

CHAPTER 19

RANDOM WAVE SIGNAL GENERATION BY MINICOMPUTER

by

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ABSTRACT

The computer programmed implementation of a pseudo-random noise generator based on the use of a binary feedback shift register and an associated digital filter is described. Descriptive details are given for the compensation of the filter for the various dynamic transfer functions in the system. The random signal generator has been programmed both for a dedicated computer (NOVA 1200 with 4K core) which can carry up to 50 different spectra in core at any one time and for the EAI 640 with 16K core and disc storage which was used to investigate the spectral and statistical properties of the generated wave. JONSWAP spectra with 0.6 and 1.0 Herz centre frequencies were generated and observed at locations from 5 to 40 meters from the wave maker using a hydraulically operated machine in a partial piston and partial hinged flap mode in 0.9 meter of water. Measurements indicate that JONSWAP spectra can be closely matched in the flume. The 1.0 Herz spectrum is less stable as a function of distance, all spectra indicate a significant decrease of the spectral peak as a function of distance from the wave maker and the ratio of H_{max}/H_{av} is in general larger than that observed by Wiegell and Kukuk.

1.0 INTRODUCTION

The term 'random wave' is used in this paper to describe a water wave which is generated to simulate a natural sea state. Strictly speaking the method used for the purpose of this study produces a 'pseudo-random' time series and as such the series is deterministic and repetitive and can be restarted from the same initial conditions. The method is logically quite similar to the hard wired devices described by Ref. 1, 2 and 3. Because of the flexibility and power of a general purpose digital computer, it was possible to compensate for several dynamic transfer functions of the wave making machinery, to monitor the generated wave at the test site and to adjust the input spectrum in order to match the spectrum of the generated wave as closely as possible to the desired reference spectrum.

It is, however, not clear how accurately one needs to reproduce the specified spectrum during the synthesis of the

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natural sea state. The reference spectrum which is to be synthesized, whether it is an observed or a theoretical spectrum, is an approximation in itself. Nevertheless, it is thought to be important to know how accurately one can match the reference spectrum by the simulation especially by using simple wave making machinery and in the absence of costly wind generating equipment.

The wave flume and the wave maker that were used in this study are described in Fig. 1. The wave maker is powered by a single hydraulic piston which is attached to the rigid board through two separate linkages on top and bottom. Individual settings on the linkages permit the board to operate either in a purely piston or a purely hinged-flap mode. Alternatively a combination of piston and flap motion is possible. For the experimental work described here, the settings were such that a 0.3 meter displacement of the top linkage corresponded to 0.173 meter displacement on the bottom. All tests were carried out with 0.9 meter of water depth in the flume. The partial flap mode of the wave board was found helpful in enhancing the generation of the higher frequencies.

The beach of the wave flume is adjustable and the top of the beach is set to be at the still water level. Reflections were measured along the flume and were found to be 12% at 1.0 Herz, from there going down to 3% at 0.6 Herz and then increasing again to 13% at 0.28 Herz.

The wave probe is a capacitive proximity gauge (Ref. 4). Its linear range of operation is restricted to 1/4 of the diameter of the outside guard ring. For the reason that this instrument has no physical contact with the water surface, it is ideally suited for wave measurements in the laboratory. However, the range over which this instrument was used exceeded the recommended linear portion and therefore it was necessary to linearize the data. This was executed by a computer program using a table look-up procedure.

Figure 2 describes the digital computer equipment which was harnessed to the project. The intent was primarily to use a small minicomputer like the NOVA 1200 as a dedicated signal generator which may then be used for all coastal studies requiring the simulation of random waves. This machine with 4000 words of core, a slow paper tape reader, a typewriter and one channel of digital to analog conversion cost approximately \$8000. It was programmed in Assembly language and can store 50 different spectral generating functions which could be selected by switch settings.

However, for the purpose of the study described in this paper a more sophisticated program was required which not only generated a random wave but also monitored the waves at the selected location in the flume and carried out the data

conversion and analysis that was required. The digital processor used for this purpose was the EAI 640 minicomputer with 16000 words of core. Figure 2 shows only those peripheral devices that were actually used in this project. Programming on the EAI 640 for this job was executed in Fortran under an in-house developed operating system which permitted the creation of separately compiled job functions which were data compatible with any other job program through a common disc storage medium. Through the use of this system a highly modular approach to programming and computer use was possible which allowed the execution of large jobs inspite of limited core memory availability. The EAI 640 program was equipped also to compute and punch on paper tape the digital filter function which may then be loaded into the NOVA 1200. This tape is only 0.4 meter long.

2.0 THE TRANSFER FUNCTIONS BETWEEN THE POINT OF SIGNAL GENERATION AND WAVE MEASUREMENT

Figure 3 summarizes the various factors that affect the wave generation signal and which must be accounted for if a wave is to be produced at some distance from the wave maker with a specified variance spectral density. This consideration applies whether the signal source is an digital or an analog random noise or is derived from an analog or digital tape of a prototype wave recording. In either case, the signal will first be affected by the analog low-pass filter which is inevitably used to smoothen out or band limit the raw noise. The low-pass filter used for this system is a third order Butterworth and is described by Ref. 5. It has a cut-off frequency set to 2 Herz.

The dynamics of the electro-hydraulic servo system of the wave maker depends on the servo valve and the design of the other electronic and hydraulic components and will have to be measured for each individual wave maker. In addition it is possible that the depth of water, the particular settings of the electronic servo loop-gain and the mechanical gains of the linkages can influence the dynamic behaviour of the system. The particular machine used for this study exhibited a first order low-pass characteristic with a cut-off at 1.5 Herz which did not change measurably when the water level was dropped from .9 to .6 meters.

