CHAPTER 9

A TRANSMISSION LINE WAVE HEIGHT TRANSDUCER

* C.M.G. Zwarts

Summary

A simple but accurate wave transducer has been developed for the measurement of waves and tidal levels on inland and coastal waters. It consists of a tunnel diode oscillator, using a transmission line to sense the water-level. The output voltage consists of a square wave, with a period linearly proportional to the water-level. In addition an analog output signal is provided. The quasi-static accuracy of the instrument is equal to or less than 0.2%.

Figure 1

Prototypes undergoing field tests in Lake Ontario.

* Instruments Section, National Research Council of Canada, Montreal Road Laboratories, Ottawa, Ontario, Canada.

Introduction

Waves have been measured, on inland and coastal waters, with various types of transducers of which the more common are:

Pressure cell, anchored to the bottom, measuring the pressure variation due to waves at the surface.

Accelerometer, mounted in a floating buoy, giving the surface-acceleration of the wave.

Wave-staff, using to sense the water-level: ohmic contacts at small regular intervals; parallel resistance wires; insulated wire as capacitance.

In this paper a wave transducer which needs a mounting frame, like the resistive and capacitive type wave-staff, is described. The instrument exploits the large change in dielectric constant from air to water (1:80)[1], as in the capacitive wire wave-staff, but in a different manner. It is the variation in electromagnetic impedance at the air-water interface, caused by the difference in dielectric constant, which is used to obtain the wave height information. An electromagnetic wave, directed towards the water surface with a transmission line eliminating focussing problems, will reflect very efficiently from the water surface. The time it takes for the wave to travel from the source to the water-level and back to the source is a direct measure for the water-level relative to the source. The method developed to extract the wave information from the time delay is however essentially different from the conventional time domain reflectrometry or echo-range finding methods, in principle and embodiment. Instead of directing a voltage step or pulse down the line and measuring the time delay, an oscillation is set up in the line, with the length of line directly determining the frequency. The oscillation is maintained with a simple tunnel diode circuit[2,3,4] over the input terminals of the transmission line. The tunnel diode is operated as a bistable device, being changed states by the reflected wave from the water surface. Since the voltage reflects with phase reversal on the water surface, the tunnel diode is triggered alternatively into both dynamicly stable states; to produce over the input terminals of the transmission line a square wave oscillation with its period being linearly related to the position of the water surface. (See Figure 2.)

The ideal transducer design, in which there is an immediate conversion from the measurand into a frequency, is closely approached by the "Transmission Line Wave Height Transducer" described in this paper. This near optimum design gives two distinct advantages, namely:

1) calibration is permanent, being nearly independent of environmental

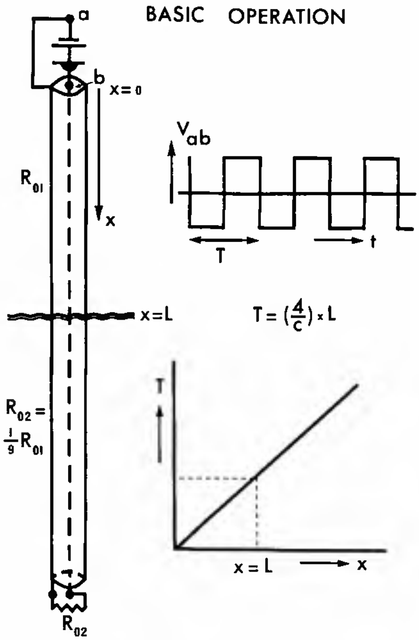

Figure 2

Top and Bottom Assembly

Tubular Transmission Line

Figure 3

and other factors.

2) the period modulated output signal is ideal for digital processing, recording, telemetry methods or transmission by cable without loss of accuracy.

The transmission line consists of two concentric aluminum tubes, approximately 1 and 2 inches in diameter, assembled in sections of ten-foot lengths. The outer tube is perforated with a regular distribution of holes, Figure 3. The rigid coaxial tubular transmission line is extended at the top with a twenth-foot length of coaxial cable to prevent the frequency of oscillation becoming too high with near complete immersion. The frequency of oscillation, in the MHz-range, is scaled down considerably with logic circuitry to make period measurement and recording more convenient and accurate. The prototypes had a down count factor of $N = 5 \times 2^{14}$, whereas the commercial instrument has $N = 2^{13}$.

