CHAPTER 20

MODEL TESTS WITH DIRECTLY REPRODUCED NATURE WAVE TRAINS

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INTRODUCTION

Hydraulic model tests are still recognized as the best and in many cases the only tool, indeed, for investigations of design criteria for harbours concerning
a) the effect of wave disturbance on moored ships in harbour basins and at offshore terminals,
b) stability of structures and wave forces on structures.

Model tests with waves have until recently usually been made with regular waves varying the wave height, wave period, wave direction for each test run. An important improvement in the model technique has been the development of irregular wave generators, capable of generating waves directly from nature wave records.

The following aspects are presented below
1) A discussion on the methodology of wave model tests.
2) A method for direct reproduction of nature wave records.
3) A method for determining the incoming wave heights in a short wave flume with a reflecting structure and reflection from the wave generator paddle.

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I. DISCUSSION ON THE METHODOLOGY OF WAVE MODEL TESTS

1.1 Introduction

In wave model tests the following types of waves may be used as input to the models:

1) Regular Waves with constant heights and periods.

2) Irregular Waves
   2.1) Artificial irregular waves generated on basis of wave spectra.
   2.2) Wave trains reproduced directly from nature wave records. (See Section 2 in this paper).
   2.3) Wind flume waves. The waves are generated either by wind alone, or by wind superimposed upon paddle generated waves.

The first section of this paper presents a discussion of which of the above methods should be employed to produce the most reliable results for different types of wave model tests. To illustrate this, the fundamental physics of three common problems from harbour engineering practice are considered. It is demonstrated that the requirements to wave reproduction depend upon the type of problem considered.

1.2 Moored Ship

One of the motions of a moored ship exposed to an external force is supposed to correspond approximately to the following equation:

\[ m\ddot{x} + b\dot{x} + kx = F(t) \]  \hspace{1cm} (1)

In the general case where \( F(t) = 0 \) for \( t < 0 \) the approximate solution to equation (1) for \( \delta < 0.6 \) is the following:

\[ x(t) = -\frac{2\pi}{\omega_0} \int_0^t \frac{F(s)}{m} \left[ e^{\frac{b}{2m}(s-t)} \sin \frac{2\pi}{\omega_0} (t-s) \right] ds \]  \hspace{1cm} (2)

where \( \omega_0 = \sqrt{\frac{k}{m}} \), \( T_0 = \frac{2\pi}{\omega_0} \)

and the logarithmic decrement

\[ \delta = \frac{bT_0}{2m} \]  \hspace{1cm} (3)

For the free damped oscillator \( \delta \) may be found through

\[ e^{\delta t} = \frac{x_n}{x_{n+1}} \]  \hspace{1cm} (4)

where \( x_n \) and \( x_{n+1} \) are two consecutive maximum amplitudes.
The equation (2) shows that because of the damping, a time scale $T$ exists for the problem

$$T = \frac{T_0}{\delta} = pT_0 \quad (5)$$

$$p = \frac{1}{\delta} \quad (6)$$

This means that when the periods of the exciting force are not too different from the natural period $T_0$, which may be the case for moored vessels, it is roughly the maximum of the mean values over this time scale $T$ of the exciting force weighted together with the response function shown below, which determine the maximum oscillation and the critical mooring and fender forces of the vessel. In other words, if, for instance, $T$ is equal to two times the natural period of the ship oscillation the maximum oscillation and forces will be reached after only two consecutive waves of that particular period.

For this type of problems it therefore seems essential to reproduce the waves with a correct succession of waves as is believed done most effectively by directly reproduced wave trains.
1.3 Vertical Face Breakwater

As another example a vertical face breakwater may be considered.

The natural period of such a structure as a whole including the "elastic" element of the soil is usually an order of magnitude smaller than the mean wave period.

This case may be illustrated by the simple system

\[ mx + kx = F(t) \]  \hspace{1cm} (7)

\[ T_0 = 2\pi \sqrt{\frac{m}{k}} \]  \hspace{1cm} (8)

where the impulse \( F(t) \) may have characteristics as shown below in Fig. 3.