The next transfer function of importance describes the relationship between the displacement of the wave board and the corresponding wave amplitude. This function is described by Ref. 6 and depends on the depth of water, the wave period and the mode of the wave board operation, i.e. piston or flap mode, or a combination mode. If the board is used in the piston mode, then for deep water and short period conditions the wave height is typically twice the stroke length of the board. For longer periods of the wave, that is lower frequencies, the stroke amplitude must be significantly amplified. The theoretical

relationship describing this transfer function, as given by Ref. 6, is based on a first order wave theory. Experimental verification agrees with this function to within 5%.

As the wave propagates down the flume, its spectrum changes. The higher frequencies appear to be dissipated and the peak of the spectrum tends to shift to a lower frequency. This phenomenon is probably due to a nonlinear transfer of energy between adjacent bands of frequencies. It is not possible to describe this transformation in terms of a simple multiplying function. Experimental measurement and adaptation of the input spectrum was used successfully to compensate for this deficiency.

3.0 THE CHOICE OF THE THEORETICAL SPECTRUM

During the initial experimental work in connection with this project, it was first decided to use the Pierson-Moskovitz spectrum for a fully developed sea (Ref. 7). Visual assessment of the waves failed to convince the observers that the result looked realistic. As a consequence, it was decided to use the more recent JONSWAP spectrum (Ref. 8, 9) with parameters $\gamma = 7$, $\sigma_a = .07$ and $\sigma_b = .09$. It was then felt that the waves generated by these specifications did resemble a natural sea state more closely. Occasional breaking waves were observed along the entire length of the flume.

4.0 THE COMPUTER PROGRAMS

Figure 4 summarizes the various computer programs that were used for the experimental work. Initially, the program *SPECTH* was called to generate a frequency discrete representation of the JONSWAP spectrum with the specified parameters.

The program *RWGEN1* has the 'real time' capability to generate the required analog signal for the wave maker and to provide data acquisition as well. However, on first entering this program, various initializations are being executed by it, including the derivation of a generating function which is then used for the purpose of digitally filtering the pseudo-random noise according to the specified reference spectrum. This digital filter function is also compensated at this stage for the various transfer functions described in Section 2.

While *RWGEN1* is in the 'on-line mode', analog data from the wave probe are read into core and recycled so that only the last 2048 samples of the water surface variation are being recorded. On depressing a switch, the program exits and subsequent programs are now called to linearize the data, to remove the mean value and to perform a variance spectral density analysis with 0.03 Herz resolution. The results are displayed for visual inspection and comparison, as a result of which the user may decide to modify the input spectrum to *RWGEN1*. This modification has been done either

by recalling *SPECTH* and specifying new and different spectral parameters or by using a differencing algorithm which compares the reference spectrum with the measured spectrum and creates from this a new input spectrum. This latter method worked quite well for spectra with peaks at 0.6 Herz but failed completely when the peak was at 1 Herz.

4.0 THE ALGORITHM FOR THE GENERATION OF THE RANDOM WAVE

In general terms, the method used to generate a 'random' signal with a specified variance spectral density consists of a random number generator algorithm which is followed by a digital filter. The digital filter is represented by its 'impulse response' function which is numerically convolved with the output of the random number generator. This method has been described by Ref. 1, 2 and 3 although the former uses analog methods to solve the convolution integral.

The random noise source for this system is a programmed implementation of the 'linear feedback shift register' which is a well known pseudo random noise generator (Ref. 3, 10, 11, 12). This register is shown symbolically in Fig. 5. Its output consists of a string of 'ones' and 'zeros' which are generated by a feedback arrangement from at least two cells of the register and which are summed by a 'modulo 2' adder (exclusive OR). Following this, all cells of the register are shifted to the right by one position with the first bit in the sequence being discarded. The order of the 'ones' and the 'zeros' is random in the sense that the outcome of each cell value is uncorrelated with the outcome of the adjacent cells at least up to the cycle length T_n of the sequence. This implies that the variance spectral density of the sequence is essentially flat over the range from $1/T_n$ to $1/(2\Delta T)$ where ΔT is the stepping interval of the shift register.

The binary number sequence of the generator is repetitive and the period of repetition depends on the particular choice of the feedback cells. Only for a few combinations of these cells will the register generate the maximum cycle length, that is to say that it will produce all possible patterns of 'ones' and 'zeros' including the 'all ones' pattern for the first n cells. The 'all zeros' pattern is not possible. It is important to realize that the flat spectral density of the binary sequence is guaranteed only if the sample length over which the spectral analysis is carried out consists of an integer multiple of the maximum cycle length. This means also that the output of the digital filter, which is driven by the binary sequence, only meets its spectral requirements over the same interval. Consequently, experiments using these random waves should be restricted to intervals corresponding to integer multiples of the maximum cycle length of the shift register. Figure 5 lists the cycle lengths of different

feedback arrangements and the corresponding periods. It may be seen that the choice is not quite as large as one might wish. This deficiency can be overcome by using a 4-point rather than a 2-point feedback (Ref. 10).

The register length of 65 cells is matched to the length of the digital filter function with which the binary random number sequence is convolved. However, the effective length of the shift register that participates in the regeneration procedure is only n cells. The remaining $(65-n)$ cells serve as a delay line memory.

