CHAPTER 26

SURF SIMILARITY

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ABSTRACT

This paper deals with the following aspects of periodic water waves breaking on a plane slope breaking criterion, breaker type, phase difference across the surfzone, breaker height-to-depth ratio, run-up and set-up, and reflection. It is shown that these are approximately governed by a single similarity parameter only, embodying both the effects of slope angle and incident wave steepness. Various physical interpretations of this similarity parameter are given, while its role is discussed in general terms from the viewpoint of model-prototype similarity.

FLOW PARAMETERS

Consider a rigid, plane, impermeable slope extending to deep water or to water of constant depth from which periodic, long-crested waves are approaching. The wave crests are assumed to be parallel to the depth contours.

The motion will be assumed to be determined wholly by the slope angle α, the still water depth d and the incident wave height H at the toe of the slope, the wave period T, the acceleration of gravity g, the viscosity μ and the mass density ρ of the water, g, μ and ρ are assumed to be constants. Effects of surface tension and compressibility are ignored.

Let X be any dimensionless dependent variable, then

$$X = f(\alpha, \frac{H}{L_0}, \frac{d}{L_0}, Re) \qquad (1)$$

in which Re is a typical Reynolds number, and

$$L_0 = \frac{gT^2}{2\pi} , \qquad (2)$$

i.e. the deep-water wavelength of small-amplitude sinusoidal, longcrested gravity surface waves with period T. The ratio H/L_0 is a wave steepness, if we define this parameter in a generalized sense as the ratio of a wave height to a wave length.

Variations in the flow regime are brought about mainly by variations of α and H/L_0, for the Reynolds number is usually larger than some minimum value above which variations in its actual value do not significantly affect the resultant motion, while for waves breaking on the slope, the value of the relative depth in front of the slope is not important either, this is well established for the relative run-up [7] and the reflection coefficient [14], for instance. So, in summary one can say that for waves breaking on the slope (1) reduces to

$$X \approx f(\alpha, \frac{H}{L_0}) \quad , \tag{3}$$

while it will be shown in the following that for many overall-properties of the breaking waves (3) reduces further to

$$X \approx f(\xi) \quad , \tag{4}$$

in which ξ is a similarity parameter, defined by

$$\xi = \frac{\tan \alpha}{(H/L_o)^{\frac{1}{2}}} \quad . \tag{5}$$

To the author's knowledge, this parameter was first used by Iribarren and Nogales [8], for determining whether wave breaking would occur. Its more general usefulness in the context of surf problems was suggested by Bowen et al [3].

FLOW CHARACTERISTICS DETERMINED BY THE SIMILARITY PARAMETER ξ

Breaking criterion

Iribarren and Nogales [8] have given an expression for the condition at which the transition occurs between non-breaking and breaking of waves approaching a slope which is plane in the neighbourhood of the still-water line. They use the shallow-water trochoidal theory for uniform, progressive waves. According

to this theory, progressive waves are at the limit of stability if their amplitude ($\frac{1}{2}H$) equals the mean depth (d). Thus, denoting the condition of incipient breaking by the index "c",

$$\tfrac{1}{2}H_c = d_c \tag{6}$$

The depth d_c at which this would occur is equated by Iribarren and Nogales to the mean undisturbed depth in the one-quarter wavelength adjacent to the still-water line, or

$$d_c = \frac{1}{2}\left(\frac{1}{4} L_c \tan \alpha_c\right) = \frac{1}{8} L_c \tan \alpha_c \quad . \tag{7}$$

The wavelength L_c is calculated as $T_c \sqrt{gd_c}$, so that

$$d_c = \frac{1}{8} T_c \sqrt{gd_c} \tan \alpha_c \quad . \tag{8}$$

Elimination of d_c between (6) and (8) gives

$$\left(T\sqrt{g/H} \tan \alpha\right)_c = 4\sqrt{2} \tag{9}$$

or, substituting (2) and rearranging,

$$\xi_c = \left(\frac{\tan \alpha}{\sqrt{H/L_0}}\right)_c = \frac{4}{\sqrt{\pi}} \approx 2.3 \quad . \tag{10}$$

Laboratory experiments by Iribarren and Nogales and others [8, 13] have confirmed the validity of (10), with the proviso that $\xi \approx 2.3$ corresponds to a regime about halfway between complete reflection and complete breaking. This quantitative agreement is considered to be fortuitous because one can raise valid objections against the derivations on several scores. These pertain to the numerical estimates used by Iribarren and Nogales, rather than to the approach as such. For instance, the limiting height for waves in shallow water is given by (6) as twice the depth, which is unrealistic. A height-to-depth ratio of order one seems more reasonable. The author has elsewhere [1] suggested more realistic values for the various numerical factors in (6), (7) and (8), which however happened to yield exactly the criterion (9) again. Even so, the fact that (9) is correct not only qualitatively but also quantitatively is still

considered to be somewhat fortuitous, because derivations such as these can be useful for indicating the form in which the respective variables are to be combined, but they cannot in general be expected to give correct quantitative predictions.