In this case, it is the characteristics of the individual wave such as the height, steepness, skewness, connections with the preceding wave, etc., which are the determining factors.

In this case the wave height distribution should be reproduced correctly, but also the shape of the waves is so important that waves generated in a wind-wave flume are required.
1.4 Rubble Mound Breakwater

A third example is the rubble mound breakwater where model tests have shown the same general stability of the armour layer on the seaward side when tested in irregular waves and in regular waves with the height equal to the significant height of the irregular waves, but different results for the requirements to the lower limit of the armour layer on the seaward side. Also the stability of the harbour side armour layer as function of crest elevation will be different for the two types of model waves.

In this case it seems likely that the wave height distribution is the determining characteristic, and either of the irregular wave generation techniques may be applied to this problem.

1.5 Conclusion

From the three cases considered it may be concluded that the directly reproduced nature wave trains (with superimposed wind for vertical face breakwater) apply in all cases, and that this method of wave generation is therefore superior to other known methods.

It should be emphasized that although a number of problems may be solved by this method, the important problem of obtaining and selecting representative wave data still remains to be solved.

A discussion of this problem, however, is beyond the scope of the present paper, and it does not give any limitations of the validity of the conclusion above.
2. REPRODUCTION OF NATURE WAVE TRAINS

2.1 General Principle

The wave generators at DHI and ISVA consist of a hydraulic servosystem, which gives a vertical wave paddle a horizontal translational movement. The position of the paddle is controlled by an electric signal from the "Wave Function Generator" (WFG). The control signal is generated on the basis of a punched tape record of nature waves. This digital record is converted to an analog electric signal, which after integration over time is fed into the servo system of the wave generator. The mean period of the model wave train is controlled by the reading speed of the punched tape, and the amplification in the WFG determines the wave heights.

2.2 Theory

In monochromatic shallow water waves (surface elevation $\eta = \frac{H}{2} \cos (\omega t)$) the horizontal particle velocity at a fixed point may be approximated with

$$u = \frac{C}{D} \cdot \frac{H}{2} \cos (\omega t) = \frac{C}{D} \cdot \eta(t)$$  \hspace{1cm} (9)

$C$ is the velocity of wave propagation, $D$ is the water depth.

Assuming that a similar proportionality between the surface elevation and the horizontal particle velocity is valid for irregular shallow water waves, also, a given wave train may be generated by moving the wave generator paddle with the velocity, $u$, and the paddle position, $x$, may therefore be written:

$$x(t) = \int_{0}^{t} \frac{dx}{dt} \, dt = \int_{0}^{t} ud\tau = \frac{C}{D} \int_{0}^{t} \eta(t) \, dt$$  \hspace{1cm} (10)

The general theory of generating monochromatic waves by means of oscillating vertical plate was described by Biesel, ref. [1]. He found that the transfer function from paddle amplitude, $x_O$, to wave amplitude is

$$\frac{a}{X_O} = \frac{2 \sinh^2 (kD)}{\sinh(kD) \cdot \cosh(kD) + kD} = k$$  \hspace{1cm} (11)

In order to see how the described method for wave generation agrees with Biesel's formula we shall apply the method to a monochromatic wave train $\eta = \cos(\omega t)$. The paddle position will be

$$x = \frac{C}{D} \int_{0}^{t} \cos (\omega t) = \frac{C}{D} \cdot \frac{a}{\omega} \sin \omega t$$  \hspace{1cm} (12)

so that

$$\frac{a}{X_O} = \frac{D \cdot \omega}{C} = \frac{D}{L} \cdot \frac{2\pi}{T} = \frac{2\pi D}{L} = k \cdot D = K$$  \hspace{1cm} (13)

where $L$ is the wave length, $T$ is the wave period, and $k$ is the wave number.
In Fig. 4 Biesel's transfer function $K$ and the simple transfer function $K_1$ are plotted against $\frac{D}{L}$. We see that $K_1$ is identical to $K$ for $\frac{L}{D} > 6$, i.e., in the shallow water region.