The digital filtering operation consists now of multiplying the value of each cell of the shift register with a corresponding weight in the filter function. Since the cell values are but one or zero, this operation requires only selected addition, an operation suitable for computers without a hardware multiply feature such as the NOVA 1200. The weights are scaled so that the summation will, at no time, result in an overflow condition.

The output of the digital filter is fed into a digital to analog converter which is then followed by the low-pass analog filter with a 2 Herz cut-off frequency (Ref. 5).

The initial condition of the binary shift register is of significance in the formation of subsequent sequences. In order to insure that the entire register of 65 cells starts off with a bit pattern that belongs to the set of all patterns in the maximum cycle length, it is best to run the generator for at least 65 steps, where the first n cells may have any starting condition except all zeros.

5.0 THE COMPUTATION OF THE DIGITAL FILTER

The digital filter consists of a string of 65 integer numbers which are the time discrete representation of the impulse response function of the equivalent analog filter. The digital filter function is obtained by the Inverse Fourier Transform of the required amplitude spectrum and an associated phase spectrum.

The amplitude spectrum is obtained directly from the reference variance spectral density which is first compensated for the various transfer functions described in Section 2. Subsequently, the square root may be applied to the variance and finally the function must be discretized by interpolation over the range from 0 to some upper frequency limit. This latter operation was found to be more important than initially expected for the following reason.

Since the Inverse Fourier Transform produces nearly twice

as many points as are contained in the input spectrum, and since the digital filter function must finally be made up of 65 points, it is necessary to represent the input function by 33 equally spaced points including one at 0 and one at the maximum frequency f_m . However wave spectra, especially the JONSWAP spectra, are very peaked. It was found that one of the 33 points of the discrete representations must fall on the peak of the input spectrum or else the output from the digital filter will not possess a spectrum with a peak frequency at the same location as the input spectrum, but rather at the frequency with the largest discrete representation.

The upper frequency limit of the discrete representation determines the stepping rate of the signal generation which is $\Delta T = 1/(2 \cdot f_m)$. The dilemma is this: If f_m is chosen too large, then the stepping interval is agreeably small but the discrete representation of the spectrum by the 33 points is too coarse to preserve some of the finer details. If the f_m is made smaller to improve the discrete representation of the spectrum, then the stepping interval is too coarse and inspite of heavy analog filtering the output signal will not be smooth. A compromise is required and it was found quite satisfactory to use the following algorithm: Find the peak frequency f_p and divide this by 14. This gives the frequency increment. Multiply this by 32 to get the upper frequency limit, f_m . With this, $f_m = 2.286 \cdot f_p$.

A phase spectrum must be assumed and there are an infinite variety of choices. The simplest phase spectra are either 90° or 0° for all frequencies. The former results in an anti-symmetric and the latter in a symmetric impulse response function. The anti-symmetric function is preferable since it is less likely to contain a bias for truncated functions and for this reason the 90° phase is used in this program. Other phase spectra, such as the random phase spectra, produce a variety of different random wave sequences. However, the impulse response functions which result from the use of random phases do not in general converge to zero at either end and this can result in excessive start-up transients.

Since the Inverse Fourier Transform is obtained from the co and quadrature spectrum, rather than the amplitude and phase spectrum, the equivalent operation to selecting the 90° phase spectrum is the clearing of the co spectrum and the equivalencing of the quadrature spectrum with the amplitude spectrum.

6.0 DESCRIPTION OF EXPERIMENTAL RESULTS

Figure 6 illustrates the generation of a JONSWAP

spectrum with 0.6 Herz centre frequency. For this sequence of tests a difference algorithm was used to improve the generation of the spectrum. All measurements were made 36 meters from the wave maker.

To start with, the reference spectrum was compensated and used to generate the random wave. After 15 minutes of operation it was assumed that steady state prevailed in the flume and 2048 samples were taken. The measured and the reference spectra were compared in the right hand column of Fig. 6. The measured spectrum was found broader than the reference spectrum and a new input spectrum was computed by the formula: $\text{New Input Spectrum} = \text{Old Input Spectrum} + \alpha \cdot (\text{Reference Spectrum} - \text{Measured Spectrum})$ where $\alpha = 0.5$ is a convergence factor. The new input spectrum was compared to the reference spectrum on the left hand column of Fig. 6. It was clearly established that the digital filter function for a JONSWAP spectrum with a 0.6 and 0.8 Herz peak frequency could be improved by a feedback arrangement based on a difference algorithm.

As a further test, a signal for a fully developed JONSWAP spectrum at 0.6 Herz was generated and the water surface variation was observed and analysed at intervals along the flume from 5 to 40 meters from the wave board at 5 meter intervals. This is illustrated in Fig. 7. The spectra exhibited variation in the peak frequency and the magnitude of the spectral peak.

Figure 8 illustrates the attempt to shape the input spectrum for a 1 Herz JONSWAP spectrum. On the left hand column of this figure, the solid line shows the shape of the desired JOHNSWAP spectrum. The top figure gives also the shape of the input spectrum which had to be made considerably more narrow. The middle figure shows the same input spectrum after its discrete representation by 33 points prior to the Inverse Fourier Transform. Because of the previously explained limitation in discretizing the input spectrum over the interval from zero to f_m the input spectrum cannot be described internally by greater definition than is shown in Fig. 8.

The result of the signal generation was measured again at 36 meters from the wave board and is shown in the bottom left illustration in Fig. 8. The measured spectrum exhibited a marked shift to the left. On the right hand side of Fig. 8 it is shown that one may compensate for this shift at the 36 meter flume location by creating a new input spectrum which has been artificially shifted to the right.

