CHAPTER 57

FORMATION OF DUNES BY TIDAL FLOWS

by

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ABSTRACT

Schematical relations for the size of dunes and for the duration of their development are derived assuming that the large scale formations on the surface of a movable bed are due to the largest eddies of turbulence. The considerations are confined to the simplest case of a two-dimensional flow and to the cohesionless granular material. The relations for tidal dunes are obtained by generalising the relations for unidirectional flow dunes. Special cases and the validity regions of the forms presented are discussed; suggestions for future measurements and model tests are included.

INTRODUCTION

The study of large scale sand waves, or shortly dunes, is becoming increasingly important in the marine engineering field. For example, there is considerable activity taking place with oil exploration in the North Sea and there is a need to bring the oil ashore by pipelines. Sometimes these will cross dune fields and there is the risk that previously buried lines will become exposed as sand waves move on. Considerable loads could then be set up on the pipes due to wave and current action with the high risk of breakages. An estimate of the UK expenditure on pipelines during the next 5 years is £ 1000M. Clearly research on the formation of sand waves is therefore justified on these grounds alone.

The second example concerns the circulation of material in relatively small oceans like the North Sea (Fig. 1). Until quite recently civil engineering works were built in estuaries without too much regard being paid to their effect on ocean circulations but we are approaching the time in the North Sea, because estuarial works are being constructed on a massive scale, when this is no longer the case. The Dutch have already completed their Delta plan, and in the long term the permanent solution to the tidal flooding problems in London might come with the construction of a causeway at the mouth of the Thames estuary. Collectively these schemes are bound to affect circulations of water and sediment in the sea. It will be out of the question to build physical models of such large areas so undoubtedly

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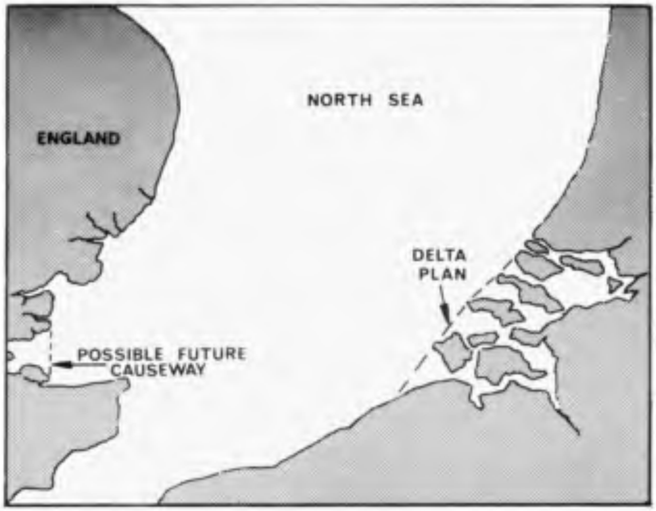

Fig. 1. Southern North Sea

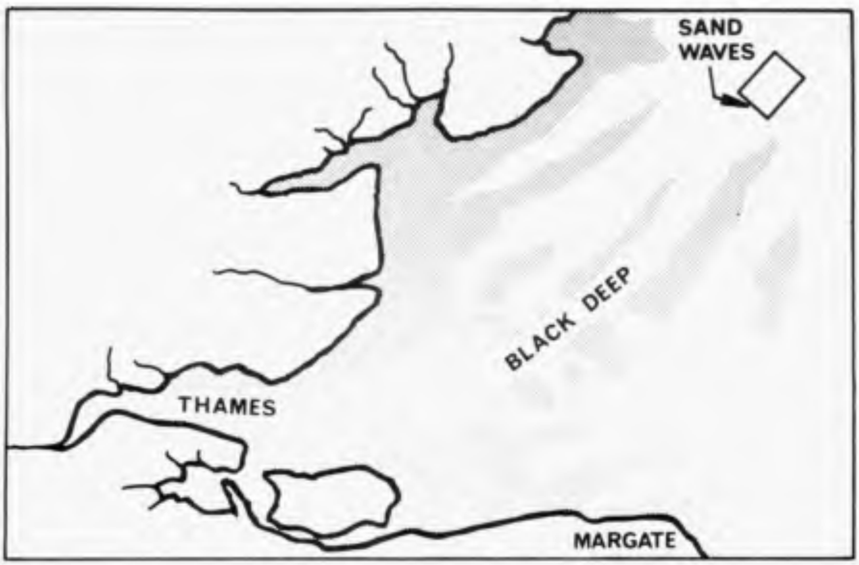

Fig. 2. Outer Thames Estuary

mathematical models will be used. The laws of sediment transport in general and the factors that affect the formation of dunes in particular will require to be better understood if reliable predictions on the behaviour of future projects are to be made.

