CHAPTER 7

WAVE GROUP FORMATION AMONG STORM WAVES

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ABSTRACT

Wave data obtained in the North Sea for stormy weather conditions are analyzed to determine the extent of wave group formation among large waves; i.e. the number of large waves succeeding each other in one single run. Three periods associated with the passage of high sea states are examined. The average correlation between succeeding wave heights is found to be +0.24, which indicates that wave heights do have a "memory". Wave group formations are found to be more pronounced when the sea is growing than decaying. The average lengths of wave runs are calculated.

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INTRODUCTION

Offshore activity in the North Sea has intensified the study of storm wave properties in this area. Such studies are important because a better understanding and description of the storm waves may improve engineering practice in this hostile environment.

The larger waves in a sea state occur more or less in a random fashion due to the statistical behaviour of the sea surface. However, it is sometimes experienced that large waves tend to stick together, as illustrated in Fig. 1 which shows waves recorded in the North Sea during a storm ($H_{1/3} \simeq 10$ m). The recording shows three large waves succeeding each other in one single run. This paper considers the extent of such wave group formation among large waves recorded during storm conditions in the North Sea.

There is an old Icelandic saying which says: "Sjaldan er ein baran stök", which means: "A large wave comes rarely alone". The offshore engineer should pay attention to the experience gained by the Icelandic fishermen, because wave group formation among large waves may be important in many engineering aspects. For example, the fact that large waves tend to stick together, may justify use of regular waves when fixed structures are tested in a wave flume. Also, for the evaluation of mooring forces, wave group formation among large waves is of interest because moored structures sometimes tend to respond to the wave height envelope rather than to the actual wave. (HSU and BLENKARN 1970, KAPLAN 1970).

Previous studies on wave group formation are relatively few. Nolte and Hsu studied the wave group formation by considering the statistical properties of the wave height envelope (NOLTE and HSU 1972), and the results were compared to actual wave measurements obtained from the Gulf of Mexico. Goda has studied the wave group formation among large waves by means of numerical experiments on wave statistics with spectral simulation (GODA 1970). Also, Wilson and Baird have presented some field results on wave group formation by considering the extent of runs of waves larger than the significant wave height (WILSON AND BAIRD 1972). The method of analysis applied by Goda has partly been adapted in this study.

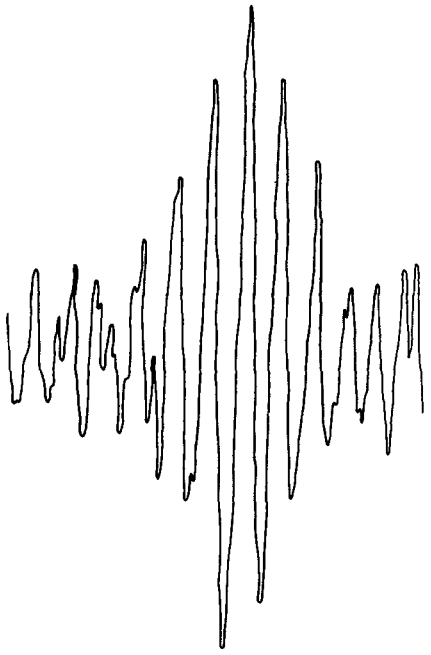


Figure 1. Part of a wave record obtained under stormy conditions in the North Sea

DATA RECORDING AND REDUCTION

A "Waverider" accelerometer buoy transmitted wave data from outside Utsira, Norway (Fig. 2). The water depth was about 100 m (Fig. 3). Waves were recorded every third hour with a duration varying between 8 and 20 minutes for each recording. The data were recorded on a strip chart and the wave heights were read off according to the zero-up-cross method.

The data applied for the analysis were collected from three storms, occuring in October, November and December 1970. 60 recordings were read off, and all of the records were selected so that some waves larger than 4 m were present in each recording. This was done to ensure that the wave records analyzed were describing relatively heavy sea states.