The same 1 Herz JONSWAP spectrum had been studied at different locations along the flume as shown in Fig. 9. In

this illustration it is quite apparent that the process of a shift in the spectral peak is more likely a matter of a transfer of energy from the original to adjacent frequency bands but predominantly in the direction of lower frequencies. In this manner the spectrum appeared to spread out and shift.

Table I summarizes various statistical properties of the random waves as observed at the test locations in the flume. The input spectrum for the 0.6 Herz wave had a $\gamma = 8.0$ (i.e. the ratio of the peak value in the JONSWAP spectrum to the corresponding peak in the Pierson-Moskovitz spectrum). It will be noticed that the γ -value decreased with increasing distance from the wave board. There also appeared to be a decreasing trend in the peak frequency. There was no significant change in the RMS-value. Other statistical wave parameters showed no significant trend but indicated inconsistency over the length of the flume by as much as $\pm 14\%$. The ratios $H_{\max}/H_{av}$, $H_{\max}/H_{1/3}$, and $H_{1/3}/H_{av}$ were observed by Ref. 13 to cover the following ranges 1.9-2.6, 1.29-1.91 and 1.37-1.85 respectively. It will be noticed that the comparable parameters for the simulated wave were generally higher.

Similar observations were made for the 1.0 Herz JONSWAP spectrum which are shown in Table II. It also appeared that the RMS-value decreased significantly with distance from the wave board while the $H_{\max}$ value tended to increase.

The input spectrum for this example had a $\gamma = 20$ and it may be seen that this extreme peak had dropped to $\gamma = 10$ within 5 meters from the board.

7.0 CONCLUSIONS

To insure consistent spectral distributions, it is considered important that experimentation with pseudo-random noise sources are carried out over a period which corresponds to an integer multiple of the maximum cycle length of the generator. Improved choice of cycle lengths could be obtained if a 4-point rather than a 2-point feedback shift register is used. Improved definition in the calculation of the digital filter is recommended and could be obtained if the shift register and the digital filter would be expanded from 65 to 129 elements.

Further details of the study including computer programs will be included in a future report.

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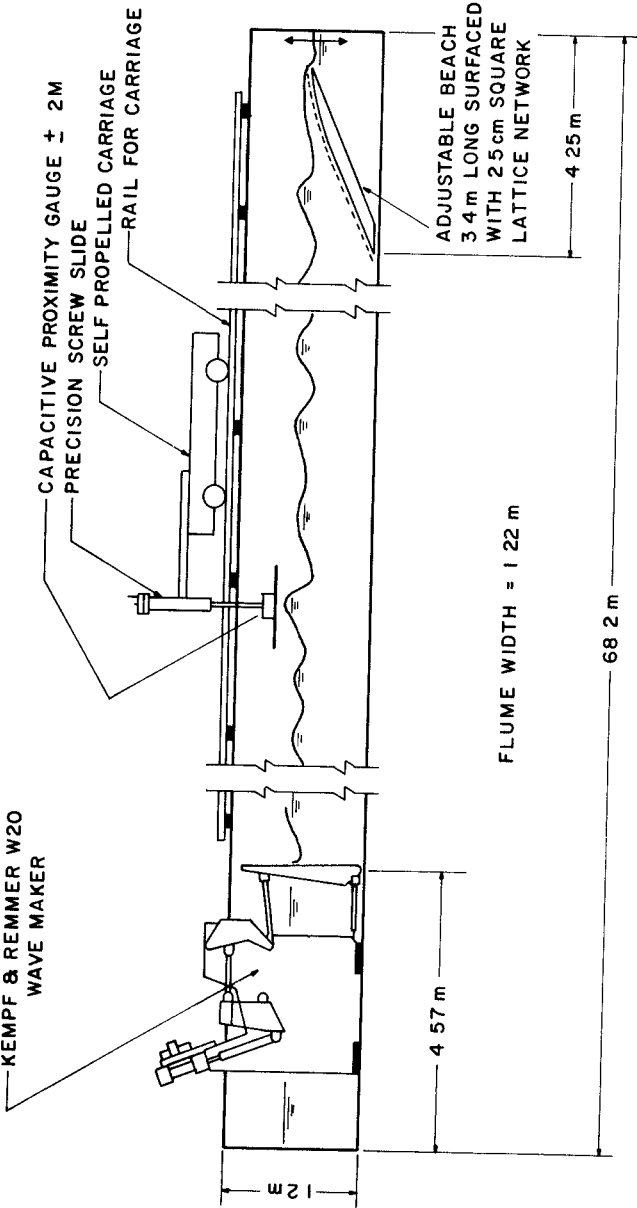