The third example, and the one that provided the impetus for the work described in this note, concerns the problems associated with dredging and maintaining large navigation channels. Figure 2 shows the Outer Thames Estuary with its complexity of banks and channels. One of these channels, the Black Deep, is the main navigation channel to the Port of London and it is interesting that at its downstream end it crosses a large sand wave field. There are proposals to deepen this channel so that engineers are interested to know the size of the features that will eventually form in the deepened area and the rate at which they will reform.

An idea of the shape and size of dunes forming in the North Sea can be conveyed with the aid of the data in Ref. [1], where it is reported that on the floor of the North Sea, south of a line from Norfolk to Den Helder in Holland, echo sounding has revealed the presence of ridges about 5 m. high and about 30 m. apart, normally orientated at right angles to the tidal streams. The example of sand waves forming in the Outer Thames Estuary can be found, e.g. in Ref. [2]. This work contains a number of informative diagrams provided by the Institute of Oceanographic Sciences (Taunton) and obtained with the aid of the side scan sonar. IOS (Taunton) have shown that although the dunes forming in the Black Deep field are not all the same, there are fairly distinct areas exhibiting sand waves of the same general characteristics. From the diagrams of Ref. [2] one can also observe that on the backs of the larger features there are sometimes smaller ones and their orientation is often different from that of larger main features.

The present paper concerns the formation of dunes by tidal flows. Attention is focussed on the following two aspects which in the authors' view are the most pertinent to the maintenance of deep navigation channels:

1. The size of dunes
2. The duration of their development

It is hoped that the relations derived may prove to be useful in the planning of future measurements aiming to reveal the correlation between the dunes forming on a sea bed and the currents generating them.

In agreement with the prevailing contemporary view, it is assumed that the large scale formations on the surface of a movable bed are due to turbulence or to be more exact to the largest eddies of turbulence.

If the flow is not uniform and the intensity in the variation of flow boundaries is appreciable, then in addition to the eddies of turbulence the flow may contain the eddies of separation. The authors think that the latter eddies also contribute to the formation of the bed surface. However, since the contribution of these eddies depends on their number, size and distribution (in the body of the flow), and since these factors are determined by the topography of the sea bed (which varies from one location of the sea bed to another in an arbitrary manner and which can thus not be described in general terms) the present study is confined to the consideration of turbulent eddies only.

EDDY STRUCTURE OF TURBULENCE

Turbulence is pictured nowadays as a multitude of eddies (or disturbances) superimposed on the time average fluid motion. These eddies have random instantaneous sizes ℓ_1 and they travel in all possible directions random distances ℓ_2 forming thereby what is called the "turbulent mixing process". It has been established that the lengths ℓ_1 and ℓ_2 are of the same order. Considering this, and also the fact that any clear distinction between ℓ_1 and ℓ_2 is impossible, it has become a custom to consider both lengths (ℓ_1 and ℓ_2) by a single length-order ℓ called the scale of turbulence. The order ℓ corresponding to a location of the space occupied by a turbulent flow is the average value of all ℓ_1 and ℓ_2 encountered at that location.

The eddies are not permanent: some eddies present at an instant t_1, are no longer present at an instant t_2. On the other hand, during the time interval $t_2 - t_1$ a set of "new" eddies is generated instead. First the largest of all eddies occur. The size L of these largest (initial) eddies is comparable with the size h of the flow (flow depth in the case of an open channel)

$$L \approx h \ . \tag{1}$$

With the passage of time the initial eddies lose their stability and disintegrate into a number of smaller eddies. These smaller eddies in turn give birth to even smaller eddies and this "cascade of disintegration" continues to take place until such a small value A, say, of the eddy Reynolds number $\ell_1 \cdot U_1/\nu$ is achieved (where U_1 is the velocity of eddy motion and ν kinematic viscosity) that the eddy becomes stable, i.e. no longer divisible. Clearly the average length ℓ_{min} of the smallest disturbances must thus be determined by the form

$$\ell_{min} = A \frac{\nu}{U} \ . \tag{2}$$

It follows that the linear scale ℓ of turbulence is the average size of the disturbances varying between the upper and lower limits L and ℓ_{min} which are given by (1) and (2) and which are referred to as the linear scales of macroturbulence and microturbulence respectively:

$$\ell_{min} < \ell < L \ . \tag{3}$$

Usually the scale ℓ is "measured" in terms of the size h of the flow, and therefore its distribution along the depth of a stationary and uniform two dimensional flow is expressed in the following manner