The output signal consists of a square wave oscillation of which the period gives the water-level or wave profile. In some cases however an analog output signal will be required. An accurate period-to-voltage converter was designed and added to the instrument to make the wave gauge useful for both digital and analog data processing and recording. The block diagram of the complete transducer is shown in Figure 4.

BLOCK DIAGRAM

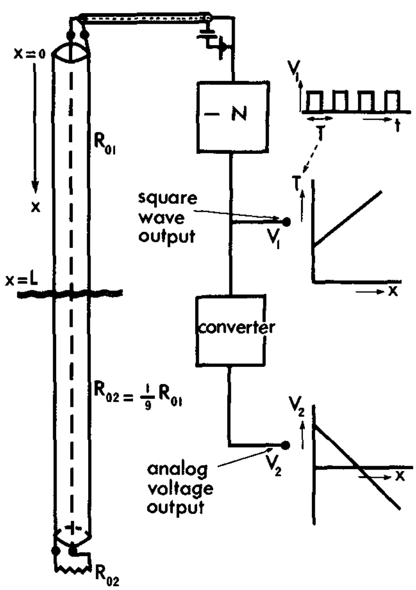

Figure 4

Analysis of Circuit Operation Using a Graphic Method

The simplified transducer circuit is shown in Figure 5. The tunnel diode circuit is connected at $x = 0$ to the transmission line, which is terminated with R_{O2}. The resistance R_{O2} represents the characteristic impedance of the transmission line in water for $x \geq L$.

The v - i characteristic curve for a tunnel diode is shown in Figure 6. The tunnel diode has a negative resistance region for $V_p < V < V_v$. The operating point will be unstable, if the load-line intersects this region, and with suitable conditions steady oscillation occurs.

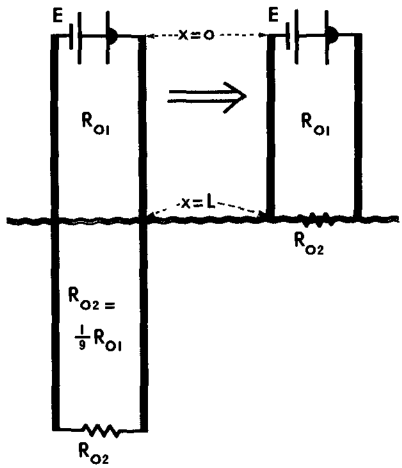

Figure 5

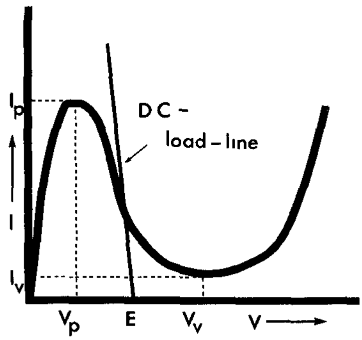

Figure 6

The problem essentially consists of finding the periodic solution

$$V = f_1\left(t - \frac{x}{u}\right) + f_2\left(t + \frac{x}{u}\right), \qquad \text{at } x = 0$$

of the linear wave equation

$$\frac{\partial^2 V}{\partial x^2} - \frac{1}{u^2} \cdot \frac{\partial^2 V}{\partial t^2} = 0$$

f_1 is an arbitrary function of the argument $(t - \frac{\hat{x}}{u})$ and represents a wave travelling with velocity u to the right. Similarly f_2 represents a wave travelling to the left. To solve this problem analytically is not easy since the tunnel diode introduces a non-linear boundary condition at x = 0. There is however a very elegant graphic method to solve this kind of problem, its major feature being the ease with which it handles non-linear boundary conditions.[5] This method allows the wave form at x = 0 for $t \geqslant 0$ to be determined in a simple manner.

