## CHAPTER 149

CONTAINING OIL SLICKS IN FLOWS OF FINITE DEPTH by

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## ABSTRACI

The hydrodynamics of oil slick containment are examined. It is shown that contajmment is only possible when a densimetric Froude number based on flow velocity and depth and the oil density is less than a critical value. Experiments confirmed that an oil slick was unable to maintain a stationary front when the Froude number exceeded the critical value.

It is also shown that viscous effects ultimately limit the thickness and therefore the length and volume of any real slick. Expressions are derived which enable these limits to be determined.

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## INTRODUCTION

It is well known that when oil spills onto a water surface it spreads rapidly to form a thin film, eventually only thousandths of an inch thick. At present there is no known means of removing an oil film as such from a water surface other than by the natural processes of evaporation and biological degradation. The efficiency of man devised removal techniques increases rapidly as the thickness of the slick itself increases so that confinement of the slick is usually the first step in the clean-up procedure.

Oil booms, which restrict the movement of oil on the water surface, are commonly used as the confining apparatus. Wind and currents will cause oil to accumulate on the upstream side of any partially submerged barrier which impedes the flow of oil on the the water surface. Figure 1 shows an oil slick contained in a flowing stream.


Fig. l. Profile of a Contained Oil Slick

Unfortunately containment of oil slicks is not always possible as winds, waves and currents can cause oil to escape past a boom or even cause structural failure of the boom itself. This paper examines the effects of current on oil contained in a horizontal two dimensional channel. It is shown that viscous shear forces at the oil-water interface ultimately limit the length of any contained oil slick. Furthermore, dynamic considerations make containment of a slick impossible if the current flow exceeds a critical Froude number. Suggestions are madc as to how slicks might be contained under supercritical conditions by judicious alignment of an oil boom.

THE PHYSICS OF A CONTAINED OIL SLICK
Figure 1 shows the profile of a typical laboratory oil slick and it should be noted that the horizontal scale has been compressed by a factor of 50 compared with the vertical scale. Starting at the upstream end $B$, the slick can be divided into two distinct areas of influence. Firstly, an abrupt thickening occurs near the very front of the slick ( $\mathrm{B}-\mathrm{B}^{\mathbf{1}}$ in Fig. 1). This is a dynamic effect brought about by the presence of a current passing beneath the slick, and in a given channel the greater the current speed, the deeper will be the slick front. Viscous effects are relatively unimportant in this frontal zone so that its thickness can be predicted with reasonable accuracy using inviscid flow assumptions.

The analysis enables the frontal thickness ratio $\phi=t / D_{0}$, (where $t$ is the frontal thickness of the slick and $D_{0}$ the current depth upstream of the slick) to be expressed as a function of a densimetric froude number $F$. The densimetric Froude number, hereafter simply termed the Froude number, is defined as

$$
\mathrm{F}=\mathrm{U} /\left(\Delta g \mathrm{D}_{\mathrm{O}}\right)^{\frac{1}{2}}
$$

where $U=$ current speed upstream of the slick
$\Delta=\rho_{\text {water }}{ }^{-\rho}{ }_{\text {oil }} / \rho_{\text {water }}$
and $\quad g=$ gravitational acceleration.
Unfortunately, solutions for $\phi$ are available only for a limited range of froude numbers less than approximately 0.5. At Froude numbers in excess of this value a stable front is unable to form and oil will pass beneath any boom, no matter how deeply it is placed. The Froude number of the flow in Fig. 1 was 0.30 and the slick was therefore stable.

The downstream region of the slick is denoted by B'C and in this zone viscous forces are responsible for the further thickening of the slick. It can be seen in Fig. l
that the interfacial slope increases with distance from the slick front. Obviously in flows of finite depth this process cannot continue indefinitely and ultimately the slick attains a critical thickness where additional oil cannot be contained and if oil is added to a slick at this stage an equal volume of oil will be displaced beneath the boom, irrespective of its depth.