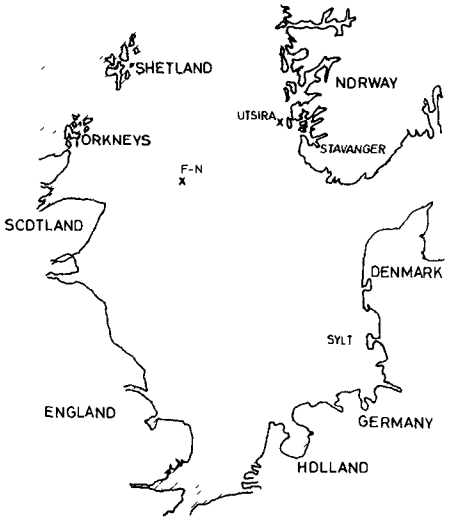


Figure 2

The location of the wave recording site

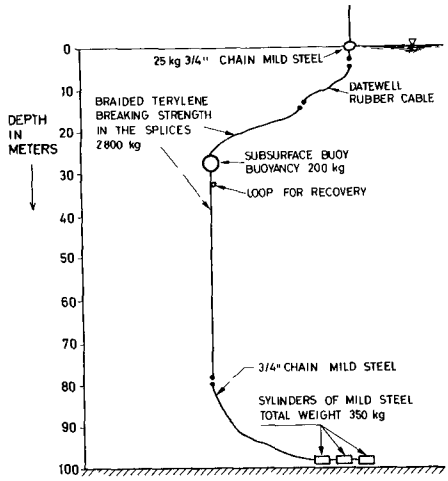


Figure 3

The wave recorder and the mooring system

Wave periods were not considered in this study because the wave heights were regarded more important. Wave heigths lower than 0.5 m were excluded because sometimes it was experienced that runs of large waves were divided by the presence of small waves which were regarded to be of secondary interest for engineering activity (Fig. 4). (The extent of small waves present in the wave record may also depend on the resolution of the wave recorder (HARRIS 1970)).

Fig. 5 shows the average wave height $\bar{H}$ for all the recordings. The 60 recordings were divided into two groups, according to whether $\bar{H}$ was recorded during growing wave height conditions (denoted by "G") or decaying wave height conditions (denoted by "D"). This was done because the statistical properties of the sea surface were different for the two cases, as will be shown later.

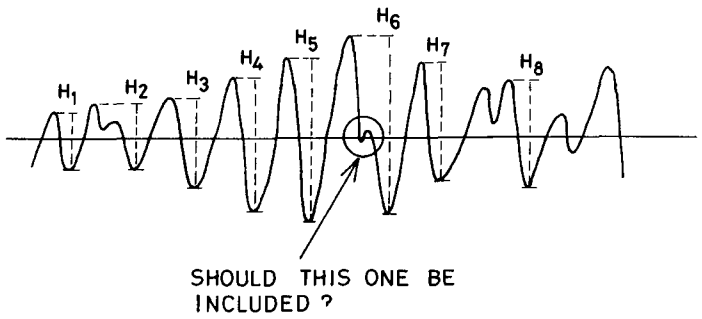


Figure 4. Illustration of the presence of relatively small waves, splitting one run of large waves into two separate runs.

WAVE HEIGHT CORRELATIONS

The first question was whether waves have a "memory" or not. A correlation coefficient between succeeding wave heights was computed according to the formula

$$\phi(k) = \frac{1}{\phi(0)} \cdot \frac{1}{N-k} \sum_{i=1}^{N-k} (H_i - \bar{H})(H_{i+k} - \bar{H}) \qquad (1)$$

