CHAPTER 32

INTERNAL VELOCITIES IN THE UPRUSH AND BACKWASH ZONE.

Patrick H. Kemp
Reader, University College, London.

David T. Plinston.
Senior Research Engineer, Institute of Hydrology, U.K.

ABSTRACT.

In an earlier paper the authors reported the results of both field and hydraulic model investigations into the reaction of beaches to the action of waves of low phase difference. These waves are those whose time of uprush on the beach is equal to or less than 0.7 T, where T is the wave period. The present paper summarises the study of the internal velocity field under similar conditions. By making certain simplifying assumptions based on observations of the profile of the wave during uprush and backwash, the surging wave is open to quantitative description. For very low phase differences the uprush is approximately the reverse of the backwash, but to a different time scale. However, as the phase difference approaches the limit for the surging condition, the uprush starts and may remain in the form of a bore

INTRODUCTION.

The essentially oscillatory nature of the uprush and backwash of waves under surging conditions bears a close resemblance to the phenomena of standing wave systems on rigid impermeable slopes. This problem has been studied by Miche (1944) and it was thus possible to compare the results of the investigation with Miche's solution for low waves on steep slopes. The theory assumed the water movement on the beach to be sinusoidal, and also that the uprush and backwash were identical in nature but of opposite sign. The phase difference according to Miche would thus always be 0.5. This is not true in the case of beaches, and in the study of water velocities on the beach outlined below, this was taken into account.

It is essential to separate uprush and backwash, since the initial boundary conditions are so different. The backwash starts from zero velocity everywhere on the beach and a 'wedge' of water then moves down the beach, the water surface remaining essentially plane from the instantaneous position of the waterline on the beach to the break-point, throughout the backwash duration. In the case of the uprush the kinematics of the breaking wave are involved. For very low phase differences (say $t/T < 0.3$) the uprush is approximately the reverse of the backwash but to a different scale. However, as the phase difference approaches the limit for the surging condition, the uprush starts and may remain in the form of a bore. Thus, there are two possible methods of solution.

The general characteristics of the flow on a beach under surging and plunging conditions are such that the retardation of the uprush and subsequent acceleration of the backwash are equal and continuous over the point of zero velocity, and are progressively greater at points further from the break-point. Most significantly, the velocity is zero at all points simultaneously and the backwash is completed before the following wave breaks, as described by Kemp and Plinston (1968).

For plunging conditions the uprush velocity is not zero at all points simultaneously. The water nearer the breakers begins to move seaward before the water further up the beach comes to rest, and the backwash is not complete before the following wave breaks.

FLOW DESCRIPTION.

Figure 1. illustrates the basis of the present formulation of flow velocities, where l_b is the total runup distance from the break-point to the limit of uprush.

Miche's theory described a symmetrical cycle of uprush and backwash, and on the basis of his analysis the component of velocity along the beach slope at a time θ measured from the instantaneous stillwater level is given by

$$V = \frac{1}{\sin\alpha}\,\frac{\pi H}{2}\left(\frac{g}{\alpha L}\right)^{\frac{1}{2}}\cos\left(\left[\frac{2\pi g}{L}\right]^{\frac{1}{2}}\theta\right)$$

The movement of the water's edge on the beach is also sinusoidal, and the distance s' from the breaker position to the edge can be expressed as

$$s' = \frac{l_b}{2}\left[1 - \cos\left(\frac{2\pi\theta}{T}\right)\right]$$

In fact, the movement of the water on the beach for the low long waves postulated by Miche is asymmetrical. However, a sinusoidal motion can be maintained by treating the uprush and backwash as two independent cycles of period t for the uprush, and $T - t$ for the backwash. For the uprush the time θ is measured from the breaking point, and for the backwash from the limit of uprush. Thus for the uprush

$$\frac{s'}{l_b} = \frac{1}{2}\left(1 - \cos\frac{\pi\theta}{t}\right) \qquad (1)$$

and for the backwash

$$\frac{s'}{l_b} = \frac{1}{2}\left(1 + \cos\frac{\pi\theta}{T-t}\right) \qquad (2)$$

By comparing curves based on these equations with the data from model experiments, it was found that the sine curve appears to be satisfactory for the backwash, but that a better description of the uprush can be developed by introducing the characteristics of a bore into the uprush equation.