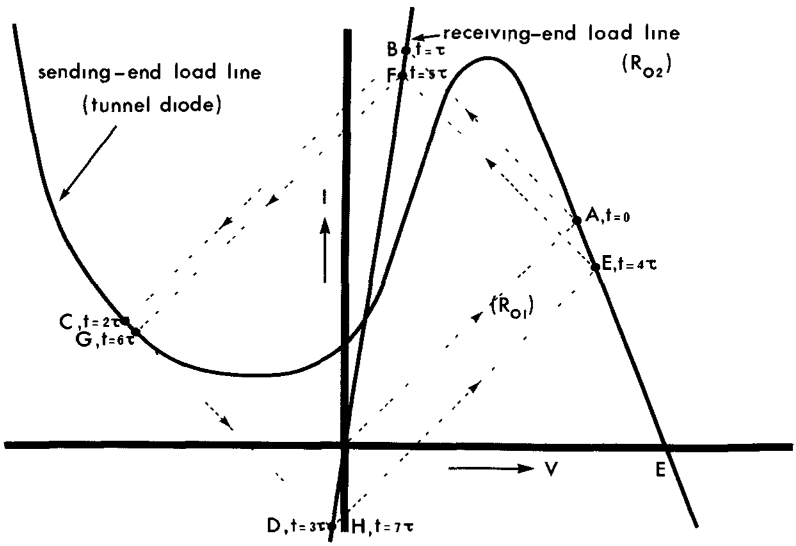

Figure 7

The graphic method, in our case, consists of drawing the load lines, presented by the sending- and receiving-end impedances, as they are seen by the transmission line, in the i-v-plane, as shown in Figure 7. The intersection of the sending-end load line with the conductance line of the transmission line, point A, gives the initial voltage and current at t = 0, travelling down the line. Projecting, starting from this point, alternatively on the receiving- and sending-end load lines give the current and voltage at intervals τ for x = 0 and x = L alternatively. Continuing this process of projections will eventually give the steady state solution. One notices that, for this particular example, the approximate steady state values are reached after only two rounds of projections, since continued projection will give the same points E, F, G, and H. The steady state values for the voltage for x = 0 and $t \geqslant 0$ can now be simply read from Figure 7, and are presented by the points E and G. The resulting voltage wave-form at the sending-end, x = 0, is therefore a square wave, with a 50% duty cycle, having a slightly different positive and negative amplitude, Figure 8.

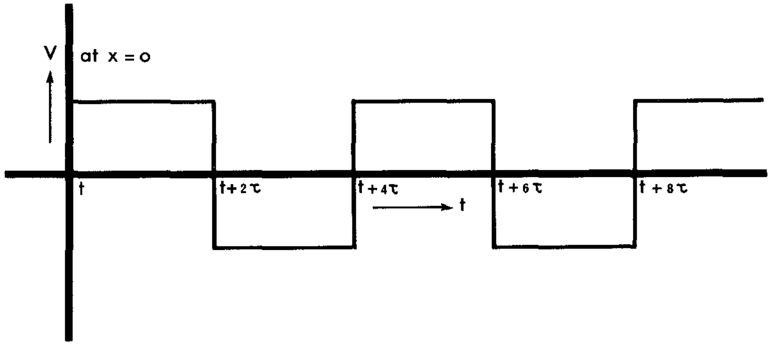

Figure 8

Since $T = 4\tau$ and $\tau = \frac{L}{c}$, c being the propagation speed and L being the length of transmission line above the water surface, it follows that

$$L = (\frac{c}{4})T.$$

The period T of the square wave oscillation is hence linearly related to the position of the water surface.

Derivation of Output Period T_0 and Sensitivity S

In the actual transducer circuit a twenty-foot length of flexible coaxial cable is inserted between the rigid transmission line (used as the sensor) and the tunnel diode to prevent the frequency of the oscillation becoming too high, Figure 9. This could occur with the rigid transmission line nearly fully immersed. The flexible coaxial cable does not alter the operation, it only shifts the operating frequency to a more convenient lower level.

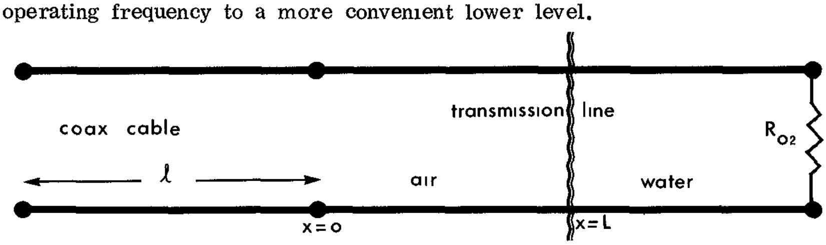

Figure 9

The one way propagation delay time is:

$$\tau = \frac{\sqrt{\varepsilon_r}\,\ell}{c} + \frac{L}{u} \simeq \frac{1}{c} \cdot (\sqrt{\varepsilon_r}\,\ell + L)$$

ε_r = relative dielectric factor of coaxial cable (= 2.25)
u = propagation speed in transmission line ($u \simeq c$)
ℓ = length of coaxial cable ($\simeq$ 20 ft.)
L = length of tubing above water surface.