The derivation given by Iribarren and Nogales suggests a physical interpretation of the parameter ξ, at least if wave breaking occurs ($\xi < \xi_c$). Consider the local steepness of the breaking waves. Their celerity is proportional to $(gd)^{\frac{1}{2}}$, their wavelength to $T(gd)^{\frac{1}{2}}$, and their steepness to $H/(T(gd)^{\frac{1}{2}})$, or to $(H/gT^2)^{\frac{1}{2}}$, since H/d is of order one for waves breaking in shallow water. Thus, the parameter ξ, given by

$$\xi \equiv \frac{\tan \alpha}{\sqrt{H/L_0}} = \frac{1}{\sqrt{2\pi}} \frac{\tan \alpha}{\sqrt{H/gT^2}} \quad , \tag{11}$$

is roughly proportional to the ratio of the tangent of the slope angle (the slope "steepness") to the local steepness of the breaking wave. The criterion for breaking given by Iribarren and Nogales can therefore be said to imply that incipient breaking corresponds to a critical value of this ratio.

Breaker types

So far the parameter ξ has been considered only in the context of a breaking criterion, that is, as an aid in answering the question whether wave breaking will occur. However, it also gives an indication of how the waves break. The main types are surging, collapsing, plunging and spilling breakers [9, 15, 4]. These occur in the order of increasing wave steepness and/or decreasing slope angle. The transition from one breaker type to another is of course gradual. The values of α and H/L_o mentioned in what follows should be considered as indicating the order of magnitude only of the values in the transition ranges.

Galvin [4] has presented criteria regarding breaker types in terms of an "offshore parameter" $H_0/(L_0 \tan^2 \alpha)$, in which H_0 is a deep-water wave height calculated from the motion of the generator bulkhead and the water depth, and an "inshore parameter" $H_b/(gT^2 \tan \alpha)$. The index "b" refers to values at the break point, which is taken to be the most seaward location where some point of the wave front is vertical, or, if this does not occur, the location where foam first appears at the crest.

Galvin's offshore parameter can be written as ξ_0^{-2}, in which the index "0"

refers to deep water (wave height). Converting the critical values of the offshore parameter given by Galvin to values of ξ_0 gives

surging or collapsing if $\xi_0 > 3.3$
plunging if $0.5 < \xi_0 < 3.3$
spilling if $\xi_0 < 0.5$

These results are based on experiments on slopes of 1 5, 1 10 and 1 20.

The inshore parameter used by Galvin, $H_b/(gT^2 \tan \alpha)$, is not equivalent to the parameter ξ_b used here. However, a re-analysis of Galvin's data in terms of $\xi_b = (H_b/L_0)^{-\frac{1}{2}} \tan \alpha$ showed that the classification of breakers as plunging or spilling could be performed equally well with ξ_b as with Galvin's inshore parameter [1]. The following approximate transition values were found

surging or collapsing if $\xi_b > 2.0$
plunging if $0.4 < \xi_b < 2.0$
spilling if $\xi_b < 0.4$

The possibility of using a parameter equivalent to ξ_b as a breaker type discriminator has also been noted by Galvin in a more recent review of breaker characteristics [5].

Phase difference across the surfzone

Not only the form of a breaking wave varies with ξ, but the distance of the break point from the mean water line as well. This distance, expressed in wavelengths, is estimated at roughly $(d_b \cot \alpha)/(\frac{1}{2}T\sqrt{gd_b}) \approx 0.8\, \xi_b^{-1}$, where we have put $H_b \approx d_b$. Observations by the author on slopes between 1 3 and 1·25, with ξ-values from 0.15 to 1.9, have indicated that his estimate is qualitatively correct, but that it is roughly 20% too high. With spilling breakers there are at least two breaking or broken waves in the surf zone simultaneously. This number ranges from zero to two for plunging breakers. Collapsing breakers occur almost at the instantaneous water's edge, so that there is at most one of these present at any one time. Reference should be made in this connection to Kemp [10], who points out that the total phase difference across the surf zone is indicative of the type of wave motion, and of the corresponding equilibrium profile

of sand or shingle beaches.