$$\ell = h \cdot f\left(\frac{y}{h}\right)\ ^{*)} . \tag{4}$$

DUNE LENGTH

Two-dimensional unidirectional flow in an open channel

As is explained in Ref. [6] the length Λ of dunes must be proportional to the size L of the largest macroturbulent eddies generating them:

$$\Lambda = \alpha \cdot L . \tag{5}$$

Since in the case of a unidirectional flow in an open channel the size of the largest eddies approximates the flow depth h (eqn (1)) the relation (5) yields

$$\Lambda \approx \alpha \cdot h . \tag{6}$$

The dimensionless proportionality factor α in general is not a constant. The field and laboratory data show (Ref. [6]) that α can only be treated as a constant if the grain size Reynolds number $X = v_* D/\nu$ is sufficiently large (larger than ≈ 45, say). In this case $\alpha \approx 2\pi$ = const and the relation (6) becomes

$$\Lambda \approx 2\pi \cdot h . \tag{7}$$

If, on the other hand, X is smaller than ≈ 45 (i.e. if the turbulent flow is not fully developed but is in a transitional or hydraulically smooth regime) then α appears to vary as a function of the Reynolds number X and the ratio h/D .

The conditions described follow from the family of curves shown in Fig. 3. This family of curves, which represents the functional relation

$$\frac{\Lambda}{D} = \psi_\Lambda\left(X, \frac{h}{D}\right) \quad \text{and consequently} \quad \alpha = \frac{\Lambda}{h} = \phi_\Lambda\left(X, \frac{h}{D}\right)$$

was determined from a very large number of field and laboratory data supplied by more than a dozen different authors. (See each of the individual curves together with the data determining it in Chapter 7 of Ref. [6].) Note from Fig. 3 that the "curve" corresponding to $X \geq 42.66$ (mentioned earlier as ≈ 45) is a straight line C implying (7). Note also that the relation (7) can already be adopted, with an accuracy sufficient for all

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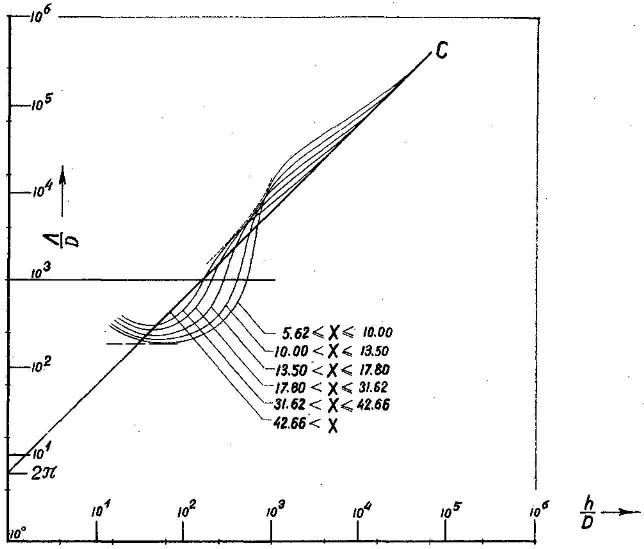
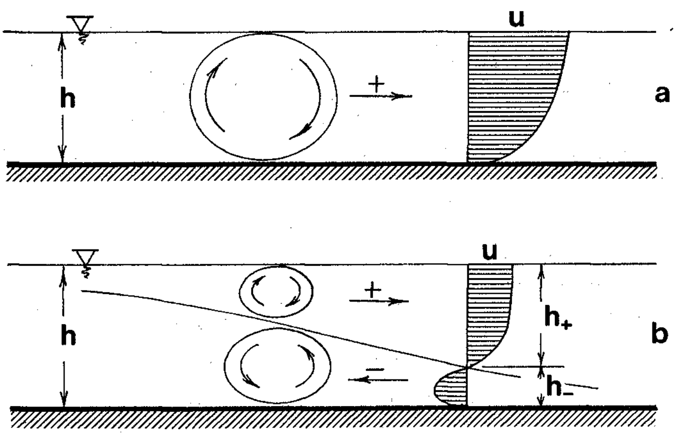
*) A general description of the contemporary approach to turbulence can be found, e.g. in Refs. [3], [4], [5].



10⁶
10⁵
10⁴
10³
10²
10¹
2π
10⁰
Λ/D
h/D
C
5.62 ≤ X ≤ 10.00
10.00 < X ≤ 13.50
13.50 < X ≤ 17.80
17.80 < X ≤ 31.62
31.62 < X ≤ 42.66
42.66 < X

Fig. 3



u
h
+
a
u
h
h+
h−
+
−
b

Fig. 4

practical purposes, when X is larger than ≈ 20, say.