FIG 1
STEEL WAVE FLUME
AT HYDRAULICS LABORATORY
N.R.C. - OTTAWA

SYSTEM CONFIGURATION OF THE EAI 640 COMPUTER AND NOVA 1200 FOR THE GENERATION OF RANDOM WAVES

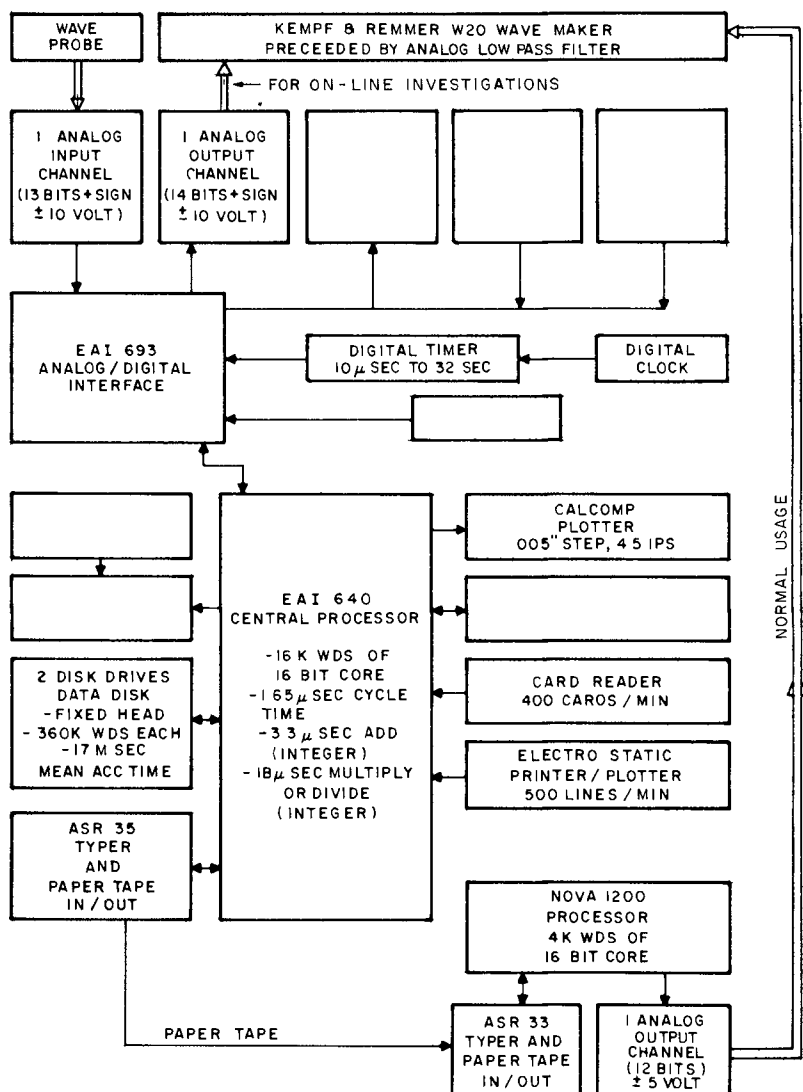


FIG 2