where

$$\phi(0) = \frac{1}{N} \sum_{i=1}^{N} (H_i - \bar{H})^2$$

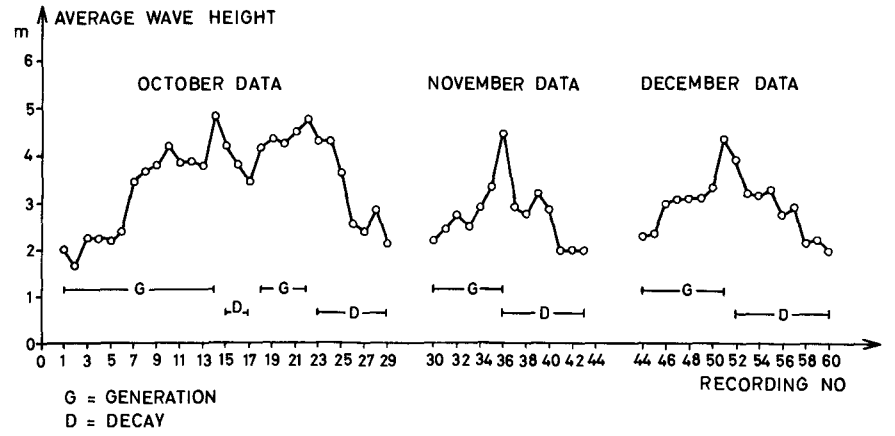
N = Number og waves in the actual recording

$\bar{H}$ = Average wave height

k = Number of lags between the waves in sequence

All the waves H_i are "sequentially" spaced because the wave periods are not considered. If succeeding waves are uncorrelated, all the $\phi(k)$ for $k \geq 1$ would approach zero when N goes to infinity. However, $\phi(1)$ was found to be different from zero, as shown on Fig. 5. An average value of +0.24 was found. The value of 0.24 indicates that waves do have a memory, and the positive sign indicates that large waves tend to be succeeded by large waves, while small waves tend to be succeeded by other small waves.

The next question is to what extent wave characteristics during growth may be distinguished from wave characteristics during decay. From Fig. 5, it is apparent that $\phi(1)$ tends to be larger during wave growth (denoted by G) than during wave decay (denoted by D). $\phi(1)$ tends to be close to 0.30 during wave growth, while during decay, $\phi(1)$ is closer to, or lower than 0.20. The wave group formation seems therefore to be more pronounced during wave growth conditions than during decay conditions. The reason for this finding will be considered later in this paper.



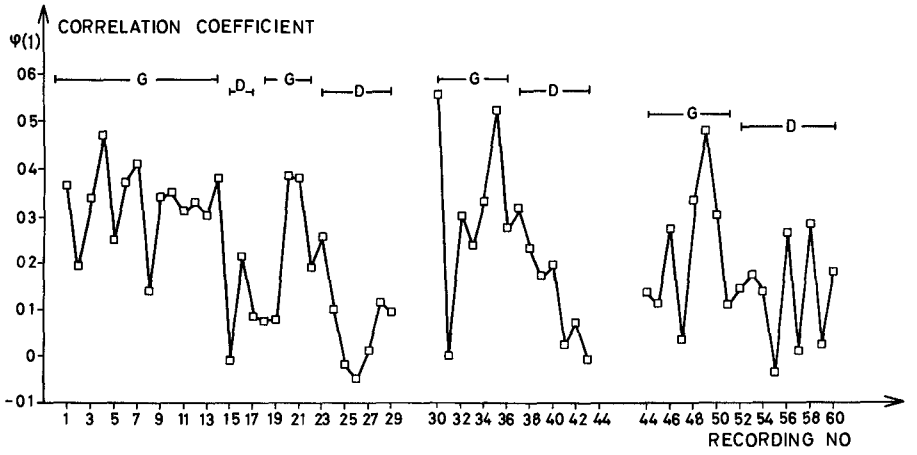


Figure 5 The average wave height $\bar{H}$ and the correlation coefficient $\phi(1)$ for 60 recordings

The correlation coefficients $\phi(2)$ and $\phi(3)$ were also computed, but they were not found to vary significantly from zero, on the average.

STATISTICS OF RUNS OF HIGH WAVES

Fig. 6 shows the wave heights to follow the Rayleigh distribution comparatively well. In order to include all of the 60 recordings on the same diagram, the average wave height was used as unity for each of the recordings.