The period of oscillation T is four times τ, hence:

$$T = \frac{4}{c} \cdot (\sqrt{\varepsilon_r}\cdot\ell + L)$$

This period T is very short (typically less than $0.2\,\mu$ sec) and considerable frequency scaling is used to increase the period to a value larger than 10 msec. This frequency scaling makes period recording and measurement much more convenient and accurate. With a frequency scaling factor N the output period of the transducer becomes:

$$T_o = N.T = \frac{4N}{c} \cdot (\sqrt{\varepsilon_r}\,\ell + L)$$

N = count down factor (= $5x2^{14}$).

The output frequency f_o is.

$$f_o = \frac{1}{T_o} = \frac{c}{4N} \cdot \frac{1}{(\sqrt{\varepsilon_r}\cdot\ell + L)}$$

The sensitivity S is

$$S = \frac{\Delta T_o}{\Delta L} = \frac{4N}{c}$$

Substituting the approximate numerical values given yields:

$$T_o \simeq (10 + \frac{L}{3}) \text{ msec.}$$

$$f_o \simeq \frac{100}{1 + \frac{L}{30}} \text{ Hz}$$

$$S \simeq \frac{1}{3} \text{ msec/ft.}$$

Dynamic Measurements

As derived, the sensitivity S is only dependent on N and c, both constants. One could state in general that S is largely independent of factors such as the temperature, pressure, humidity, dielectric constant, conductivity, cleanness of tube surfaces, etc.

Static Measurements

The stability of the device becomes an important factor when measuring slowly varying levels. The main factor in the expression for T_0 accounting for possible temperature effects is the relative dielectric factor of the coaxial cable. Another factor (which however does not appear in the expression for T_0 since we have assumed negligible switching times for the tunnel diode) to be considered is the capacitance of the tunnel diode and the effect of variations in it, due to temperature changes, on the output period. Both effects remain small resulting in a very stable device relative to temperature variations.

Design of Transmission Line

A transmission line can be made up of two parallel conductors, irrespective of the particular shape as long as their cross section remains constant; there is thus in principle a very large choice[6]. The prototype was made up of standard aluminum conduit, with the inner tube centered inside the outer tube. The inner tube is held concentric with sets of 1/4 inch diameter nylon screws (3 scews, 120°) at intervals of about three feet. This method allows sturdy positioning of the inner tube with negligible interference with the operation of the transducer. A small disadvantage of tubing is that openings have to be made in the outer tube to allow the water to freely flow in and out the space between the two tubes to assure equalizing of outside and inside water-level. The tubular construction still seems to be a rather optimum structure·

1. It presents a very strong mechanical structure, needing a limited amount of supports and spacers.
2. Eliminates electrical interference from external sources, due to its enclosed structure.
3. The circular form has little interference with waves and is moreover equal for waves from any direction.
4. The space between the two tubes remains dark enough to eliminate biological growth.

5. The tubular construction with holes in the outer tube will reduce erroneous readings due to splashing and wave build up.

6. Any length of transmission line can be readily made up joining additional sections with screw couplings.

The connection between the transmission line and the electronic circuitry is made via a coaxial cable of 50 ohms. It is therefore required that the characteristic impedance of the transmission line is approximately 50 ohms. Using for the outer tube an ID of 2 inches and an inner tube with an OD of 1 inch gives a characteristic impedance:

$$R_o = 60 \ln \left(\frac{r_2}{r_1}\right) \simeq 42\,\Omega$$

It was experimentally determined that this small mismatch can be tolerated since it did not interfere with the operation of the transducer.

Parameters Affecting Dynamic Accuracy

An interesting feature of the tubular structure chosen for the transmission line is that it allows the outer tube to act as a filter by the appropriate choice of the number and the diameter of the holes in the outer tube. One could, in general, distinguish three different time constants for the filter.

level recording and tidal waves: time constant in fraction of hours.
tsunamis and harbour seiches· time constant in fraction of minutes.
wind generated wave time constant in fraction of seconds.