Breaker height-to-depth ratio

The ratio of wave height to water depth at breaking is an important parameter of the surf zone, it is here denoted by the symbol γ_b.

$$\gamma_b = \frac{H_b}{d_b} \tag{12}$$

The depth d_b is here defined as the still-water depth at the break point.

Values of γ_b generally range between 0.7 and 1.2. Bowen et al [3] suggest that γ_b may be a function of ξ_o only. The data presented by them are given in fig. 1. In addition, data have been plotted from Iversen [9], from Goda [6], and from unpublished results obtained by the present author.
It can be observed that the results from Bowen et al [3] form a separate group, outside the range of the others. The reason for this is not known. The other points in fig. 1 show a weak trend with ξ_0. For values of ξ_0 less than about 0.2, in the range of spilling breakers, they are scattered about a value of $\gamma_b \approx 0.8$, while there is a slow increase with ξ_o for higher values.

The scatter in the results may partly be due to the fact that for this purpose the independent variables H/L_0 and α cannot adequately be combined in the single parameter ξ. However, even the values of γ_b presented by various authors for the same values of α and H/L_0 show considerable scatter. This is undoubtedly to some extent due to the difficulties and ambiguities inherent in defining (experimentally) and measuring breaker characteristics. Another factor contributing to the scatter may be the occurrence of parasitic higher-harmonic free waves which are often inadvertently generated along with the intended wave train. The secondary waves affect the breaking process in a manner depending on the phase difference with the primary wave, which in turn depends (among others) on the distance from the wave generator. This distance is not commonly introduced as an independent variable, so that any effects which it may have on the results can appear as unexplained scatter.

Set-up, run-up and run-down

The set-up is defined as the wave-induced height of the mean level of the water surface above the undisturbed water level. Theoretical and experimental

results [11, 3] indicate that the gradient of the set-up in the surf zone on gently sloping beaches is proportional to the beach slope, the coefficient of proportionality is a function of γ, the average height-depth ratio of the waves in the surf zone. The maximum set-up is roughly equal to $0.3\ \gamma\ H_b$.

The run-up height R_u is defined as the maximum elevation of the waterline above the undisturbed water level. A simple and reliable empirical formula for the run-up height of waves breaking on a smooth slope has been given by Hunt [7]. It can be written as

$$\frac{R_u}{H} = \xi \qquad \text{for } 0.1 < \xi < 2.3 \quad . \tag{13}$$

An investigation by Battjes and Roos [2] of some details of the run-up of breaking waves on dike slopes (1:3, 1:5, 1 7), such as the mean velocity of advance, particle velocities, layer thickness and so on, has shown that many of these parameters are functions of ξ only if normalized in terms of the incident wave characteristics.

Measurements of the run-down height R_d (minimum elevation of the waterline above the undisturbed water level) are very scarce, and, if available, not very accurate since run-down is rather ill-defined experimentally. An analysis of the measurements by Battjes and Roos [2], supplemented with unpublished data gathered for this study, has indicated that in the experimental range ($\cot \alpha = 3,5,7,10$; $0.02 < H/L_0 < 0.09$, $0.3 < \xi < 1.9$) the run-down height R_d is roughly equal to $(1 - 0.4\ \xi)R_u$. In other words, the ratio of the variable part of the vertical motion of the waterline ($R_u - R_d$) to the maximum elevation above S.W.L. is approximately $0.4\ \xi$. It has a maximum value of about 1 for waves in the transition from non-breaking to breaking, and decreases with decreasing ξ. For very small ξ the set-up constitutes the greater part of the run-up height.

Reflection and absorption

The relative amount of wave energy that can be reflected off a slope is intimately dependent on the breaking processes and the attendant energy dissipation. Because of this, and in view of the fact that these processes appear to be governed to such a large extent by the parameter ξ, it is natural to try to relate the reflection coefficient to ξ.

The reflection coefficient r is defined as the ratio of the amplitude of the reflected wave to the amplitude of the incident wave. The estimation of r

on a slope generally takes place according to a procedure given by Miche [13] who assumes that the reflected wave height equals the maximum height possible for a non-breaking wave of the given period on the given slope, in other words, only the energy corresponding to the height in excess of the critical height is assumed to be dissipated. This gives

$$r_{th} = \frac{(H_0/L_0)_c}{H_0/L_0} \quad \text{if this is less than 1}$$
$$= 1 \quad \text{otherwise,} \tag{14}$$

in which $(H_0/L_0)_c$ is the critical steepness for the onset of breaking, for which Miche uses an expression derived previously by him [12]. The index "th" refers to "theoretical". The actual reflection coefficient will be smaller than r_{th} due to effects of viscosity, roughness, and permeability. Miche recommends a multiplication factor of 0.8 for smooth slopes.