Backwash velocities. The derivation of the instantaneous velocities from the changing geometry of the water level on the beach is made possible by making assumptions based on observations of surging waves. They are:

1. The beach is plane, of slope $\tan\alpha$.
2. The velocity of water on the beach is everywhere zero at the temporal limit of uprush.
3. There is no loss or gain of water into or out of the beach.
4. The water surface remains plane from the limit of uprush to the break-point throughout the backwash period, though the slope can vary.
5. The velocity is uniform from bed to water surface, for a given position 's' from the breakers.
6. The position of the edge of the backwash, s' from the break-point

can be described by a function of l_b, θ, and $T - t$

7. The backwash is complete in the time $T - t$, i.e. surging conditions only are considered.

The symbols used are defined in the glossary and in Figure 1.

The expressions for the internal velocities associated with both uprush and backwash are based on the normal equation of continuity, on the geometry of the system, and on the observed variations in water level at the breakers during the wave cycle.

The continuity equation is given by

$$\frac{\partial(z'v')}{\partial s} = \frac{\partial z'}{\partial \theta} \qquad (3)$$

and if it is assumed that the beach and water surface are plane, then $z' = y'\left(1 - \frac{s}{s'}\right)$. The distance from the breakers to the edge position is given by equation (2) in the form

$$s' = \frac{l_b}{2}\left(1 + \cos\frac{\pi\theta}{T - t}\right) \qquad (2)$$

An experimental study of the variation in water level at the break-point can be closely described by

$$y' = y_0 \cos\frac{\pi}{2}\frac{\theta}{(T - t)} \qquad (4)$$

where y' is the water depth at any time θ from the commencement of breaking.

From the assumption that the water surface and beach are plane it follows that

$$z' = y'\left(1 - \frac{s}{s'}\right) \qquad (5)$$

Differentiating equation (3) with respect to θ

$$\frac{\partial z'}{\partial \theta} = \frac{\partial y'}{\partial \theta}\left(1 - \frac{s}{s'}\right) + y'\frac{\partial}{\partial \theta}\left(1 - \frac{s}{s'}\right) \qquad (6)$$

Differentiating equation (2) and writing

$$\frac{\partial}{\partial \theta}\left(1 - \frac{s}{s'}\right) = \frac{s}{(s')^2}\frac{\partial s'}{\partial \theta}$$

enables equation (6) to become

$$\frac{\partial}{\partial s}(z'v') = \frac{\partial z'}{\partial \theta} = \frac{\partial y'}{\partial \theta}\left(1 - \frac{s}{s'}\right) - \frac{y's}{(s')^2}\left(\frac{l_b}{2}\cdot\frac{\pi}{T-t}\sin\frac{\pi\theta}{T-t}\right) \qquad (7)$$

Integrating with respect to s gives

$$z'v' + f(\theta) = \frac{\partial y'}{\partial \theta}\left(s - \frac{s^2}{2s'}\right) - y'.\frac{s^2}{2(s')^2}\left(\frac{l_b}{2}.\frac{\pi}{T-t}\sin\frac{\pi\theta}{T-t}\right) \quad (8)$$

where $f(\theta)$ can be defined from the boundary conditions by putting $s = 0$ or $s = s'$. Using $s = s'$ i.e. $z' = 0$,

$$0 + f(\theta) = \frac{\partial y'}{\partial \theta}\frac{s'}{2} - \frac{y'}{2}\left(\frac{l_b}{2}\frac{\pi}{T-t}.\sin\frac{\pi\theta}{T-t}\right) \quad (9)$$