For level recording, measuring tidal waves, tsunamis or harbour seiches one hole in the bottom part of the outer tube suffices.

The appropriate number, size and distribution of the holes for wind generated waves has been determined experimentally, simulating wave conditions by moving the tube sinusoidally up and down in still water.

The rate of change of the water surface, for steep waves with an amplitude of several feet and higher, is considerable. With falling water-level, a film of water is left behind on the "inner" tube surfaces, which falls at a slower rate than the water surface. This causes the effective dielectric constant in the transmission line, above the water surface, to be somewhat larger, resulting in a slight decrease of the propagation speed in the transmission line over the wetted length of line. The measured wave height is now somewhat larger than the actual wave height.

The dynamic simulation (vertical sinusoidal movement of transmission line) shows therefore the net result of the restriction in equalizing flow through the holes and the wetting of the "inside" surfaces of the transmission line. Figure 10 shows an increase in response, due to wetting, before the response rolls off, indicating that the restriction in flow through the holes becomes the dominant factor at the higher rates of vertical water movements. The dynamic tests were conducted with a length of twenty feet of transmission line, of which half was immersed in the water. The hole distribution, on the outer tube over the twenty-foot length, consisted of two 3/4 inch holes, directly opposite, spaced vertically at intervals of 6 inches, with every successive pair being rotated over 90°.

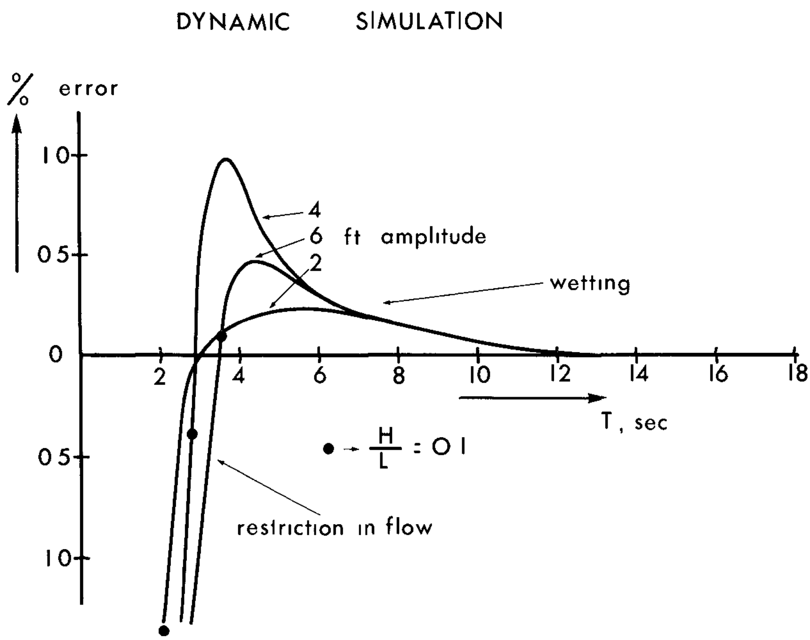

Figure 10

The down counting of the oscillator frequency (several MHz) by a large factor N, to produce a low output frequency, has a certain effect on the dynamic response of the transducer, especially if N is taken too large. The transducer has a dynamic limit resolution since one has to wait one output cycle before information is available of the water-level over that period of time. Each output cycle measured constitutes an average of the wave input signal over that period of time. This filtering action, with N normally chosen such that the output frequency varies between 50 to 100 Hz, can be considered negligible. However if the output period is made to be of the same order of magnitude as the wave period a considerable filtering action results. This effect can be analysed analytically if we assume the waves to be sinusoidal, evaluating the integral,

$$\int_{t_n}^{t_{n+1}} f(t)\,dt = N = \text{constant} \qquad \text{in which,}$$

$f(t)$ = frequency before scaling down
N = count down factor
$t_{n+1} - t_n$ = instantaneous period of scaled down frequency.