Tidal Flow

In the case of a tidal flow it is often observed that the length Λ of dunes is about six times the flow depth (i.e. as implied by the relation (7)). However, it would be wrong to assume that the dune length formula for unidirectional flow should be regarded as applicable to all cases of tidal flow. The authors prefer to believe that the basic mechanism of the formation of dunes is the same, i.e. that they are also formed by the largest macroturbulent eddies as in the unidirectional case and consequently their length should still be given by a relation such as eqn (5). On the other hand, one has no reason to think that the values of the dimensionless proportionality factor α should necessarily be the same as those for unidirectional flow, nor has one any reason to believe that the time average size $\bar{L}$ of the largest eddies (averaged over one periodic cycle) should necessarily approximate to the time average flow depth $\bar{h}$. In fact one would expect that $\bar{L}$ should be smaller than h. Indeed in the case of a tidal flow, the largest eddies of the size comparable with h (Fig. 4a) can only form if the whole thickness (h) of the fluid is moving in only one direction (i.e. when the tidal flow is virtually a unidirectional flow). When the flow is reversing, then one flow takes place on top of the other moving in the opposite direction (Fig. 4b) and the size of the largest eddies forming at such times must be smaller than h. But if in some parts of the periodic cycle the size L approximates h, while in others it is smaller than h, then the time average value $\bar{L}$ should be smaller than the time average flow depth $\bar{h}$.

The size of the largest eddies can always be "measured" in terms of the flow thickness:

$$\bar{L} = \beta \cdot \bar{h} \quad , \tag{8}$$

In the case of a unidirectional flow $\beta \approx 1$ (and the bars signifying the time average over the period T become redundant); in the case of a tidal flow $\beta < 1$. In the latter case the value of β varies depending on the nature of tidal flow. The conditions reflected by Fig. 4b suggest that the value of β should increase with the decrement of the ratio $T_{\pm}/T$ where $T_{\pm}$ stands for that part of the period T in which the fluid is moving in both directions (where both h_+ and h_- (in Fig. 4b) are different from zero). According to Ref. [7] for a given geometry of the flow boundaries and a specified fluid the ratio $T_{\pm}/T$ must decrease with the increasing values of the dimensionless parameters

$$[\,(\text{tidal wave length})/\bar{h}\,] \quad \text{and} \quad [\,(\text{effective bed roughness})/\bar{h}\,]. \tag{9}$$

DURATION OF DEVELOPMENT OF DUNES

General (Unidirectional flows)

In the case of a bed covered by (two-dimensional) sand waves (Fig. 5) the elevation z of the bed surface and the transport rate p (per unit width) vary as functions of position x and time t :

$$z = f_z(x, t)$$
$$p = f_p(x, t) \quad . \tag{10}$$

As is well known these functions are interrelated by the Exner-Polya eqn

$$\frac{\partial z}{\partial t} + \frac{1}{\gamma_s}\frac{\partial p}{\partial x} = 0 \tag{11}$$

i.e.

$$-\frac{(\partial z/\partial t)}{(\partial p_v/\partial x)} = 1 \tag{12}$$

($p_v = p/\gamma_s$ volumetric transport rate).

It would be only reasonable to express x, z and t as some fractions of the wave length Λ , of the wave height Δ and of the development duration T respectively. Similarly it would be only natural if the local transport rate p is expressed in terms of the space average transport rate p_m which for the two dimensional case under consideration implies

$$p_m = \frac{1}{\Lambda}\int_x^{x+\Lambda} p \, dx \quad . \tag{13}$$

Introducing the dimensionless variables

$$\xi = \frac{x}{\Lambda} \quad , \quad \Theta = \frac{t}{T}$$

and functions (14)

$$\zeta = \frac{z}{\Delta} \quad , \quad \Pi = \frac{p}{p_m} = \frac{p_v}{(p_v)_m}$$

the relation (12) can be expressed as follows

$$-\left[\frac{\Delta\Lambda}{T\,p_m}\right]\cdot\frac{(\partial\zeta/\partial\Theta)}{(\partial\Pi/\partial\xi)} = 1 \quad . \tag{15}$$

If the ratio of $-(\partial z/\partial t)$ to $(\partial p_v/\partial z)$ is a constant, namely unity (eqn (12)) then the ratio of the dimensionless versions of these quantities