Two practical points concerning boom immersion depths may be made at this stage. Firstly the skirt depth must at least exceed the dynamically induced thickness $t$ otherwise virtually no oil will be contained at all. Secondly even when the froude number is less than 0.5 there is a limit to the volume of oil which can be contained behind a single boom. This volume is dependent upon the Froude number of the flow and the interfacial stress. An increase in magnitude of either of these factors will reduce the volume of oil which can be contained. In order to retain the maximum amount of oil, that is achieve a slick of critical length, the skirt depth must exceed the critical slick thickness at C in Fig. 1. The ratio of this thickness to the upstream flow depth $D_{0}$ can be expressed as a function of the Froude number $F$ and is quite independent of the magnitude of the viscous forces even though it is these forces which cause the slick thickness to reach its limiting value. If the skirt depth of the boom is intermediate between the dynamically induced thickness $t$, and the maximum slick thickness, oil will be contained but less than if the skirt extended past the critical thickness of the slick.

## THEORY

Benjamin (1968) examined the problem of an air cavity advancing into a horizontal box as fluid drained from its single open end. The fluid flow around the air cavity is similar in many respects to the passage of water beneath an oil slick contained in a horizontal channel. The main point of difference between the two phenomena is the existence of the stagnation point at the very front of the slick ( $B$ in Fig. 1) results in a superelevation ( $\varepsilon$ ) of the level of the water surface. The superelevation is small compared with the slick thickness $t$ but is important in determining the pressure distribution beneath the slick. However $\varepsilon$ may be neglected when summed with $t$ in the convective terms of the force balance equations. Benjamin's solution can be readily adapted to describe a contained slick (see Appendix l) and the frontal thickness ratio ( $\phi$ ) is related to the Froude number of the flow by the equation

$$
\begin{equation*}
F^{2}=\phi\left[\frac{2 \phi}{I-\phi}+\frac{1}{I-\Delta}\right]^{-1}(2-\phi) \tag{1}
\end{equation*}
$$

Since the slick thickness ratio $\phi$ is derived from a momentum
pressure force balance relationship, Eq. l yields a $\phi$ value which in an inviscid fluid would be approached asymptotically. In real flows the thickness predicted by Eq. l is achieved 15 to 30 slick thicknesses from the front.

It will be apparent later that physical solutions are limited to thickness ratios between zero and approximately one third, depending upon the specific gravity of the oil. This range of solutions for Eq. 1 is plotted as the curve $A B$ in Fig. 2. Frontal thickness of a number of laboratory oil slicks are in close agreement with Eq. l.


Fig. 2. Frontal Thickness Ratio as a Function of Froude Number
The significant point to note is that no solutions exist to the force balance equation for froude numbers greater than approximately one half, and in the experiments it was found that slick containment became impossible at Froude numbers in excess of this value.

The relationships describing the eauilibrium of the viscous zone of an oil slick, where flow conditions are slowly varying, are derived in Appendix 2. Briefly, an equation describing the flow force balance at a section is combined with the continuity equation to yield a relationship between the upstream Froude number $F$, the free surface slope $\frac{\partial D}{\partial x}$, the interfacial slope $\frac{\partial y}{\partial x}$, the slick thickness ratio at
the section of interest and the boundary shear stress ${ }^{\top} b$ acting on the flow at the channel bottom and sides.
second relationship is derived giving conditions for equilibrium of the slick itself and this equation involves all the previous variables except that the boundary stress is replaced by an interfacial stress $\tau_{i}$ which acts at the oil-water interface. The slope of the free oil surface, which is of no practical interest, can be eliminated between the above two equations leaving a single relationship for the interfacial slope given by:
$\Delta \rho g y \frac{\partial y}{\partial \mathrm{x}}\left\{1-\left(\phi+\left[\frac{F}{1-\phi}\right]^{2}\right)\right\}=\frac{\tau i}{1-\Delta}\left\{1-\Delta\left(\phi+\left[\frac{F}{1-\phi}\right]^{2}\right)\right\}+\tau_{b} \phi$
where $\rho$ is the density of the water and
$y$ is the slick thickness.
The two stress terms can be expressed in terms of a friction coefficient, the upstream Froude number and the local slick thickness ratio as below.
and

$$
\begin{equation*}
\frac{\tau_{i}}{\Delta \rho g D_{o}}=\frac{f_{i}}{8}\left(\frac{F}{1-\bar{\phi}}\right)^{2} \tag{3a}
\end{equation*}
$$

$$
\begin{equation*}
\frac{{ }^{\tau} b}{\Delta \rho g D_{o}}=\frac{f_{b}}{8}\left(\frac{F}{1-\phi}\right)^{2} \tag{3b}
\end{equation*}
$$

Values of the interfacial friction coefficient $f_{i}$ must be determined from experiments while the boundary friction coefficient $f_{b}$ is well documented elsewhere (Streeter, 1961).

