remarks

The ever increasing requirements concerning the accuracy of depth charts of the ELBE Estuary are the motive for these investigations. The tidal area of the Outer Elbe with its transition into the open sea and the adjacent tidal flats with their various hydraulic and morphologic problems provide the opportunity to deal thoroughly with the problem of reduction. This study aims at diminishing the gap between the tideland survey by levelling which can be done with high precision and the hydrographic survey so far often carried out unsatisfactorily. By further refinement of the technical position fixing and sounding devices better results are possible and desirable.

In the authors' opinion, the new method could also be applied in other tidal coastal areas with due consideration of the respective local conditions. This will no doubt require critical examination with regard to such tidal areas. The first examinations according to the new method have been carried out by RICHTER in the area of the WASSER- SCHIFFFAHRTSAMT TÜNNING and has found as a result high reduction accuracies.
Map of the Outer Elbe

Reduction according to the variation in Travel Time and Height Reduction according to the M.H.W./M.L.W. gradient

fig. 1

fig. 2
Determination of the water level at the sounding place

\[ H_T = H_r \pm \Delta h \]
\[ H_T' = H_T \pm \Delta h \]

- \( H \) = water level at the reference gauge
- \( H_r \) = water level at the sounding place
- \( \Delta h \) = height difference according to the mean HW/LW gradient
- \( T \) = actual temporal distance between TLW/THW reference gauge and hour of sounding at the sounding place
- \( T_L \) = theoretical temporal distance (const.) between TLW/THW reference gauge and hour of sounding at the sounding place

**fig. 3**

Determination of the „mean tidal curve“

**fig. 4**
Effects of the location of the reference gauge

Maximum reduction errors

fig. 5

fig. 6
Effects of the reduction

fig. 7

Deviations of reduced and observed water levels