Substituting into (8) and putting $z' = y'(1 - \frac{s}{s'})$ and reerranging and dividing by $y'(1 - \frac{s}{s'})$ gives

$$v' = -\frac{1}{y'}\frac{\partial y'}{\partial \theta}\left(\frac{s' - s}{2}\right) + \frac{l_b}{4}\frac{\pi}{T-t}\left(\frac{s' + s}{s'}\right)\sin\frac{\pi\theta}{T-t} \quad (10)$$

Using the water level equation (4)

$$\frac{\partial y'}{\partial \theta} = -\frac{\pi}{2}\frac{1}{T-t}\, y_o \sin\frac{\pi}{2}\frac{\theta}{T-t} \quad (11)$$

Substituting (4) and (11) into (10), gives

$$v' = \frac{\pi}{2(T-t)}\left(\frac{s'-s}{2}\right)\tan\frac{\pi\theta}{2(T-t)} + \frac{l_b}{4}\frac{\pi}{T-t}\left(\frac{s' + s}{s'}\right)\sin\frac{\pi\theta}{T-t} \quad (12)$$

This is the horizontal component of the velocity parallel to the beech, so the velocity v parallel to the beach is $\frac{v'}{\cos\alpha}$.

Equations (12) and (2) can be combined to give

$$v = \frac{1}{\cos\alpha}\frac{\pi l_b}{.4(T-t)}\left[\frac{3}{2}\sin\frac{\pi\theta}{T-t} + \frac{s}{l_b}\tan\frac{\pi\theta}{2(T-t)}\right] \quad (13)$$

Clearly a velocity only exists when there is water on the beach at the particular value of s/l_b chosen. This means that s/l_b must be less than or equal to s'/l_b. When $s/l_b = s'/l_b$ then the edge of the backwash has reached the position chosen. Immediately the velocity falls to zero as the beach becomes dry. If in equation (13) s/l_b is put equal to s'/l_b and the substitution from equation (2) used in the form

$$\frac{s'}{l_b} = \cos^2\frac{\pi\theta}{2(T-t)} \quad (14)$$

$$v = \frac{1}{\cos\alpha.2(T-t)}\frac{l_b}{}\sin\frac{\pi\theta}{T-t} \quad (15)$$

and the time of occurrence is found from equation (14)

$$\frac{s}{l_b} = \frac{s'}{l_b} = \cos^2 \frac{\pi\, \theta}{2(T-t)} \tag{16}$$

Thus using equation (16) for the chosen s/l_b , θ can be computed for the condition $s = s'$, hence using (15) the velocity at this value of s and θ can be found. This is the backwash velocity immediately before the beach becomes dry at this s/l_b.

To find the maximum backwash velocity for a particular position s/l_b and the absolute maximum backwash velocity occurring on the beach at any s/l_b and any θ :

abbreviating equation (13) for convenience, the velocity just before the beach becomes dry is

$$v_1 = \frac{3}{2} \sin R\,\theta + \frac{s}{l_b} \tan R\,\theta \tag{17}$$

where $R = \frac{\pi}{2}\left(\frac{1}{T-t}\right)$ and $v_1 = v \cos\alpha . \frac{4(T-t)}{\pi\, l_b}$

For a particular value of s/l_b in equation (17) $\frac{\partial v}{\partial \theta} = 0$ will give the value of $R\theta$ for the maximum velocity.

$$\frac{\partial v}{\partial \theta} = 3R \cos 2R\theta + R\frac{s}{l_b} \sec^2 R\theta. \tag{18}$$

This has been plotted as line ABC in Figure (2), s/l_b plotted against $\frac{\theta}{T-t}$. The condition that there is water on the beach i. e. the limiting s/l_b , $\frac{\theta}{T-t}$ relation, equation (16) is also plotted as line DBE on the graph. For values of s/l_b from 0 up to the value corresponding to B on the graph, the backwash velocity reaches a maximum before the beach becomes dry. For larger values of s/l_b up to 1 the maximum velocity is not reached before the beach becomes dry at that point.

Hence the line ABD gives the time $\frac{\theta}{T-t}$ for the maximum velocity at any chosen s/l_b.