As a nature wave train most often has its energy distributed over a wide range of periods (frequencies), this method of reproducing waves will sometimes lead to a distortion of the wave train, because not all of the component waves are shallow water waves. Fig. 4 indicates that the heights of these short period components will be reproduced with too small wave heights in the model.

![Graph](image_url)

**Fig. 4.** Transfer functions from paddle movement to wave profile.

### 2.3 Testing of the Method of Reproduction

Tests have been run in the 0.6 m wide wave flume at ISVA. To reduce reflection from the end of the flume opposite the wave generator a wooden slope with an inclination of 1.17 was placed in the flume.

A total of 36 tests have been run varying wave height and period and the water depth. The prototype wave record was the same for all tests namely a record from the Danish North Sea Coast, recorded near Hanstholm during a storm in February 1973 by a waverider accelerometer buoy.

The two wave trains, model and prototype respectively were recorded simultaneously on an analog tape recorder, and later analyzed on a digital computer. The prototype record was taken from the WFG and the model record from a conductivity wave gauge in the flume, 10.16 m from the mean position of the paddle.
The recorded wave trains have been compared by means of cumulative wave height distribution, H-T-distributions (H versus T) and wave energy spectra with corresponding coherence function. A sample of each of these graphs is shown in Figs. 5, 6 and 7.

Fig. 5. Cumulative wave height distribution.

Fig. 6. H-T-distribution
In order to summarize all the results it was decided to calculate the percentage of the total prototype wave energy being reproduced with a coherence greater than 0.8. This quantity, which is of course an arbitrary measure for the goodness of the reproduction, has been plotted against $\frac{L}{D}$ in Fig. 8 and against $\frac{H}{D}$ in Fig. 9.
Fig. 8. Percentage of total prototype wave energy reproduced with coherence greater than 0.8.

Fig. 9. Percentage of total wave energy reproduced with coherence greater than 0.8.

It is obvious that the best reproduction is obtained with long and low waves. General limits for the validity of the applied method of reproduction cannot be given because the range of $\frac{L}{D}$ and $\frac{H}{D}$, for which the method produces reliable results, depends very much upon the accuracy that is needed for a particular problem.

Maximum accuracy (app. 80% of the total energy reproduced with a coherence greater than 0.8) is obtained with $\frac{L}{D} \geq 10$ and $\frac{H}{D} < 0.05$. 
A visual correlation between the input prototype wave train and the generated model wave train may be obtained from Fig. 10. It is possible to identify each of the longer waves, whereas details are blurred. This is probably because short period waves are not reproduced correctly and that they move with another celerity than the longer period waves.

![Prototype and model wave trains.](image)

**Fig. 10.** Prototype and model wave trains.

### 3. IRREGULAR WAVES IN SHORT FLUME WITH A REFLECTING STRUCTURE

#### 3.1 Introduction

Model tests with reflecting structures have always given rise to a lot of problems. In tests with regular waves one has had to use only the results from the first few waves before the re-reflection from the wave generator or other boundaries begin.

The problem with irregular waves may be solved in long wind wave flumes, in which the reflected wave energy is more or less suppressed by the wind stress. This is, however, a very expensive solution, and a method of controlling the waves in a short flume was therefore developed at DHI.

Irregular waves in short flumes require a reproduction method for irregular waves of a quality such as presented in Section 2, because even with the application of wind, the length of the flume is not great enough to allow the wind to change the waves generated by a wave paddle to "nature waves". The waves must in other words be born as nature waves.

When the waves hit a reflecting structure they are more or less re-lected and the reflected waves travel backwards in the flume and are partially re-reflected from the wave generator. This re-reflection of
energy implies that the resultant incident wave energy is larger than the energy generated initially. Hence it is essential to determine the reflection characteristics at both ends of the flume. The method developed for this is described in the following section.