It will be noted that Eq. 2 is singular when

$$
\begin{equation*}
\left\{1-\left(\phi+\left[\frac{F}{1-\phi}\right]^{2}\right)\right\}=0 \tag{4}
\end{equation*}
$$

and at this point the interfacial slope becomes vertical. The gradually varied flow assumption upon which Eq. 2 is based is no longer valid and infact the slick thickness reaches its maximum value just before this critical point is reached. Experiments have shown that if oil is added to a slick which has reached critical length, oil will commence to flow beneath the barrier and escape, irrespective of how deeply the barrier is immersed in the flow. Oil will continue to be lost until the slick is again of critical size.

It can be seen in Eq. 4 that the critical thickness ratio is independent of the magnitude of the viscous stresses and depends only upon the upstream Froude number. The curve AO in Fig. 2 shows the critical thickness ratio plotted as a function of Froude number. The range of thickness of a slick possible at any given Froude number, can be determined
from the ordinate distance between the frontal thickness curve $A B$ and the critical thickness curve $A O$. It could be anticipated from the decreasing ordinate distance that as the Froude number increases in value the volume of oil which may be contained in any given channel will also decrease. Experimental values of the critical slick thickness are in good agreement with Eq. 4.

Profiles of oil slicks can be calculated by numerical integration of Eg. 2 using Eq. 1 to predict the initial frontal thickness. Figure 3 shows the results of such a calculation for the laboratory oil slick shown in Fig. 1.


The Froude number in this experiment was 0.30 and the mean Reynolds number in the flow passing beneath the slick was 5,400 yielding a friction coefficient of 0.038. Calculations were made for the profiles of slicks having the above Froude number and boundary friction coefficient and were repeated for four values of the interfacial friction coefficient. Each profile is shown in Fig. 3 together with the experimental profile. It appears that the interfacial friction coefficient decreases slightly in value along the length of an oil slick. Similar behaviour was noted in other oil slick experiments and is not unexpected as thickening of a slick results in a convective velocity increase along the length of a slick and hence a similar increase in the Reynolds number of the flow passing beneath the slick. Reynolds numbers of the flows in the experiments were in the range

5,000 to 15,000 where an increase in Reynolds number leads to a decrease in the boundary, and probably the interfacial friction coefficients. Similar effects would probably not be observed in large scale slicks. Interfacial friction coefficients measured in laboratory experiments are generally of the order 0.01 , however it is expected that the development of interfacial waves in prototype slicks could result in an appreciable increase in this coefficient.

## OPTIMISING BOOM LAYOUTS IN RIVERS AND ESTUARIES

Very few rivers or estuaries will conform to the highly idealised geometry considered so far in this paper. Generally velocities will vary considerably in the horizontal so that in a single river cross-section there might be areas where conditions are critical for containment, and areas where containment is feasible. The problem then becomes that of directing oil out of the critical areas into the quieter regions where it can be removed using skimmers or oleophilic belts. Figure 4 shows a typical Froude number distribution across a river channel and two possible boom layouts.


Fig. 4. Typical Boom Configurations
If a boom is set across the channel as in $A A$, oil would tend to accumulate in the area where the current speed was greatest, and as in this particular case the Froude number in this area
exceeds the critical value, the oil will pass beneath the boom and will be lost. The second layout BB is far preferable since by angling the boom across the maximum velocity region, the velocity component normal to the boom is substantially reduced and the oil will be diverted to quieter regions of the river where containment is possible.

CONCLUSIONS

1) Oil slicks cannot generally be contained in areas where the densimetric Froude number of the current exceeds approximately 0.5 .
2) In any channel there is a limit to the volume of oil which can be contained as a single slick.
3) Judicious alignment of oil booms will often enable oil to be diverted from supercritical areas of a channel to quieter regions where containment and removal is possible.

APPENDIX 1
Analysis of the Frontal Region of a Contained oil slick
An expression is derived which relates flow conditions in a two-dimensional channel of finite depth, to the equilibrium thickness of an oil slick which can be contained on the water surface in the channel. The flow beneath the oil slick is assumed to be inviscid.