Provided that the wave heights follow the Rayleigh distribution, the number of waves that exceed some fixed level (say the level of $H_{1/3}$) may be evaluated from the formula

$$\text{Prob} \quad (H > H_{1/3}) = \exp\left(-\frac{H_{1/3}^2}{8m_o}\right) = \exp(-2) \simeq 0.134 \qquad (2)$$

where m_o is the variance of the wave record and $H_{1/3}$ is equal to four times this variance.

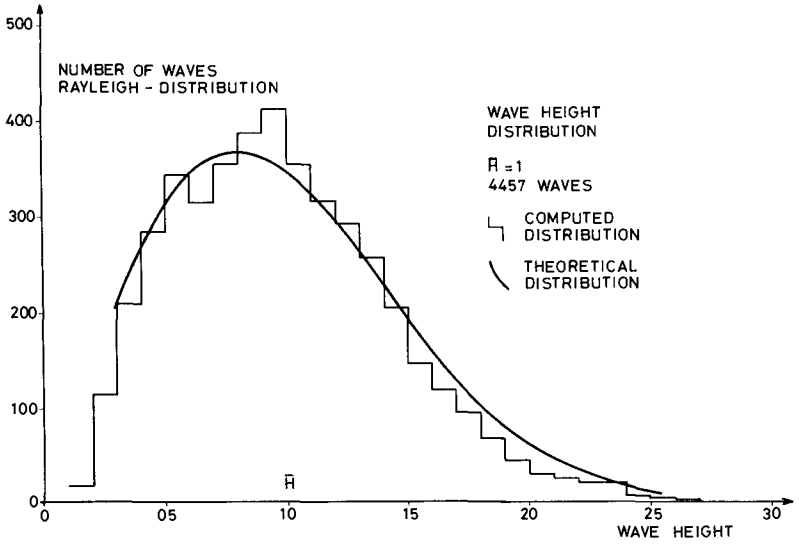


Figure 6. The average distribution of the wave heights. Waves lower than 0.5 m are excluded. The average wave height H is unity for each recording.

The probability of the occurrence of waves larger than $H_{1/3}$ may be treated as illustrated in Fig. 7, which shows two waves (H_3 and H_4) larger than $H_{1/3}$ occurring in one run and another single wave (H_8) larger than $H_{1/3}$ which occurs in another run. The probability that j waves larger than $H_{1/3}$ succeed each other in one single run is denoted P(j). Provided that only waves larger than the significant wave height are considered, the relation

$$\sum_{j=1}^{\infty} P(j) = 1 \qquad (3)$$

holds. The results from a calculation of P(j), averaged for all the 60 recordings, are shown in Fig. 8.

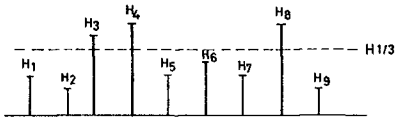


Figure 7

Illustration of two runs of waves larger than the significant wave height

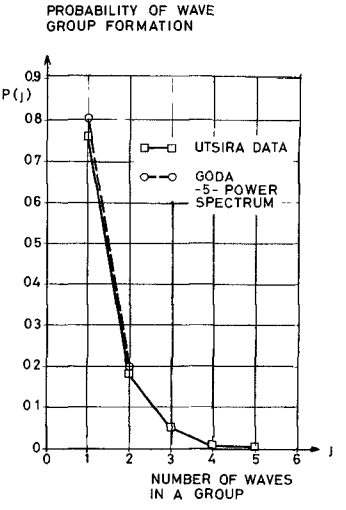


Figure 8

The average probability of occurrence P(j) of runs of waves larger than $H_{1/3}$ for the field data and from the numerical simulation by Goda applying a Pierson - Moskowitz type spectrum

P(j) may be determined analytically provided that the wave heights are assumed to succeed independent of the preceding wave height; i.e. the waves are assumed to have no "memory". This case would correspond to $\phi(1)$ equal to zero on Fig. 5. For this case P(j) is given by

$$\begin{aligned} P(2) &= P(1) \cdot Q \\ P(3) &= P(1) \cdot Q^2 \\ &\vdots \\ P(j) &= P(1) \cdot Q^{j-1} \qquad \text{for } j \geq 1 \end{aligned} \tag{4}$$

and the relations (3) and (4) lead to

$$P(1) = 1 - Q \tag{5}$$

where Q is the probability of occurrence of the event. (For the event (2), Q is equal to 0.134). From (5), all the P(j) will be given by (4).