3.2 Controlling Principle

As mentioned in Section 2 the reproduced wave heights may be controlled by changing the amplification factor in the wave function generator and the mean period may be changed by varying the reading speed of the punched tape. With fixed amplification factor and reading speed, tests were run for the following three situations (Fig. II):

1. Sloping beach providing almost complete absorption.
2. Completely reflecting structure (vertical plate).
3. Partly reflecting structure (object to be tested).

Fig. II. Test program with the same wave input.

In all these tests the zero crossing period $T_z$, the mean wave height $\bar{H}$ and the root mean square value of the water surface elevation $h$ are recorded at a number of positions along the flume.

The total potential energy of a wave motion is represented by the quantity $h^2_{\text{rms}} = h^2$. The kinetic energy may be assumed to be equal to the potential energy.

In the second test (with the fully reflecting plate) the function
$h_{rms}^2(x)$ is determined at a number of points at various distances $x$ from the plate ($x = 0$) where each "wave component" is reflected creating "standing waves". The function slopes off fairly rapidly with increasing values of $x$, because the various "components" have different wave lengths. At some distance from the plate the function $h_{rms}^2(x)$ reaches a stationary value (with little scatter) due to the continuous (non-discrete) nature of the spectrum.

If there were no re-reflection from the paddle the fully reflecting plate would give a stationary value of $h_{rms}^2$ equal to twice the value of $h_{rms}^2$ for the purely progressive waves advancing towards the flume beach in the first series. The actual stationary value is somewhat larger because of the re-reflection.

In the tests with the sloping face structure the reflection is less than 100% because of overspill and energy loss.

The influence of re-reflection may be estimated by the following calculations.

Calculation of reflection and re-reflection

$\bar{h}_2^2 = \text{initially generated energy}$

$\bar{h}_1^2 = \text{incoming energy (total)}$

$\bar{h}_r^2 = \text{reflected energy}$

$\beta_r = \text{reflection coefficient for energy}$

$\beta_{rr} = \text{re-reflection coefficient for energy}$

$\bar{h}_2^2 = \bar{h}_1^2 + \bar{h}_r^2$

$\bar{h}_r^2 = \bar{h}_2^2 - \bar{h}_1^2$

$\beta_r = \frac{\beta_{rr} \bar{h}_1^2}{\bar{h}_2^2 - \bar{h}_1^2}$

$\bar{h}_2^2 = \frac{1 + \beta_r \bar{h}_1^2}{1 - \beta_{rr} \beta_r} \bar{h}_2^2$

$h_{rms,i} = \sqrt{1 - \beta_{rr} \beta_r} h_{rms,g}$

<table>
<thead>
<tr>
<th>No reflection</th>
<th>Total reflection</th>
<th>Breakwater section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{h}_2^2$</td>
<td>$E_1$</td>
<td>$E_2$</td>
</tr>
<tr>
<td>$\bar{h}_g^2$</td>
<td>$E_1$</td>
<td>$E_1$</td>
</tr>
<tr>
<td>$\beta_r$</td>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\beta_{rr}$</td>
<td>$- \beta_{rr}$</td>
<td>$\beta_{rr}$</td>
</tr>
</tbody>
</table>
By measuring $E_1$, $E_2$, and $E_3$ the coefficients $\beta_r$ and $\beta_{rr}$ and hence the correction coefficient $\alpha = \frac{1}{\sqrt{1 - \beta_{rr}\beta_r}}$ may be calculated.

In tests with a partly sloping face breakwater performed in a 20 m long wave flume $\alpha = 10\%$ is a typical value.

Under the basic assumption that the re-reflection does not change the characteristics of the wave essentially, the other linear parameters for the wave height are also increased with the correction coefficient $\alpha$.

In order to check the reliability of this method extensive tests have been performed in a 70 m long and in a 20 m long programmed paddle wind-wave flume with wind at natural scale and the same generated wave trains. The test program contained detailed measurements of shock pressures on circular caissons and the results obtained were in good agreement in the two types of tests.

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REFERENCE