TRANSFER FUNCTIONS FROM SIGNAL SOURCE TO TEST SITE

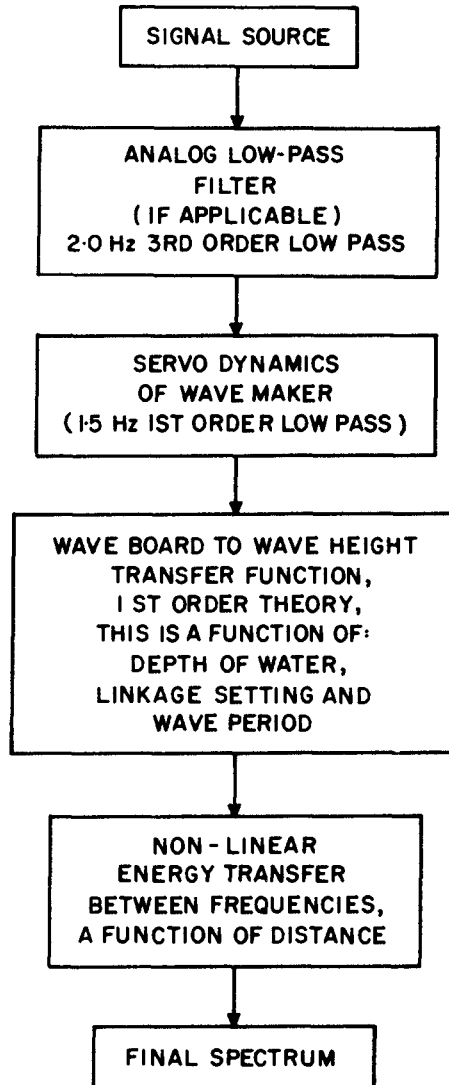


FIG 3

FLOW CHART OF
COMPUTER OPERATIONS FOR
RANDOM WAVE GENERATION

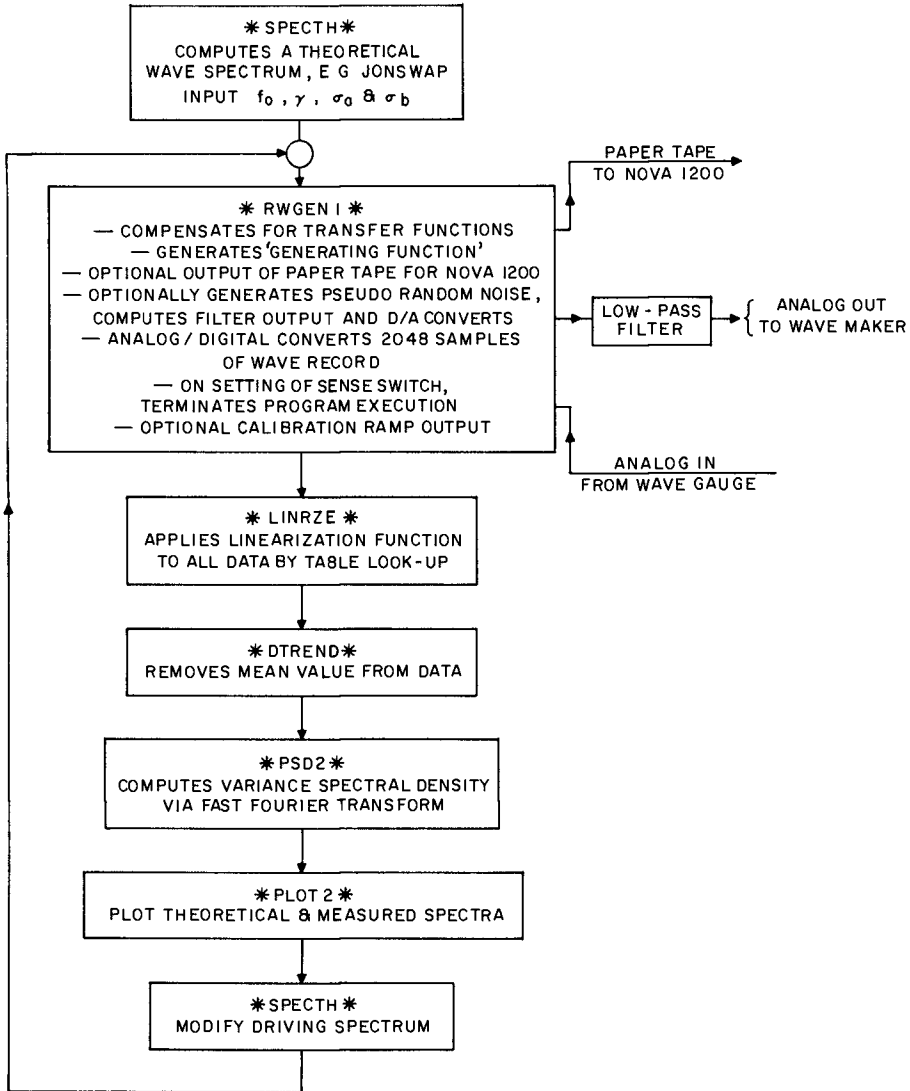
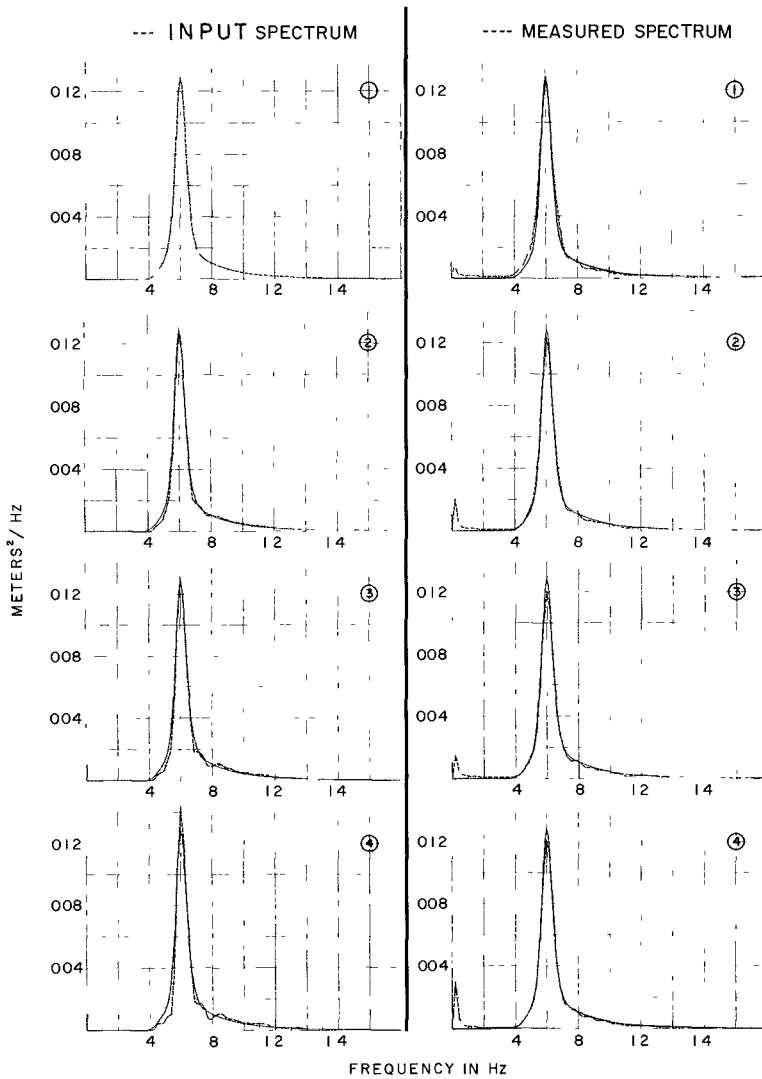