Working out the integral to obtain an expression $t_{n+1} = F(t_n)$ gives

$$t_{n+1} = \frac{1}{\pi f}\operatorname{arctg}\left[(1-\alpha^2)^{\frac{1}{2}}\ \operatorname{tg}\left\{\frac{\pi f \beta}{\alpha}(1-\alpha^2)^{\frac{1}{2}} + \operatorname{arctg}\left(\frac{\operatorname{tg}(\pi f t_n) + \alpha}{(1-\alpha^2)^{\frac{1}{2}}}\right)\right\} - \alpha\right]$$

in which:

$$\alpha = \frac{\frac{1}{2}H}{(\sqrt{\varepsilon_r}.\ell + L)}, \qquad \beta = \frac{2NH}{c}$$

$\sqrt{\varepsilon_r}.\ell$ = electrical length of coaxial cable
L = half the total length of transmission line
N = frequency scaling factor
c = propagation speed of light
H = total amplitude

For a transmission line 20 ft. long, with 20 ft. of coaxial cable and $N = 5 \times 2^{14}$,

$$\alpha = \frac{H}{80} \qquad \text{and} \qquad \beta = \frac{H}{6000}$$

The relation $t_{n+1} = F(t_n)$ is illustrated in Figure 11 for the worst case, i.e.: waves with a steepness factor $\delta = 0.14$.

EFFECT of FREQUENCY SCALING on WAVE HEIGHT RESPONSE

for Scaling Factor $N = 5 \times 2^{14}$

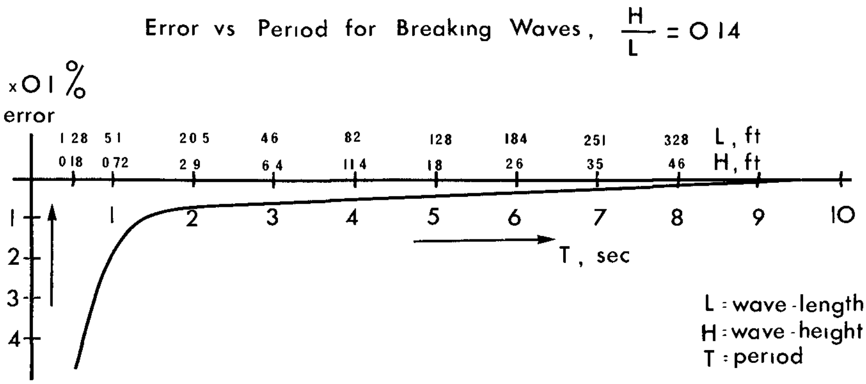

Figure 11

Period to Analog Voltage Converter

Since in some cases an analog output voltage might be required for recording the wave data, a circuit has been designed to convert accurately the output period into an analog voltage. The essential feature of the designed circuit is that it converts the wave height information from period into voltage in the same format, i.e.. for each output cycle a proportional constant voltage level is produced.

The operation of the converter consists basically of a precision ramp signal which is swtiched back to its reference level for every completed output cycle. The ramp voltage level is sampled just prior to switching by a sample-and-hold-circuit to produce a constant output voltage, updated at every successive completed output cycle. Since the output frequency is about two orders of magnitude higher than the wave frequencies, the output voltage of the converter appears as a continuous signal when measuring waves. Except for a slight smoothing, no filtering is required.

Discussion of Accuracy and Experimental Results

The wetting of the "inner" surfaces of the transmission line is the major source of error. There is a static and dynamic wetting error. For a slowly falling water-level a very thin stable film of water clinges to the "inner" surfaces, but with fast rates of change (several ft./sec) a film of water runs off the tube surfaces. Both of these errors increase the response the water-level measured is lower than the actual level and the wave height measured is larger than the actual wave height (wetting increases slightly the effective dielectric constant in the line above the water surface, decreasing the propagation speed).

The restriction in equalizing flow through the holes, for the pattern chosen, has to be taken into account for wave surfaces having a rate of vertical change beyond several ft./sec.

The transmission line is made up of two concentric aluminum tubes. Aluminum can be used very successfully in sea water[7,8]. If aluminum is corroded by water, the attack takes the form of pitting and there is no general corrosion or gradual thinning as occurs with steel. The rate of pitting decreases rapidly with time and follows a relation: $d = Kt^{1/3}$, d = max. pit depth, K = constant, t = time. Pitting should have little effect on the accuracy since the tubes are only used to guide the electromagnetic waves. Galvanic corrosion has been greatly reduced by DC-isolation of the tubes from the electronic circuitry and the use of non-conducting mounting brackets.

The first prototype built has been field tested, summer of 1971, in Lake Ontario, Figure 1, and the results obtained compared with a resistive step type wave-staff, closely mounted together. Figure 12 shows the good agreement obtained for the power spectra of these two completely dissimilar wave measuring instruments.