Miche's assumption regarding the reflection coefficient can be expressed in terms of ξ and Iribarren and Nogales' breaking criterion. Substitution of (5) into (14) gives

$$r_{th} = (\xi/\xi_c')^2 \quad \text{if this is less than 1}$$
$$= 1 \quad \text{otherwise,} \tag{15}$$

in which ξ_c' is the critical value of ξ for the <u>onset</u> of breaking, as distinguished from ξ_c, the value given by Iribarren and Nogales for the condition <u>halfway</u> between the onset of breaking and complete breaking ($\xi_c \approx 2.3$), for which the reflection coefficient is about 0.5 [7, 8]. So (15) becomes

$$r \approx 0.1\ \xi^2 \quad \text{if this is less than 1}$$
$$= 1 \quad \text{otherwise} \tag{16}$$

An extensive series of measurements of the reflection coefficient of plane slopes has recently been presented by Moraes [14]. His results for slopes with $\tan\alpha = 0.10$, 0.15, 0.20 and 0.30 have been replotted in terms of r vs ξ in fig. 2. The experimental points for the four slopes more or less coincide with each other and with the curve representing eq. (16) for $\xi < 2.5$, i.e. as long as the waves break. For $\xi < 2.5$ they diverge, gentler slopes giving less reflection than steeper slopes (at the same value of ξ).

GENERAL COMMENTS REGARDING THE PARAMETER ξ

In the preceding paragraphs examples have been given of a number of characteristic surf parameters for the determination of which it is not necessary to specify both α and H/L_0, but only the combination $\tan\alpha/(H/L_0)^{\frac{1}{2}}$. It may be useful to summarize them here a breaking criterion, the breaker type, the breaker height-to-depth ratio, the number of waves in the surf zone, the reflection coefficient (therefore also the discrimination between progressive waves and standing waves), and the relative importance of set-up and run-up. They have been collected in Table 1. Characteristic values of ξ are given in the upper row of the table. Each of the following rows indicates how one of the parameters just mentioned varies with ξ.

The recognition of the possibility that several properties can roughly be expressed as functions of ξ alone contributes to a more unified understanding of the phenomena involved. Such understanding would be deepened by further insight in the nature of the parameter ξ itself. One interpretation has already been mentioned in the preceding, when it was observed that ξ is approximately become proportional to the ratio of the tangent of the slope angle to the shallow-water wave steepness. Also, ξ^{-1} had been seen to be approximately proportional to the number of wavelengths within the surf zone. This is in essence equivalent to saying that ξ is approximately proportional to the relative depth change across one wavelength in the surf zone. This interpretation is obviously relevant to the dynamics of the breaking waves, particularly with regard to their rate of deformation. It makes it plausible that ξ is of importance, but it does not prove that ξ serves as the sole determining factor for the (suitably normalized) parameters of the surf. Indeed, there are valid arguments which throw doubt on this possibility of full similarity. In this regard it is useful to consider two situations of different slope angle and wave steepness as a prototype and a distorted scale model thereof. It is well known that Froudian model-prototype similarity can be obtained even in distorted models, provided the assumption of hydrostatic pressure distribution is valid both in the prototype and in the model. Pertinent scale ratios (λ) are given in Table 2, expressed in terms of the horizontal and vertical geometrical scales and the scale of the gravitational acceleration (unity).

ξ	$\approx$ 0.1	0.5	1.0	2.0	3.0	4.0	5.0
	breaking						no breaking
	spilling		plunging		collapsing/surging		
H_b/d_b	0.8	1.0	1.1	1.2			
N*)	6-7	2-3	1-2	0-1	0-1		
r	10^{-3}	10^{-2}	10^{-1}	4×10^{-1}	8×10^{-1}		
	absorption						reflection
	progressive wave						standing wave
	set-up predominant			run-up predominant			

*) number of waves in surf zone

Table 1

Variable	Symbol	Scale ratio
horizontal length	ℓ	λ_ℓ
depth	d	λ_d
gravity acceleration	g	$\lambda_g = 1$
bottom slope	$\tan \alpha$	$\lambda_{\tan \alpha} = \lambda_d \lambda_\ell^{-1}$
wave height	H	$\lambda_H = \lambda_d$
wave length	L	$\lambda_L = \lambda_\ell$
wave celerity	$c \approx (gd)^{\frac{1}{2}}$	$\lambda_c = \lambda_d^{\frac{1}{2}}$
wave period	$T = L/c$	$\lambda_T = \lambda_\ell \lambda_d^{-\frac{1}{2}}$

Table 2 - Scale ratios for a distorted long-wave model.