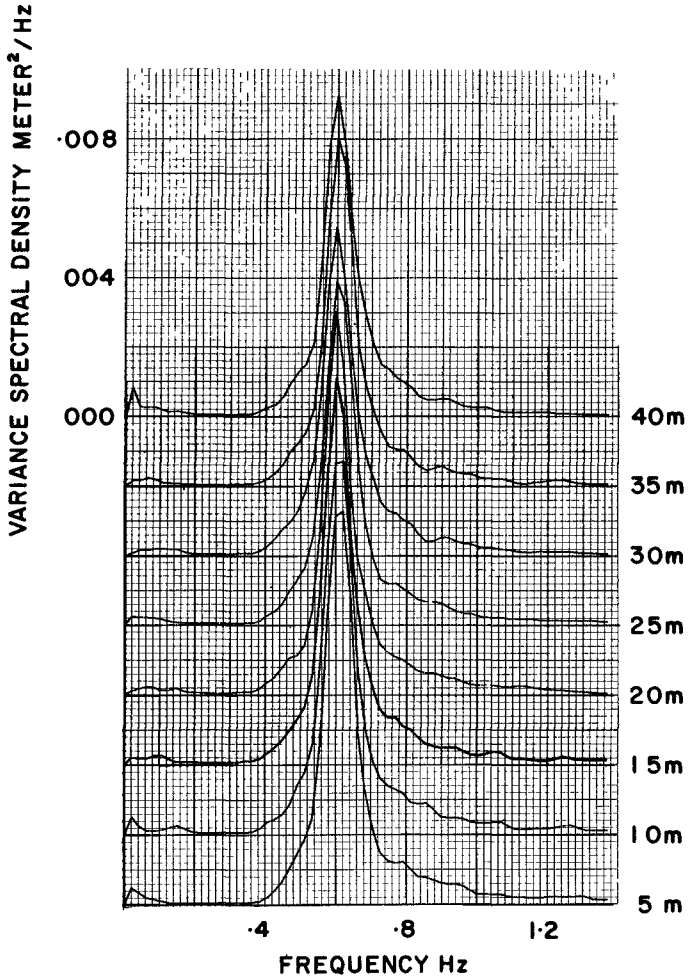
FIG 4

— REFERENCE SPECTRUM JONSWAP WITH
 $\gamma = 7.0$, $\sigma_a = 0.07$, $\sigma_b = 0.09$ AT 36 METERS FROM WAVE BOARD