<u>Uprush velocities.</u>

The uprush can be in the form of either

(a) a simple surge similar to the backwash in reverse,

or

(b) a bore or

(c) a combination of (a) and (b), the bore form giving way to the simple wedge-type surge.

Considering the uprush to be in the form of a wedge, the result can be shown to be,

$$v = -\frac{1}{\cos\alpha}\cdot\frac{l_b}{4T}\left(\frac{3}{2}\sin\frac{\pi\theta}{t} + \frac{s}{l_b}\tan\frac{\pi\theta}{2t}\right) \tag{19}$$

This is the expression for backwash velocity (equation 13) with T-t replaced by t .

If it is assumed that the height of the bore front z' decreases linearly with distance up the beach, then

$$z' = y_o\left(1 - \frac{s'}{l_b}\right) \tag{20}$$

if the bore height is zero at the limit of uprush.
If it is assumed that the velocity of the bore front is given by

$$v' = k(gz')^{\frac{1}{2}} \qquad \text{where } k \text{ is a constant,}$$

and since

$$\frac{\partial s'}{\partial\theta} = v' \tag{21}$$

substitution gives

$$\frac{\partial s'}{\partial\theta} = k\left(gy_o\left(1-\frac{s'}{l_b}\right)^{\frac{1}{2}}\right)$$

Integrating with respect to θ and noting the bound $s' = 0$ when $\theta = 0$, gives

$$s' = k(gy_o)^{\frac{1}{2}}.\theta - k^2 gy_o\frac{1}{4l_b}\,\theta^2 \tag{22}$$

Substituting $s' = l_b$ when $\theta = t$, gives

$$l_b = k(gy_o)^{\frac{1}{2}}t - k^2\, gy_o\frac{1}{4l_b}.t^2 , \qquad \text{a solution of}$$

which is

$$k(gy_o)^{\frac{1}{2}} = \frac{2l_b}{t} \tag{23}$$

Substituting this in equation (22) gives

$$\frac{s'}{l_b} = \frac{2\theta}{t} - \left(\frac{\theta}{t}\right)^2 \tag{24}$$

From equations (21) and (24) $\qquad v' = \frac{2\,l_b}{t}\left(\frac{1-\theta}{t}\right)$ (25)

On comparing equation (24) with experimental values it was found that the experimental points lay between this curve and the sine curve from the linear theory of Miche, indicating that the uprush is a combination of the two conditions.

In order to find the internal velocity v' at any other position in the uprush , it is neceesery to look at tha behaviour of the water surface. The experimentsl results showad that the depth of water st the breek-point rapidly echievee e level corresponding to the height of the bore et breaking, and that this depth remains constant throughout the uprush. It followe from the assumption that the bore front diminishes in height linesrly with distance up the beach, that if the water surface is assumed plsne et any time, ssy at the temporal limit of uprush, then it csn be said that the watar surface lies on the same plane in epace throughout the uprush.

It follows from continuity of volume and from the essumption of the plane water surfsce that

$$v' = \frac{2\,l_b}{\cos\alpha t} \cdot \frac{(1 - \frac{\theta}{t})^3}{1 - s/l_b} \tag{26}$$

This ie the horizontal component of velocity, and the velocity parallel to the beach will be,

$$v' = \frac{2\,l_b}{\cos\,\alpha t} \; \frac{(1 - \frac{\theta}{t})^3}{1 - s/l_b} \tag{27}$$

Figure 3 illustrates the form taken by typical resulte from model teets, compared with the theoretical curves for a bore type and wedge type uprush.

Field obeervations mede with emall propellor current meters fixed to stakes set into the beech ehowed that the flow broadly followed the same pattern.

The limitations of the predicted velocitiee both for uprush and beckweeh sre those imposed by the assumptions. The backwesh seems to be edequately described by the eine function expression. So far as the uprush is concerned, long low wavee seam to be described by the wedge form. Tha complete bore form ie echieved towards the limit of eurge conditions. Between the two there is e wide range when both forms are combined. Obeervations on the coast showed that the uprush was of the bore form during the observational periods.