A somewhat similar computation was carried out by GODA (1970) by means of a spectral simulation on a computer applying a Pierson - Moskowitz type spectrum as the spectral input. The runs of high waves computed by Goda have been replotted in Fig. 8 and appear to be relatively similar to the results obtained from the field data. However, the data material applied by Goda is relatively sparse and only two points are shown.

It is noted that the power dependency of j in (4) indicates that on a semi-logarithmic plot, the P(j)'s will follow straight lines, as shown in Fig. 9.

The P(j) values computed from the field data are also shown in Fig. 9. They show that the wave group formation is more pronounced than would be expected from a completely random distribution of the wave height successions. This result is in accordance with Goda's results (which are shown in Fig. 8 and replotted on a semi-logarithmic plot in Fig. 10), and also in accordance with Wilson and Baird's conclusions from their study of the wave climate outside Nova Scotia (WILSON and BAIRD 1972). Also, the results shown in Fig. 9 are consistent with the positive correlation found for $\phi(1)$ shown in Fig. 5.

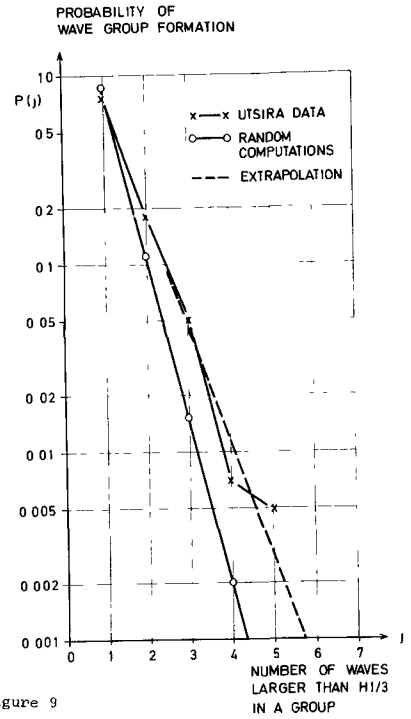


Figure 9

The average occurrence P(j) of runs of waves larger than $H_{1/3}$ for the field data plotted on semi-logarithmic paper as compared to P(j) computed from equation (4) (Q = 0 134)

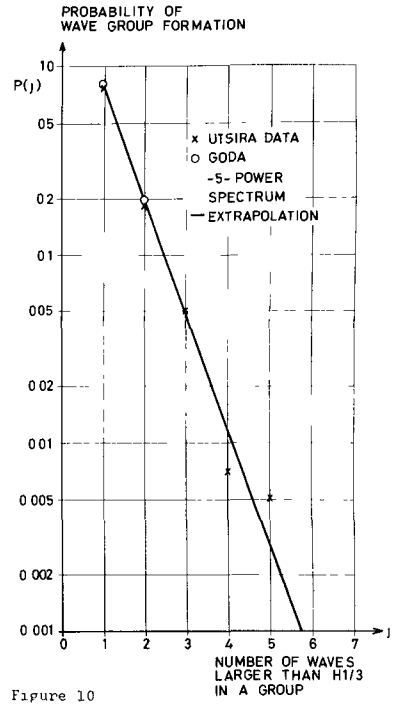


Figure 10

The results from Figure 8 replotted on a semi-logarithmic plot

If the field data are separated into the cases for wave growth (G) and wave decay (D) conditions, the average values of P(j) differ, as shown in Fig. 11. The wave group formation appears to be more pronounced during wave growth conditions than during wave decay. It is recalled that the same tendency was noted for the $\phi(1)$-values; $\phi(1)$ tends to be larger during wave growth conditions than during wave decay.