IMPROVING THE SPECTRUM BY
 FEED BACK

FIG 6



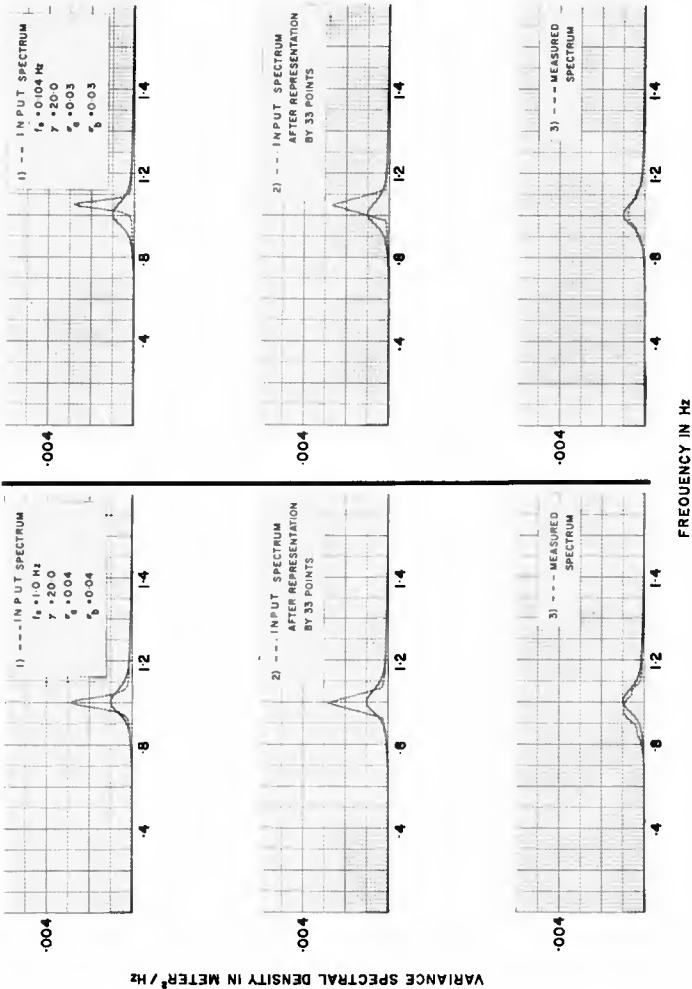
THE PROPAGATION OF THE SPECTRUM

AS MEASURED FROM 5 TO 40 METERS FROM THE WAVE BOARD,

INPUT SPECTRUM : JONSWAP WITH

$\gamma = 8.0$, $\sigma_a = 0.04$, $\sigma_b = 0.08$

FIG 7

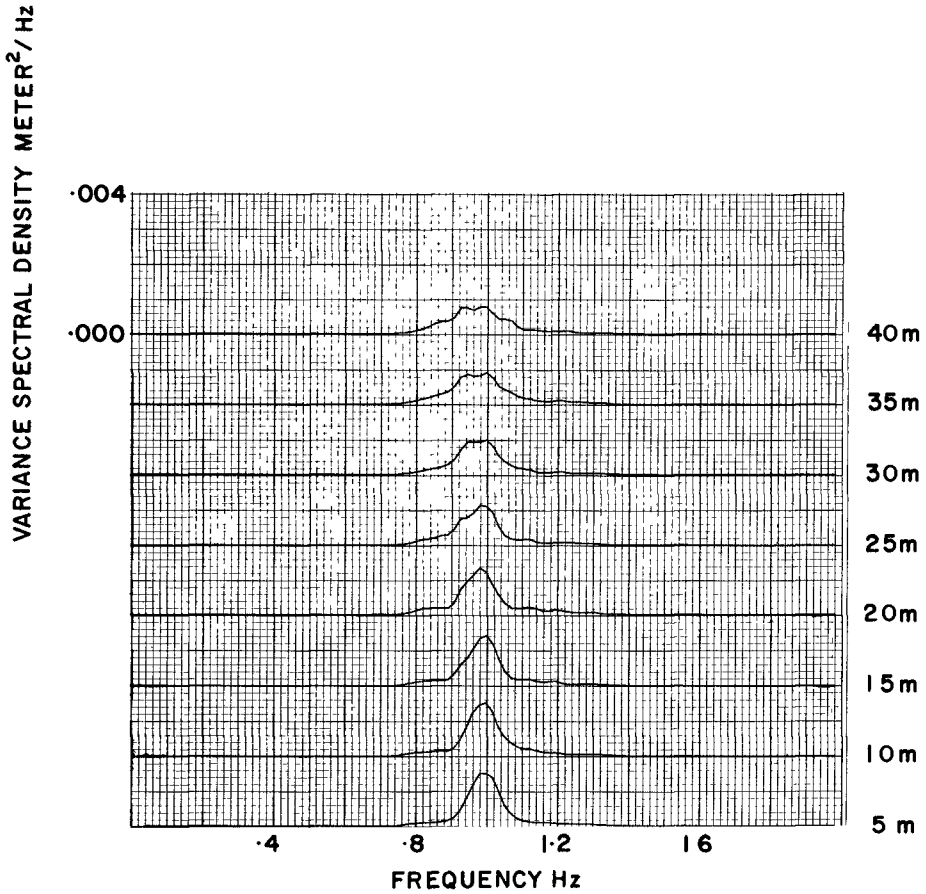


GENERATION OF A JONSWAP SPECTRUM WITH

$f = 1.0 \text{ Hz}$ AT 36 METERS FROM WAVE BOARD AND

$\gamma = 20.0, \sigma_a = 0.07, \sigma_b = 0.09$

FIG. 8



THE PROPAGATION OF THE SPECTRUM

AS MEASURED FROM 5 TO 40 METERS FROM THE WAVE BOARD,

INPUT SPECTRUM - JONSWAP WITH

$$\gamma = 20.0, \quad \sigma_a = 0.04, \quad \sigma_b = 0.04$$

FIG 9

TABLE I

WAVE PROPERTIES AS A FUNCTION OF DISTANCE FROM WAVE BOARD FOR AN
INPUT JONSWAP SPECTRUM WITH

$$F_{\text{peak}} = 0.6 \text{ Hz}, \gamma = 8.0, \sigma_a = 0.04, \sigma_b = 0.08$$

Y	f_o	γ	RMS	$H_{1/3}$	$H_{1/10}$	H_{max}	H_{av}	H_{rms}	T_{av}	T_{max}	$\frac{H_{\text{max}}}{H_{\text{av}}}$	$\frac{H_{\text{max}}}{H_{1/3}}$	$\frac{H_{1/3}}{H_{\text{av}}}$
5	.611	6.75	.039	.14	.18	.25	.09	.10	1.55	1.52	2.79	1.8	1.6
10	.610	6.35	.039	.14	.18	.26	.09	.10	1.54	1.59	2.89	1.85	1.6
15	.603	6.23	.040	.14	.18	.25	.09	.10	1.57	1.62	2.50	1.8	1.6
20	.601	6.08	.039	.14	.18	.28	.09	.10	1.61	1.63	3.11	2.0	1.6
25	.605	5.65	.039	.13	.18	.24	.08	.09	1.56	1.57	3.00	1.84	1.63
30	.600	5.17	.038	.13	.18	.22	.08	.09	1.58	1.59	2.85	1.80	1.63
35	.604	5.68	.039	.13	.18	.27	.08	.09	1.54	1.61	3.37	2.07	1.63
40	.599	4.99	.037	.13	.16	.21	.08	.09	1.57	1.62	2.60	1.60	1.63

TABLE II

WAVE PROPERTIES AS A FUNCTION OF DISTANCE FROM WAVE BOARD FOR AN
INPUT JONSWAP SPECTRUM WITH

$$F_{\text{peak}} = 1.0 \text{ Hz}, \gamma = 20.0, \sigma_a = 0.04, \sigma_b = 0.04$$

Y	f_o	γ	RMS	$H_{1/3}$	$H_{1/10}$	H_{max}	H_{av}	H_{rms}	T_{av}	T_{max}	$\frac{H_{\text{max}}}{H_{\text{av}}}$	$\frac{H_{\text{max}}}{H_{1/3}}$	$\frac{H_{1/3}}{H_{\text{av}}}$
5	.989	10.08	.016	.051	.073	.086	.035	.039	1.00	.98	2.45	1.68	1.46
10	.993	10.33	.015	.056	.070	.092	.034	.039	1.00	.98	2.70	1.64	1.65
15	.993	9.81	.015	.055	.070	.089	.032	.037	1.01	1.01	2.78	1.62	1.72
20	.983	8.69	.015	.055	.072	.105	.031	.037	1.00	.95	3.39	1.91	1.77
25	.987	7.56	.014	.053	.072	.098	.030	.036	1.01	.95	3.27	1.85	1.77
30	.995	6.83	.014	.052	.072	.109	.029	.035	1.01	.96	3.76	2.10	1.79
35	.995	6.37	.014	.054	.075	.106	.030	.036	1.03	.99	3.53	1.96	1.80
40	.990	5.20	.013	.051	.073	.104	.028	.034	1.00	1.03	3.71	2.03	1.82

Y = Distance from Flap in Meters.

T_{max} is the period of H_{max} in seconds.