Beckwash measuremente on the coast gave good qualitstive agreement with theory, but generally the maximum predicted velocitiee were higher than those observed. For practical purposed, the significant weve period and mean uprush time were used in the theory, and individual wave velocity measurements could be expected to show considerable varistions from the mean predicted valuee. Tha variations from weve to wave were of the eame order es the differences between measured and predicted valuee.

Referencea.

Kemp P.H. & Plinston D.T. 1968 Beaches produced by waves of low phaae difference. Proc.Amer. Soc. Civ. Engrs. Jour. Hydr. Div. HY5. pp 1183-1195.

Miche, M. 1944. Mouvements ondulatoires de la mer. Annales des Ponts et Chausses.

Glossary.

l_b = breaker length.

s = distance from the break-point.

a' = variable diatance from the break-point.

T = wave period.

v = velocity, as defined in the text.

t = time of uprush, or time.

x = horizontal coordinate,

y = water depth at break-point.

y_o = water depth at break-point at temporal limit of uprush.

z = vertical coordinate

= beach slope measured at mean water level.

O = time

' = (suffix) denoting variable quantity.

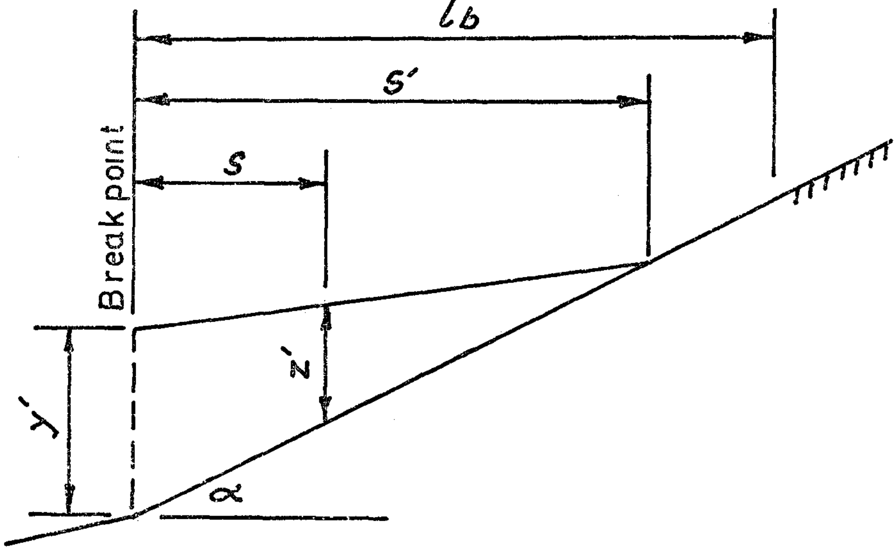
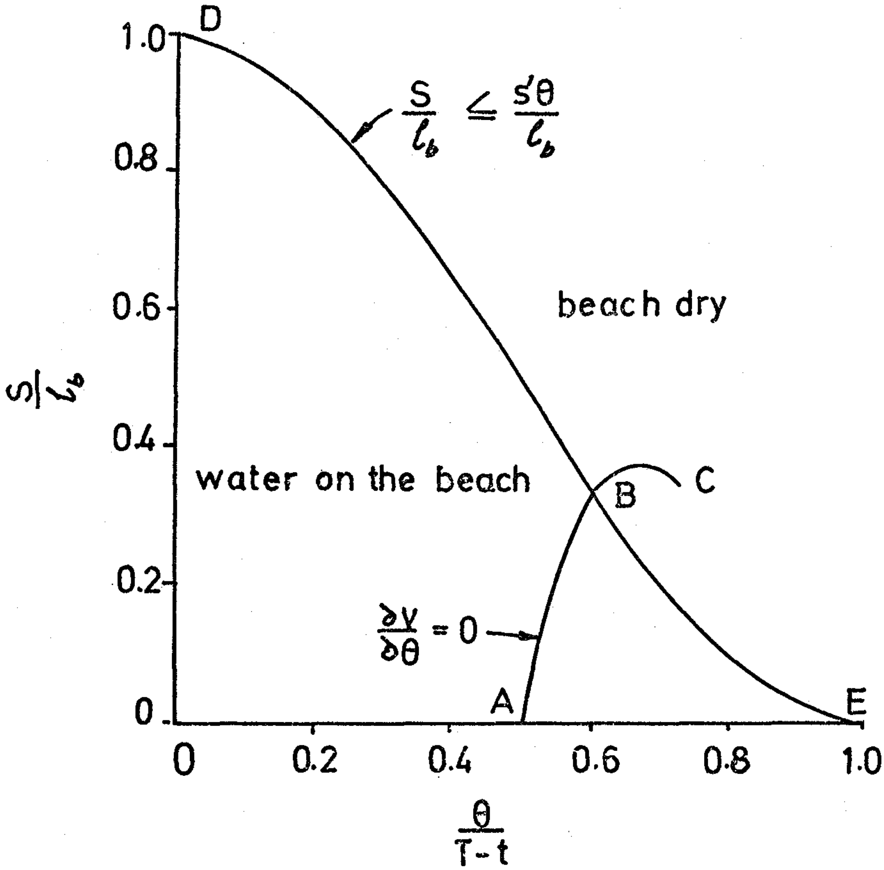