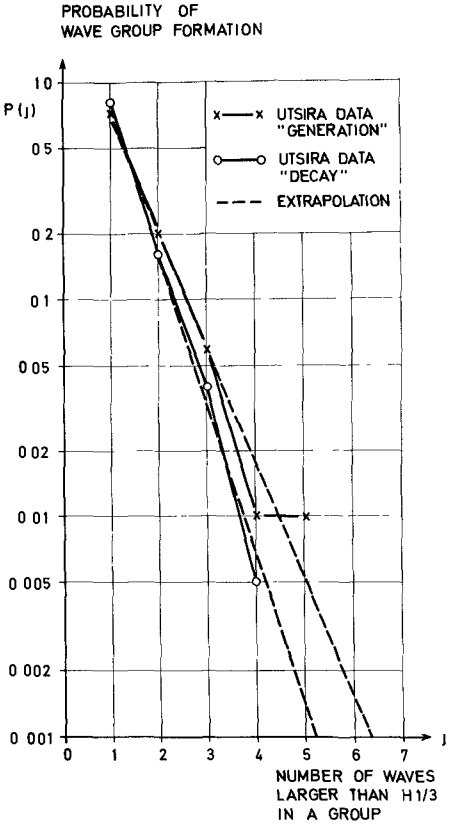


Figure 11. The average probability of occurrence P(j) of runs of waves larger than $H_{1/3}$ for the field data when they are divided into the case for wave growth and wave decay

The explanation for this finding may not be obvious. Longuet-Higgins has shown theoretically that wave group formation tends to be more pronounced for a narrow-band spectrum (LONGUET-HIGGINS 1957). Because swell is described by a narrow-band spectrum, one might expect that the wave group formation would be more pronounced during wave decay than during wave growth. However, the result shown in Fig. 11 is contrary to this expectation.

On the other hand, the beginning of the wave decay is usually due to changes in the wind field, both in strength and direction. This changing is likely to lead to a relatively "confused" sea state; the state of "regular swell" is likely to occur at a later stage of the decay (say, when the maximum waves of the sea state have passed below 4 m).

In addition, recent research results have shown the wave energy spectrum to be much more sharply peaked than would be expected from a Pierson - Moskowitz type spectrum (The JONSWAP Project, HASSELMANN et. al. 1973). Fig. 12 shows the average JONSWAP spectrum and Fig. 13 shows an example of a spectrum recorded during wave growth at Utsira. They look very similar. Goda concluded that the controlling factor for the length of the wave runs appears to be the spectral peakedness (GODA 1970). Thus the sharply peakedness of the wave spectra during the wave growth stage may be another reason why the group formation tends to be more pronounced during wave growth than during decay.

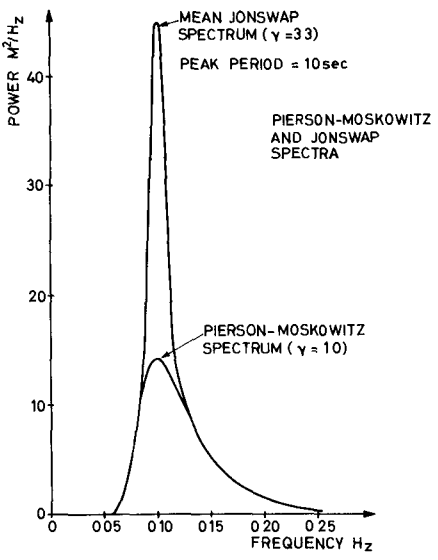


Figure 12

The average JONSWAP spectrum recorded outside the island of Sylt under fetch-limited, wave-growth conditions as compared to the Pierson-Moskowitz spectrum