Since ξ is defined as

$$\xi = \left(\frac{gT^2}{2\pi H}\right)^{\frac{1}{2}} \tan \alpha \quad , \tag{17}$$

its scale ratio is

$$\lambda_\xi = \lambda_T \lambda_H^{-\frac{1}{2}} \lambda_{\tan \alpha} \, , \tag{18}$$

which becomes, using the values given in Tabel 2,

$$\lambda_\xi = (\lambda_\ell \lambda_d^{-\frac{1}{2}})(\lambda_d^{-\frac{1}{2}})(\lambda_d \lambda_\ell^{-1}) = 1 \quad . \tag{19}$$

In other words, a distorted long-wave model which is dynamically similar to its prototype necessarily has the same ξ as this prototype. Conversely, a distorted wave-model with the same value of ξ as its prototype is similar to this prototype if the pressure distribution in both is hydrostatic. This is not the case in breaking waves in shallow water, where some effects of the vertical accelerations must be taken into account due to the fact that the surface curvature is locally strong. Thus, the existence of similarity of the

surf in distorted models is not proved, and must be doubted to the extent that deviations from the long-wave approximations have a significant effect. Such effects are certain to be of importance for the details of the local flow patterns, but this is not necessarily the case for overall properties of the surf. The final check on this must of course be obtained empirically. In this regard it appears justified to draw the conclusion from the data presented above that the factor ξ is a good indicator of many overall properties of the surf zone, and may indeed be called a similarity parameter. The importance of this parameter for so many aspects of waves breaking on slopes appears to justify that it be given a special name. In the author's opinion it is appropriate to call it the "Iribarren number" (denoted by "Ir"), in honor of the man who introduced it and who has made many other valuable contributions to our knowledge of water waves.

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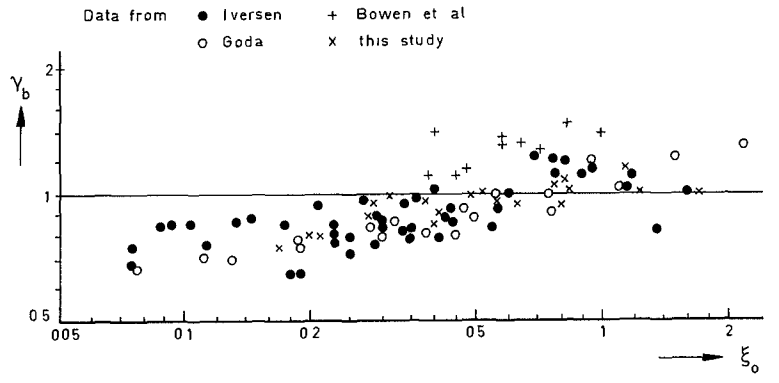

Figure 1 - Height-depth ratio at breakpoint vs. the similarity parameter.

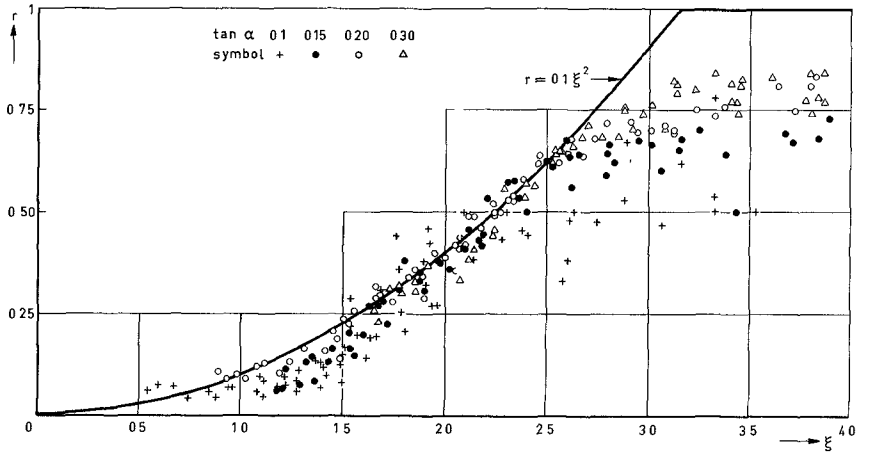

Figure 2 - Reflection coefficient vs. the similarity parameter.