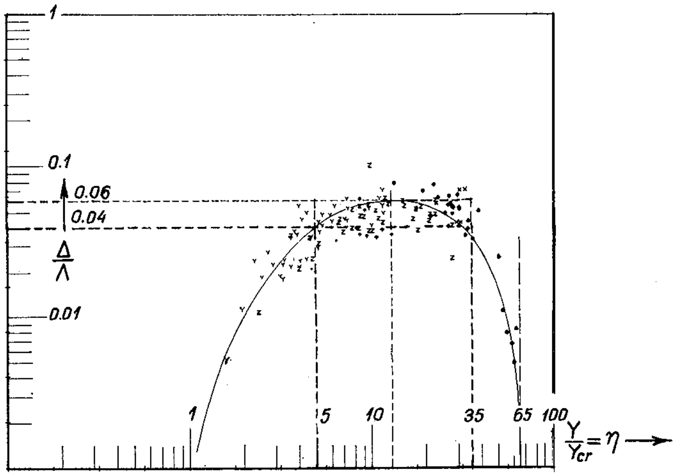


Fig. 5

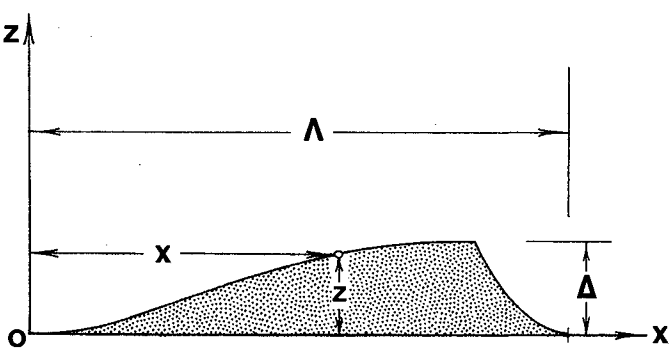


Fig. 6

(i.e. of $-(\partial\zeta/\partial\Theta)$ to $(\partial\Pi/\partial\xi)$) must also be a constant (even if not unity):

$$-\frac{(\partial\zeta/\partial\Theta)}{(\partial\Pi/\partial\xi)} = \text{const} \quad . \tag{16}$$

Substituting (16) in (15) one arrives at the relation

$$T = \text{const} \cdot \frac{\Delta\Lambda}{p_m} \tag{17}$$

which indicates that the duration T of the development of a sand wave is directly proportional to its volume per unit width $\approx (1/2) \cdot \Delta\Lambda$ and inversely proportional to the transport rate p_m (per unit width).

Since in the present derivations the quantity p_m is treated as a constant the space averaging procedure (13) must be associated with a specified instant. Accordingly p_m must be interpreted, for example, as the overall rate of sediment transport which takes place at $t = 0$ (i.e. over the flat initial bed just before the growth of sand waves) or as that taking place at $t \geq T$ (i.e. over the bed covered with developed sand waves) and so on. The difference in the decision on this score can affect only the numerical value of the constant in the expression (17). The exact value of this constant (corresponding to an adopted definition of p_m) can be revealed only by experiment. For the present, however, one can simply take const $\approx 1/2$.*

The values of Λ , Δ and p_m , which appear in (17), must either be revealed by measuring these quantities in the area of interest or they must be estimated with the aid of appropriate graphs and/or relations. For example, knowing the values of the characteristic parameters h, D, S and ν and thus of the dimensionless variables h/D and $X = v_* D/\nu$ (where $v_* = gSh$) the value of Λ/D and thus the length of dunes Λ can be predicted from the graph in Fig. 3. Similarly, knowing the values of $Y = (\gamma/\gamma_s)\, hS/D$ and Y_{cr} (from the Shields' curve) and consequently of the relative transport intensity $\eta = Y/Y_{cr} = \tau_o/(\tau_o)_{cr}$ one can predict the steepness Δ/Λ and thus the dune height Δ from the graph in Fig. 6.

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* The value of the constant would then be exactly 1/2 if the two dimensional sand waves were strictly triangular, and if the transport rate which figurates in (17) were the time-space average rate

$$\frac{1}{T}\int_0^T p_m \, dt = \frac{1}{T\Lambda} \cdot \int_0^T \int_x^{x+\Lambda} p \, dx \, dt \quad .$$

The transport rate p_m can be predicted by means of a bed load formula. The least complicated (and yet not the least reliable) would be a bed load formula of the following (Meyer-Peter/Bagnold) form

$$p = C \rho v_*^3 (1 - \frac{1}{\eta})^m \tag{18}$$

where $C = 8$ and $m = 1.5$ (Meyer-Peter) *
or $C = 0.4$ and $m = 1.0$ (Bagnold) .