FIG 1.



D
1.0
0.8
0.6
0.4
0.2
0
$\frac{S}{l_b} \leq \frac{s'\theta}{l_b}$
beach dry
water on the beach
B
C
$\frac{\partial v}{\partial \theta} = 0$
A
E
$\frac{S}{l_b}$
0
0.2
0.4
0.6
0.8
1.0
$\frac{\theta}{T-t}$

FIG.2.

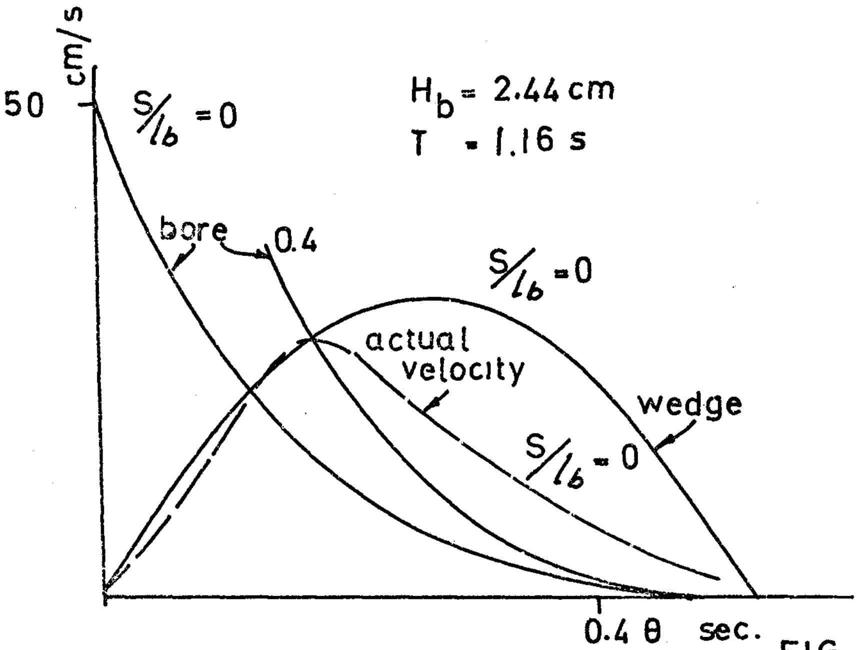

cm/s
50
S/lb = 0
Hb = 2.44 cm
T = 1.16 s
bore
0.4
S/lb = 0
actual
velocity
wedge
S/lb = 0
0.4 θ
sec.

FIG.3.