Four other units were field tested in Lake Ontario. during the summer of 1973. The instruments worked reliably throughout the season but the wave data gathered have not been processed and are as yet not available. There was considerable algae growth on the outer tube, the holes remained open however and there was no sign of growth inside, between the tubes. No field data available as yet on fouling in sea water.

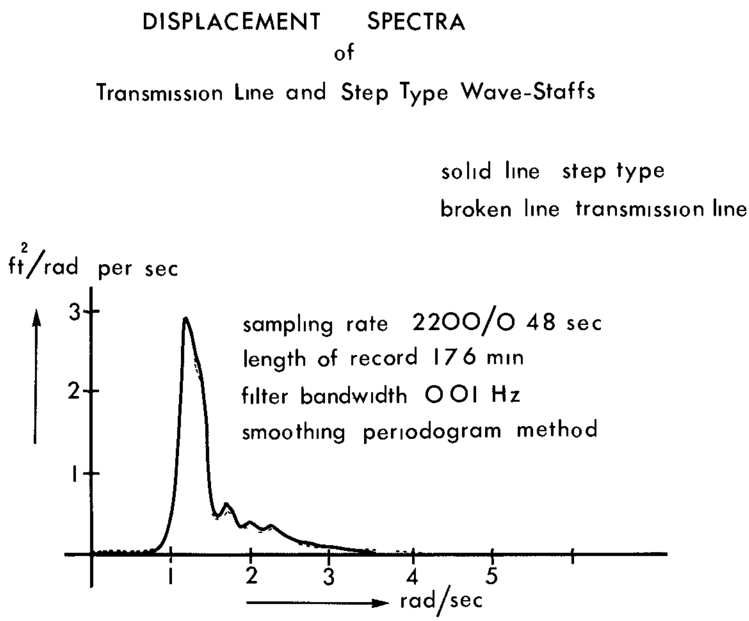

Figure 12

Quasi-static conditions, in fresh and salt water (sp.gr.1.025), were simulated in the laboratory using a transmission line, 20 ft. long. Figure 13 shows the laboratory set-up used for these tests. The accuracies attained are demonstrated by plotting the differences between measured and theoretical values for the period, Figure 14 and Figure 15 (the measurements were obtained with a commercial version of the transducer, Model P116).

The small error developing, with the transmission line moving out of the water, is due to the wetting of the tube surfaces. This error represents the maximum possible quasi-static error for each level of immersion, since it implies wetting over the total length of the transmission line above the water surface at each measured level. From the results obtained one can define the accuracy to be better than 0.2% for quasi-static measurements. (Use of a non-wetting coating on the tube surfaces will substantially improve the accuracy; the maximum wetting error, 0.2%, being the dominant error.)

Transducer in Laboratory Set-up

Figure 13

It might be noted that under typical laboratory conditions it takes about 4 to 5 hours for the wetting effect to completely disappear. Under field conditions it can be expected to disappear much more quickly due to air circulating between the tubes, stimulating evaporation, resulting in a negligible error and improved accuracy.

The results of the dynamic tests are shown in Figure 10 (the period-to-voltage converter was used for the dynamic tests and its error is therefore included in the total dynamic measuring error). Waves were simulated moving the transmission line vertically, in a sinusoidal fashion. The restriction in equalizing flow starts to dominate the wetting effect, for the amplitudes 2, 4 and 6 ft. chosen, for very steep (close to breaking) deep water waves of equal amplitudes. This would indicate that for normal wave activities the error in the wave height measurement is not appreciable.

The transducer has a temperature coefficient of 510^{-3} inch/°F, which applies only when measuring still water-levels, since temperature variations only produce a parallel shift and hence keeping the dynamic sensitivity S constant. The maximum length for the transmission line is 90 ft.; "dry" tests with metal bar electrical shorting showed no deterioration of accuracy over that length. The period-to-voltage converter is very accurate and its output voltage signal has essentially the same accuracy as given by the period information of the square wave output signal. The transducer operates from a 12 volt battery and has a total current consumption of 32 mA.