Substituting in (17) the values

$$\text{const} \approx \frac{1}{2} \quad , \quad \Lambda = h\, \phi_\Lambda (X, \frac{h}{d}) \quad , \quad \Delta = \Lambda\, \phi_\Delta (\eta) \quad \text{and} \quad (18)$$

one determines the expression of the duration of development in terms of the characteristic parameters

$$T \approx \frac{1}{2C} \cdot \frac{\left[h \cdot \phi_\Lambda (X, \frac{h}{D})\right]^2 \cdot \phi_\Delta (\eta)}{\rho v_*^3 (1 - \frac{1}{\eta})^m} \tag{19}$$

(where the functions ϕ_Λ and ϕ_Δ are given by the graphs in Fig. 3 and Fig. 6 respectively).

<u>Special cases</u> (unidirectional flows)

(i) Observe from Fig. 6 that dunes can be present only if

$$1 < \eta < \approx 65 \ . \tag{20}$$

In the great majority of practical cases, dunes are likely to occur in the following "inner part" of the "existence region" (20):

$$\approx 5 < \eta < \approx 35 \ . \tag{21}$$

But, as can be seen again from Fig. 8, in this "inner part" the value of Δ/Λ varies within the relatively narrow interval

$$\approx 0.04 < \frac{\Delta}{\Lambda} < \approx 0.06 \ . \tag{22}$$

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* The fact that the (apparently different) bed load formulae of Meyer-Peter and Bagnold can be written in the unified manner (18) is demonstrated in Ref. [6].

Hence, for any η larger than ≈ 5 and smaller than ≈ 35 one can take simply

$$\frac{\Delta}{\Lambda} = \phi_\Delta(\eta) \approx 0.05 \quad . \tag{23}$$

(ii) As has been pointed out earlier, when $X > \approx 20$, then the dune length, i.e. the expression in the square brackets in (19), can be simplified as follows

$$\Lambda = h \cdot \phi_\lambda (X, \frac{h}{D}) \approx 2\pi \cdot h \quad . \tag{24}$$

Substituting (23) and (24) into (19) one arrives at the reduced version

$$T \approx \left[\frac{\pi^2}{10\ C}\right] \frac{h^2}{\rho\ v_*^3\ (1 - \frac{1}{\eta})^m} \tag{25}$$

(which is valid for $\approx 5 < \eta < \approx 35$ and $X > \approx 20$).

(iii) If η is sufficiently large (larger than ≈ 10, say) then $1/\eta$ becomes negligible in comparison to unity and the expression for T reduces even further. It becomes

$$T \approx \left[\frac{\pi^2}{10\ C}\right] \frac{h^2}{\rho\ v_*^3} \tag{26}$$

(which is valid for $\approx 10 < \eta < \approx 35$ and $X > \approx 20$). It follows that for sufficiently large values of the Reynolds number X, and the relative transport intensity η the duration of development of dunes tends to become proportional to the square of the flow depth and inversely proportional to the cube of the flow velocities (as any u is proportional to v_*).

Tidal flows

The eqn (19), and thus (25) and (26), were derived by evaluating (17) with the aid of the unidirectional flow relations. Hence (19), (25) and (26) should be generalised to tidal flows with some reservation and only if the factor β in eqn (8) is near unity (i.e. if the numerical values of the parameters (9) are rather large). If $\beta << 1$ then the relation (17) can still be regarded as valid, but Λ, Δ and p_m should be evaluated differently:

$$\left.\begin{aligned} \Lambda &= \alpha_1 \bar{L} = \alpha_1 \beta \bar{h} \\ \Delta &= \psi_\Delta(\eta) \cdot \Lambda \\ \bar{p}_m &= C_1\ \rho\ \bar{v}_*^3\ (1 - \frac{1}{\eta})^n \end{aligned}\right\} \quad . \tag{27}$$

Here the to date unknown values of α_1 , ψ_Δ , C_1 and n may not be the same as those of α , ϕ_Δ , C and m respectively. *

Substituting (27) in (17) (and assuming again that const ≈ 1/2) one obtains

$$T \approx \frac{1}{2} \frac{[\alpha_1 \bar{L}]^2 \cdot \psi_\Delta(\eta)}{C_1 \rho \bar{v}_*^3 (1 - \frac{1}{\eta})^n} \tag{28}$$

i.e.

$$T \approx \frac{1}{2} \frac{[\alpha_1 \beta] \cdot (\bar{h})^2 \cdot \psi_\Delta(\eta)}{C_1 \rho \bar{v}_*^3 (1 - \frac{1}{\eta})^n} \; . \tag{29}$$

This relation will also yield special cases analogous to (25) and (26). However, the numerical values of the validity regions must be expected to differ from those of (25) and (26) (the more so, the smaller is the factor β).