Consider the contained oil slick shown in Fig. 1. If one assumes steady uniform flow conditions exist at $A$ and $B^{\prime}$ then the sums of horizontal momentum and pressure force per unit width will be equal at both sections and are given by:

$$
\rho U^{2} D_{0}+\frac{\rho g D_{O}^{2}}{2}=\frac{\rho U^{2} D_{O}^{2}}{D_{0}+\varepsilon-t}+\frac{0(1-\Delta) g\left(D_{0}+\varepsilon\right)^{2}}{2}+\frac{\Delta \rho g\left(D_{0}+\varepsilon-t\right)^{2}}{2}
$$

where $U=$ velocity of flow upstream of the slick
$\rho=$ density of the water
$D_{0}=$ depth upstream of the slick (section 0 )
$g=$ gravitational acceleration
$\rho(1-\Delta)=$ density of the oil
$\mathrm{t}=$ frontal thickness of the slick
and $\quad \varepsilon=$ difference in elevations of the free surface at points $A$ and $B^{\prime}$.

In any real problem $\Delta \rho$ is small compared with $\rho$, and $\varepsilon$ is very small compared with $D_{0}$. Consequently pressure force terms involving $\triangle \rho E$ and $\varepsilon^{2}$ and momentum terms having $\varepsilon$ as a numerator will be neglected in the expansion of the flow force equation above. The flow force equation then reduces to the simpler form:

$$
\rho U^{2} D_{0}+\frac{\rho g D_{O}^{2}}{2}=\frac{\rho U^{2} D_{O}^{2}}{D_{0}-t}+\frac{\rho g D_{O}^{2}}{2}+\rho g D_{O} \varepsilon-\Delta \rho g D_{O} t+\frac{\Delta \rho g t^{2}}{2}
$$

The difference in levels of the free surface at sections 0 and $l$ can be obtained by considering the pressure at the stagnation point $B$ on the streamline $A B$. As there is no energy dissipation between points $A$ and $B$, the pressure $(p)$ at $B$ is given by the Bernoulli relationship

$$
\mathrm{p}=\frac{\rho U^{2}}{2}
$$

This pressure is balanced by an elevation in the surface level of the oil slick,
and therefore

$$
p=\rho(1-\Delta) g \varepsilon
$$

$$
\varepsilon=\frac{1}{1-\Delta} \frac{U^{2}}{2 g}
$$

The above equation can now be combined with the earlier flow force expression and non-dimensionalized to give

$$
F^{2}=\frac{F^{2}}{1-\phi}+\frac{1}{1-\Delta} \frac{F^{2}}{2}-\phi+\frac{\phi^{2}}{2}
$$

and re-arranging we get

$$
F^{2}=\phi(2-\phi)\left[\frac{2 \phi}{1-\phi}+\frac{1}{1-\Delta}\right]^{-1}
$$

where

$$
F=\frac{U}{\left(\Delta g D_{0}\right)^{\frac{1}{2}}}
$$

and

$$
\phi=\frac{t}{D_{0}}
$$

The flow force equation above relates the equilibrium thickness of a contained oil slick from conditions upstream of the slick.

## APPENDIX 2

Analysis of the Viscous zone of a Contained Oil slick
The viscous zone of a contained oil slick may be analyzed by considering the equilibrium of the streamwise gradient of pressure and intertial forces and the boundary shear stress. Consider the slick shown in Fig. l. The equilibrium of section $P R$ is given by

$$
\frac{\partial}{\partial x}\left\lceil\frac{\rho g(1-\Delta) D^{2}}{2}+\frac{\Delta \rho g(D-y)^{2}}{2}+\frac{p q^{2}}{D-y}\right\rceil=-\tau_{b}
$$

where $\tau_{b}$ is the boundary shear stress and $q$ is the flow per unit width beneath the slick. A similar expression can be derived for equilibrium of the slick itself ( QR ) yielding

$$
(1-\Delta) g y \frac{\partial D}{\partial x}=\tau_{i}
$$

where ${ }^{\top}{ }_{i}$ is the interfacial shear stress.
$\frac{\partial D}{\partial x}$ can be eliminated between the above two equations and non-dimensionalising gives

$$
\Delta \rho g y \frac{\partial y}{\partial x}\left\{1-\left[\phi+\left(\frac{F}{1-\phi}\right)^{2}\right]\right\}=\frac{\tau}{1-\Delta}\left\{1-\Delta\left[\phi+\left(\frac{F}{1-\phi}\right)^{2}\right]\right\}+\tau_{b} \phi
$$

## REFERENCES

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