Conclusions

In this paper a very simple, rugged, but accurate wave measuring device has been described, which forms a very attractive alternative for existing wave gauges. The output signal format, a low frequency, period modulated signal-derived directly without intermediate conversion steps from the psotion of the water surface - is very convenient for recording, transmitting and digital processing. The sensitivity S of the transducer as derived earlier, is only a function of the propagation speed u of the electromagnetic signal and the frequency division factor N of the electronic circuitry. The calibration is hence permanent and largely independent of changes in environmental and circuit parameters. Patent applications have been filed and a license granted to a Canadian firm for the manufacturing of the instrument.

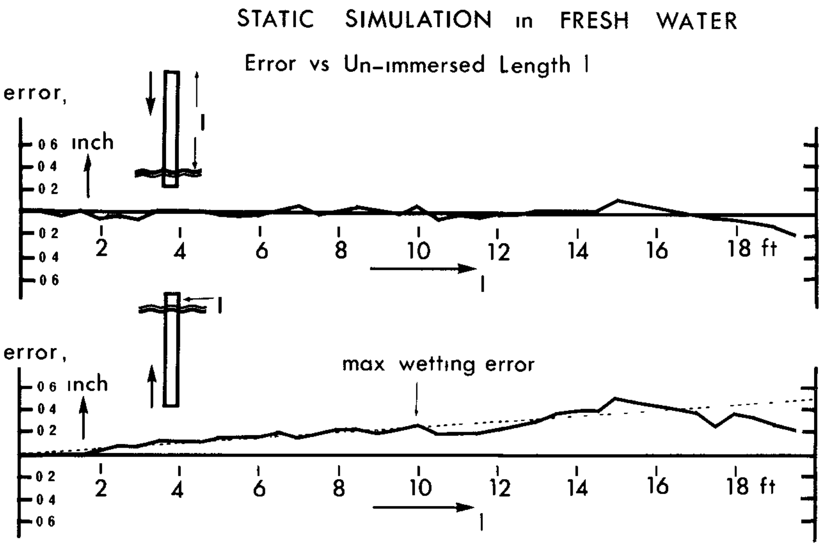

Figure 14

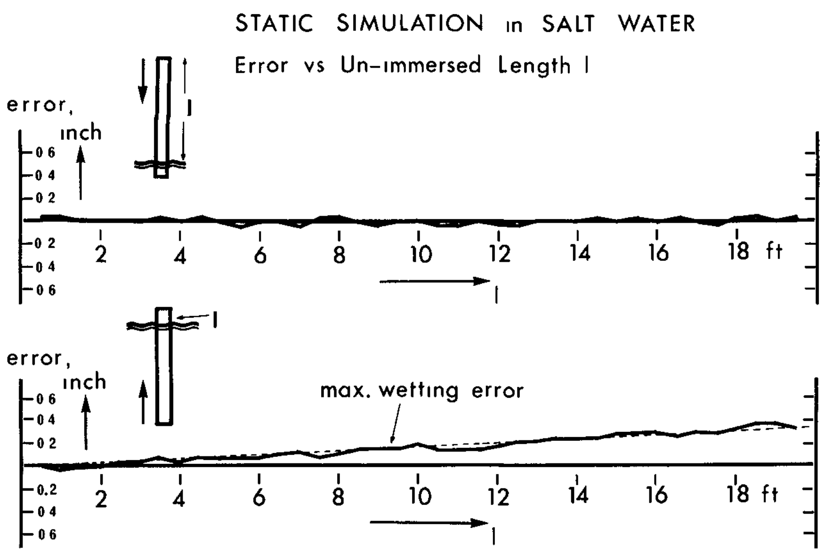

Figure 15

References

1. Handbook of Chemistry and Physics,
Chemical Rubber Publishing Co., 1953.

2. Transistor Manual, General Electric,
New York, 1964.

3. Tunnel Diodes, M.A. Lee et al, Chapman and Hall Ltd.,
London, 1967.

4. Transmission Lines with Pulse Excitation,
G. Metzger, J.D. Vebre, Academic Press Inc.,
New York, 1969.

5. R.S. Singleton, Finding Transmission Reflections,
Electronics, October 28, 1968.

6. The Physics of Transmission Lines,
P. Grivet, Academic Press Inc.,
New York, 1970.

7. H.P. Godard, The Corrosion Behaviour of Aluminum in Natural Waters,
The Canadian Journal of Chemical Engineering,
October, 1960.

8. E.D. Verink and P.F. George, Aluminum Alloys for Desalination
Service, Materials Protection and Performance,
May, 1973.