In closing this section it should be mentioned that even the values of C given for unidirectional flows are far from reliable. Indeed the values of C given above, (eqn (18)), strictly speaking, correspond to a "flat bed". If the bed is covered with sand waves, then C varies, and in fact decreases, with the increasing value of the sand wave steepness Δ/Λ (and perhaps with some other factors). In other words, the factor C is not a constant but a certain function:

$$C = \phi_C(\Delta/\Lambda , \ldots\ldots) \; . \tag{30}$$

The values of C given by (18) correspond merely to a special value of this function

$$C = \phi_C(0 , \ldots\ldots) \approx \text{const} \; . \tag{31}$$

The variation of C due to Δ/Λ is not "negligible" as it has often been observed that the discrepancy between the actual (measured) values of the transport rate over an undulated bed and those predicted by a transport formula can reach an order of magnitude. No transport formula has been produced to date (even for unidirectional flows) that contains a factor (analogous to C) given as a function of (at least) Δ/Λ in a clear and numerically usable manner.

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* In the relations (27) v_* and η signify some typical values over a periodic cycle.

SUGGESTIONS FOR FUTURE MEASUREMENTS, MODEL TESTS

The relations presented in the preceding section are rather schematical in their nature and the regions of their existence are either based on some estimated orders or they are not known at all. However, in spite of these shortcomings these relations can at least give some "hints" as to how future measurements aiming to correlate dunes and currents should be orientated.

The most fundamental point that follows from Section 3 is that the length Λ of dunes is determined by the size of the largest eddies, i.e. by some "lengths" inherent in the internal structure of the flow rather than by those characterising its external geometry (like flow depth, width and so on); the fact that the size of the largest eddies is determined by (and thus is proportional to) the external dimensions of the flow is beside the point for (as can be inferred e.g. from Fig. 4) the proportionality factors can vary from one case to another considerably.

The best way of determining the size $\bar{L}$ of the largest eddies would be with the aid of oscillograms of velocity fluctuations in space (by means of samples of the stochastic process $u = f(x)$ corresponding to various instants). However, the realisation of such a method would require the simultaneous operation of a number of current meters placed along the flow direction x near the bed, which is certainly very costly and difficult. Hence, it would perhaps be more reasonable to make an attempt to determine $\bar{L}$ with the aid of the velocity fluctuations in time (from the samples of the process $u = \phi(t)$ corresponding to various fixed points in space). In this case, the following procedure (which can be applied to both model and prototype) can be suggested:

1. Specify a location M in the plane of the flow area,* and record in a continuous manner (for a few periodic cycles) the behaviour of the velocity u along the flow direction x (in that location) at a level y near the bed. The recorder should have a sufficiently good response as to register at least those fluctuations of u which are due to macroturbulence. The recorded oscillograms will appear (for one periodic cycle) as the random function (curve) $u = \phi(t)$.

2. Divide the period T into n intervals and determine for each interval i the average duration $(t_L)_i$ corresponding to the longest, i.e. slowest, fluctuations (caused by the passage of the largest macroturbulent eddies). Multiply these values of $(t_L)_i$ with the corresponding average value of the flow velocity $\bar{u}_i$ of the same interval i . The product $(t_L)_i \cdot \bar{u}_i$ can be regarded as a sufficiently accurate approximation of the maximum eddy size L_i

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* Of course, the "location M" in the plane of the prototype may itself be an area of a considerable size.

corresponding to phase i of the periodic cycle

$$L_i = (t_L)_i \cdot \bar{u}_i \quad . \tag{32}$$

3. Determine the average value of L_i over the periodic cycle

$$\bar{L} = \frac{1}{n} \sum_{i=1}^{n} (t_L)_i \cdot \bar{u}_i \quad . \tag{33}$$

This is the value with which the length of dunes is expected to be correlated.

As is well known the large scale sand waves are often accompanied by smaller waves superimposed on them. Is this because the spectral density function of the process $u = f(x)$ has perhaps two, or more, peaks? (at least at some parts of the periodic cycle and/or for certain values of the flow Reynolds number and the relative roughness). It should be worthwhile to carry out the spectral analysis of the process $u = f(x)$ (via $u = \phi(t)$) to clarify this point.