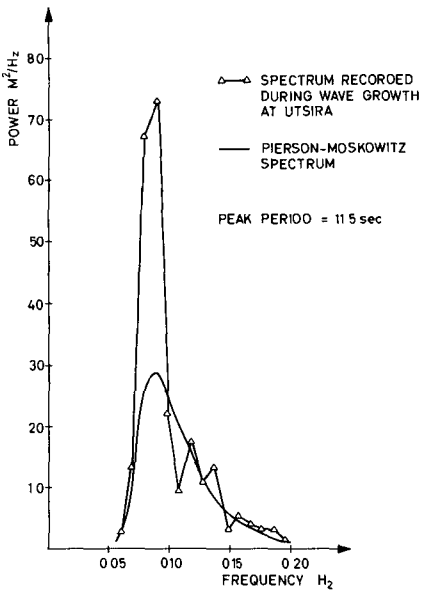


Figure 13

Wave spectrum recorded during wave-growth conditions at Utsira

At the presentation of this paper it was suggested that the extent of wave group formation might correlate to the spectral width parameter ε. However, the ε parameter for a Pierson - Moskowitz type spectrum appears to be highly sensitive to the choice of high-frequency cut-off to the spectrum: ε varies between 0.4 and 0.8, depending on the cut-off frequency applied (GODA 1970). This is so because ε is dependent on the fourth moment of the spectrum while the Pierson - Moskowitz spectrum varies with the minus fifth power of the frequency at the high-frequency range. No correlation between the $\phi(1)$ (given by (1)) and ε was found.

Fig. 14 compares the wave group formation recorded during wave growth to the wave group formation from a very narrow-banded spectrum simulated on the computer by Goda. The simulated waves described by the narrow-band spectrum (which has a high-frequency tail falling off as frequency to the minus tenth power) shows a much larger extent of wave group formation than the field data.

Fig. 15 is an extrapolation based on the straight line drawn for the field data (wave growth data) shown in Fig. 11. The line denoted A represents the probabilities P(j) based on the computations given by (4), while the line denoted B represents the field data extrapolated for the wave-growth case. Note that P(6), for instance, is about 40 times larger for the field data than for the evaluation based on the theory. This leads to the conclusion that the probabilities P(j) for wave group formation among large waves are much more pronounced than would be expected from a completely random distribution of the wave heights for large j.

Similar computations were also carried out for the event of H_i lower than $H_{1/3}$. For this case Q in (4) equals to $1 - 0.134 = 0.866$.

Based on the random theory, the average length E_M of wave runs was computed according to the formula

$$E_M = \sum_{j=1}^{\infty} jP(j) = \frac{1}{1-Q} \tag{6}$$

where (4) and (5) have been used. The second moment of P(j) is

$$E = \sum_{j=1}^{\infty} j^2 P(j) \tag{7}$$

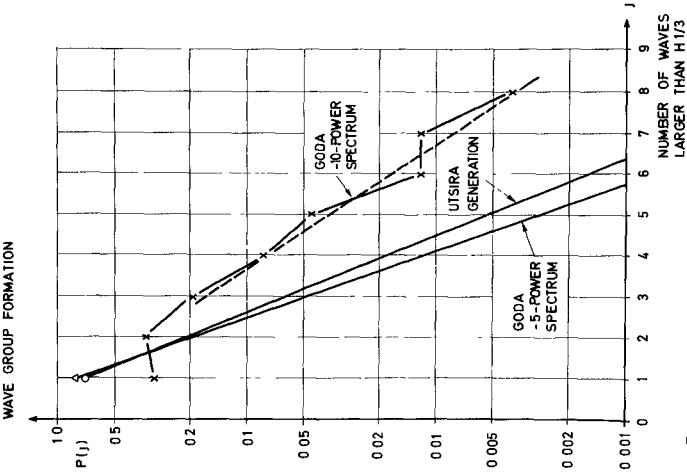


Figure 14

The average probability of occurrence P(j) of runs of waves for the wave growth data as compared to the wave runs obtained with spectral simulation applying a spectrum with a high-frequency tail falling off as frequency to the minus tenth power