Once the values of $\bar{L}$ for various locations of the prototype sea area are determined and the corresponding values of Λ are measured, one should be able to see how $\bar{L}$ and Λ are correlated: i.e. whether the ratio $\Lambda/\bar{L} = \alpha$ remains constant over the area subjected to measurements, and if not how it varies ... and so on. The same applies to $\bar{L}$ and $\bar{h}$, i.e. to the ratio $\bar{L}/\bar{h} = \beta$. If the distribution of the values of α and β over the sea area under investigation are revealed, then the dune length Λ can be correlated (in that area) with the aid of the flow depth (h) only:

$$\Lambda = (\alpha\beta) \cdot \bar{h} \quad . \tag{34}$$

If from the prototype measurements a satisfactory correlation between, say, Λ and $\bar{L}$ is achieved, then it may be helpful for model tests. Indeed, in this case some predictions can be made even with the model having rigid flow boundaries (provided, of course, that the model is sufficiently large and that its roughness is adequate). Since in a Froudian model the Reynolds numbers are smaller than their prototype counterparts, only the macroturbulent eddies will be reproduced in the vertical model scale λ_h, i.e. correctly; the microturbulent eddies will be distorted. Indeed, as can be seen from eqn (2) which can be written as

$$l_{min} \sim \frac{\nu}{\bar{U}} \sim \frac{\bar{h}}{\left(\frac{v_* \bar{h}}{\nu}\right)} = \frac{\bar{h}}{Re_*} \tag{35}$$

it is clear that if $\lambda_{Re_*} < 1$ then $\lambda_{\ell_{min}} > \lambda_h$. The fact that the relative size of microturbulent eddies (i.e. $\ell_{min}/\bar{h}$) in the model will thus be larger means that the model oscillogram u'' will not possess as fine a structure as its prototype counterpart u'. However, this should hardly be relevant with regard to the reproduction of dunes (which are caused by the largest eddies). The following tests can be carried out:

(i) Construct the model as to be geometrically similar to the existing prototype, i.e. without introducing any of the planned modifications. Run the flow, measure the oscillograms u'' (in the locations M''_j corresponding to the prototype locations M'_j) and determine the distribution of the model values $(\bar{L})''_j$ in plane. If the model roughness and the flow are adequate the field of $(\bar{L})''_j$ will be similar to the prototype field $(\bar{L})'_j$. If it is not, then the model roughness must be adjusted until $(\bar{L})''_j / (\bar{L})'_j \approx \lambda_h$ is achieved (theoretically for all j).

(ii) Modify the model according to the planned development (introduce the planned navigation channels, etc.) and determine again the values of $(\bar{L})''_j$. This time they will be different (at least in some locations), for the alteration of the flow boundaries will inevitably alter the flow and its macroturbulent structure (varying with the external dimensions of the flow). The ratio of the "new" and "old" values of $(L)_j$ will signify how many times the size of dunes will become larger or smaller as the result of the modification.

(iii) Similarly, determine the "new" time average velocities $\bar{u}''$ at the bed of the model navigation channel, and compare them with the "old" values of $\bar{u}''$ in the same location. The introduction of a larger depth (navigation channel) will almost certainly yield $u''_{new}/u''_{old} = N < 1$. Knowing N one can predict that the dunes on the bed of the proposed navigation channel will grow $\approx N^3$ times more slowly (see eqns (27) to (29)) than they grow on the plane bed at the same area, ... and so on.

In the case of a movable bed model the most important dimensionless variable is the relative flow intensity $\eta = \tau_o/(\tau_o)_{cr}$. The model bed material and the distortion must be selected so that the ratio η is identical in model and prototype, i.e. so that

$$\lambda_\eta = 1 \qquad (36)$$

is valid. Note that the dune steepness Δ/Λ , the dimensionless transport rate $p_m / \rho v_*^3$ and the duration T of the dune development are those aspects (functions) of the two-phase motion which vary most intensively with η ; no similarity of the aspects mentioned can be achieved if $\lambda_\eta = 1$ is not provided. On the other hand if $\lambda_\eta = 1$ is provided, then the error in similarity of these aspects can only be due to the non-equality of the remaining variables determining them. Since the role of the remaining variables in determining Δ/Λ , $p_m / \rho v_*^3$ and T can be regarded as "small" (in comparison to that of η) the error due to the omission of their identity should also be "small". This, however, does not mean that one should not even try to achieve the identity (of at least some) of the remaining variables wherever possible.

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LIST OF RELEVANT SYMBOLS

x	direction of flow	τ_0	bed shear stress (at $y = 0$)
y	direction perpendicular to x	$v_* = \sqrt{\tau_0/\rho}$	shear velocity
t	time	u	flow velocity at a level y
g	acceleration due to gravity	γ	specific weight of fluid
ℓ	scale of turbulence	γ_s	specific weight of grains
L	size of the largest eddies	D	representative grain size
ρ	fluid density	Λ	dune length
ν	kinematic viscosity	Δ	dune height
h	flow depth	T	duration of dune development
S	slope of the free surface	p	sediment transport rate

If A is a quantity then:

A_{cr} is the critical value of A (at the beginning of sediment transport)

$\overline{A}$ is the time average value of A (averaged over a periodic cycle)

A' and A'' prototype and model values of A respectively

$\lambda_A = A''/A'$ the scale of A .

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