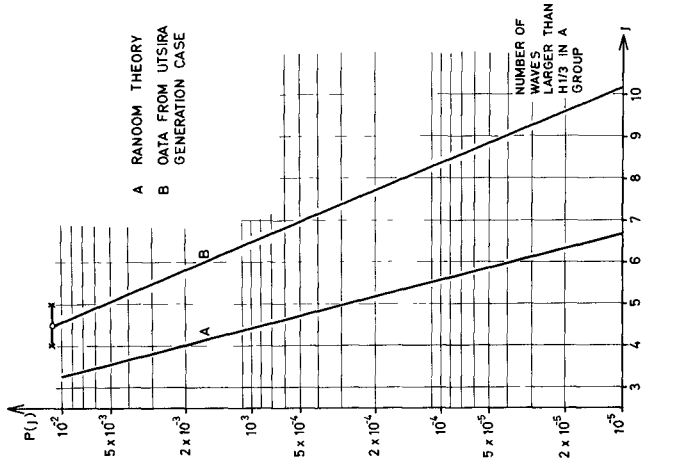


Figure 15

The average probability of occurrence P(j) for the wave growth data extrapolated as compared to the determination of P(j) according to equation (4).

The standard deviation σ is therefore

$$\sigma^2 = E - E_M^2 \tag{8}$$

where E and E_M are evaluated from (7) and (6).

The results from the computations based on (4) as compared to the field results are shown in Table I.

CASE	AVERAGE	STANDARD DEVIATION
1. UTSIRA - DATA	1,35	0,61
2. WAVE - GENERATION	1,41	0,69
3. WAVE - DECAY	1,26	0,51
4. RANDOM - COMPUTATIONS	1,15	0,42

Table I. The average duration E_M and the standard deviation σ for runs of waves larger than $H_{1/3}$.

A similar computation was also carried out for the event that H_i is lower than $H_{1/3}$. The results are shown in Table II.

CASE	AVERAGE	STANDARD DEVIATION
1. UTSIRA - DATA	7,71	6,23
2. WAVE - GENERATION	8,05	6,63
3. WAVE - DECAY	7,26	5,70
4. RANDOM - COMPUTATIONS	7,46	6,95

Table II. The average duration E_M and the standard deviation σ for runs of waves smaller than $H_{1/3}$.

The tables show that the standard deviations of the runs of high (or low) waves are relatively large, especially for the runs of low waves.

Table III shows the mean number of waves between two runs of waves larger than $H_{1/3}$.

	E_M $H>H_{1/3}$		E_M $H<H_{1/3}$		MEAN LENGTH OF TOTAL RUN FOR $H_{1/3}$ TO $H_{1/3}$
RANDOM COMPUTATIONS	1 15	+	7 46	=	8 61
UTSIRA AVERAGE	1 35	+	7 71	=	9 06
UTSIRA GENERATION	1 41	+	8 05	=	9 46
UTSIRA DECAY	1 26	+	7 26	=	8 52

Table III. The mean number of waves between two runs of waves larger than $H_{1/3}$.

This number was simply evaluated by adding the E_M for $H_i < H_{1/3}$ and for $H_i > H_{1/3}$ together. The table shows the average number of waves between two runs of waves larger than $H_{1/3}$ to be 9, on the average, for all the cases considered. This result is also in accordance with Goda's computer simulations, applying a Pierson - Moskowitz type of spectral input.

It should, however, be stressed that this result is dependent on the specific wave height level chosen. Say, if a level above the significant wave height level had been chosen, the mean length between two runs of large waves would have been larger.

CONCLUSIONS

The results derived in this paper may be summarized as follows:

1. The wave group formations among large waves are found to be larger than would be expected from an estimate based on a completely random successions of the wave heights.

2. The field results compare well to the results obtained numerically by Y. Goda.

3. Wave group formations tend to be more pronounced for a growing sea than for a decaying sea.

For offshore practice, conclusion No. 1 will be of particular importance because a large probability of a single run with many large waves might justify the use of regular waves when fixed structures are tested in a wave flume.

ACKNOWLEDGEMENTS

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