CHAPTER 134

DISTORTED MODELING OF DENSITY CURRENTS

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ABSTRACT

The paper describes a method of choosing scales for the correct representation of convective spread in a hydraulic model involving densimetric phenomena. Although most prototype spreads are non-viscous in nature it is possible for small scale laboratory spreads to become affected by viscous influence when the spreading density front is remote from the outfall. This must be avoided if correct representation is to be achieved and the scales for the model must be chosen with this in view. It is often impossible to avoid viscous influence in a natural scale model of reasonable size and distortion of the vertical scale must then be employed.

INTRODUCTION

In attempting to design a physical model of phenomena involving densimetric spread, there are a number of aspects of the dispersion and diffusion process which must be considered. Ackers(1) has listed these as; jet diffusion, buoyant rise, convective spread, mass transport by ambient currents, ambient turbulent mixing and surface cooling (if applicable). Each of these has distinct scaling requirements and this paper concentrates on the requirements for convective spread with particular reference to outfalls located at the surface.

An existing procedure for convective spread is that due to Barr(2,3) and makes use of design diagrams obtained from lock exchange flow. Since the method to be described has been developed along lines suggested by Barr's approach, it is worthwhile to outline the basis of the lock exchange diagrams before proceeding to an evaluation of the new method.

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SCALING PROCEOURE BASEO ON LOCK EXCHANGE FLOW OATA

Figure 1 represents an illustration of basic exchange flow and it has been shown (2,3) that the distance travelled by the overflow, may be related to the time of travel by:

$$T/T_{\Delta} = \phi\left[L/H,\ \frac{(g')^{1/2} H^{3/2}}{\nu}\right] \quad \ldots\ldots\ldots (1)$$

where $T\Delta = (H/g')^{\frac{1}{2}}$; H = depth of flow; g' = 'reduced' nett gravitational acceleration at the outfall, $g\Delta\rho/\rho$; $\Delta\rho$ = density difference; ρ = base density; ν = kinematic viscosity; L = length of travel; T = time of travel.

Considerable experimental work resulted in a graphical representation of the relationship among these parameters, and this is shown in Figure 2, which forms the design diagram for choosing model scales.

A detailed explanation of Barr's method has been given elsewhere (1) but the method basically seeks to first establish details of an analogous, large scale, lock exchange mechanism which would give rise to the conditions expected to occur in the three dimensional prototype. Figure 2 can then be used to obtain scales which would allow of correct reproduction of rate of spread in a model lock exchange flow. It is surmised that if the scales are correct for the two dimensional phenomena they will also be correct for the three dimensional phenomena. Figure 2 indicates the effect of viscous influence on the rate of spread, the horizontal part of each curve of constant T/TΔ representing the region in which the spread is unaffected by viscous influence. Since most prototype spreads will be non-viscous, the objective is to ensure that the model relationship among the three parameters of Equation. 1 is such that the model spread also lies outside the viscous zone.

Consider an analogous lock exchange flow in which

$(g')^{1/2} H^{3/2}/\nu = 1.06 \times 10^6$ and L/H = 267. (Longannet data:(2) $\Delta\rho$ = .0015 gm./ml., H = 15 ft., L = 4,000 ft., and ν = 1.2×10^{-5} ft.^{2}/sec.). From figure 2, T/TΔ = 500. In a natural scale model, the distance and time parameters must have the prototype values and, to avoid viscous influence, $(g')^{1/2} H^{3/2}/\nu$ must therefore be greater than approximately 2×10^4. (From Figure 2)

The limiting scale of the non-viscous, natural scale is thus given by

$$\frac{1}{x} = \frac{H_m}{H_p} = \left(\frac{2 \times 10^4}{1.06 \times 10^6}\right)^{2/3} = 1/14 \quad \ldots\ldots\ldots (2)$$

where 1/x = horizontal scale and suffixes m and p refer to model and prototype respectively. A larger scale would allow the spread to travel further than the design extension with no viscous influence and a smaller scale would have the opposite effect.

If distortion is employed

$$(L/H)_m = (L/H)_p \cdot \frac{1}{e} \qquad \dots\dots\dots (3)$$

$$(T/T_\Delta)_m = (T/T_\Delta)_p \cdot \frac{1}{e} \qquad \dots\dots\dots (4)$$

$$\frac{(g')^{1/2} H^{3/2}}{\nu}_m = \frac{(g')^{1/2} H^{3/2}}{\nu}_p \cdot \left(\frac{x}{e}\right)^{3/2} \qquad \dots\dots\dots (5)$$

The required horizontal scale is normally fixed by space considerations, and equations 3 - 5 and Figure 2 are sufficient to calculate the minimum distortion necessary to ensure viscous flow in the model. (3) Greater distortion has the same effect as increasing the scale of a non-distorted model.

For the Longannet model, the horizontal scale was set at 1/400 and on this basis the required distortion was found to be 6.7.

CRITICISM OF THE DESIGN METHOD

Various assumptions are implicit in the method, and these are worth comment.

1. The spread of a lock exchange flow is entirely convective compared with the normal circumstances of an outfall where there is almost certainly some jetting action. However, any element of forced flow in the model will inhibit viscous influence. Thus, if model scales are chosen to ensure that a convective flow is non-viscous, a forced flow in the model must also be non-viscous and would remain so at greater extensions than the design extension. Barr's design method is, therefore, conservative and errs on the safe side.

2. Figure 2 was prepared from experiments on two dimensional data and doubts have been raised (4) as to the applicability of that data to three dimensional spread. The validity of this criticism depends, to some extent, on site conditions which govern the shape of the spread, but the use of Figure 2 could be justified simply by the absence of more relevant information.

3. Before model scales can be chosen, it is necessary to assume a large scale lock exchange which is thought to be analogous to the prototype spread, i.e. one which will produce the same spread conditions as are expected to occur in the prototype. There can be

no doubt that this must be extremely difficult unless considerable first-hand experience of exchange flow is available and this severely limits application of the method.

DESIGN DIAGRAMS BASED ON THREE DIMENSIONAL SPREAD

A simple representation of three dimensional spread from a buoyant surface discharge is shown in Figure 3. It is assumed that an effluent discharge, Q, of density, ρ, flows through a circular pipe of diameter D into a quiescent homogeneous body of receiving fluid of density $\rho + \Delta\rho$. The effluent and the receiving fluid are miscible, and the kinematic viscosity of each, ν, is assumed to be the same.

By similitude reasoning it can be shown [5] that

$$\phi\left[\frac{L(g')^{1/5}}{Q^{2/5}},\ \frac{T(g')^{3/5}}{Q^{1/5}},\ \frac{Q(g')^{1/3}}{\nu^{5/3}},\ \frac{D(g')^{1/5}}{Q^{2/5}}\right] = 0 \quad \ldots\ldots (6)$$

where L = distance from source to tip of front
and T = time of travel.

The parameter $D(g')^{1/5}/Q^{2/5}$ is a form of outfall densimetric Froude number and, for constant values of $D(g')^{1/5}/Q^{2/5}$

$$\phi\left[\frac{L(g')^{1/5}}{Q^{2/5}},\ \frac{T(g')^{3/5}}{Q^{1/5}},\ \frac{Q(g')^{1/3}}{\nu^{5/3}}\right] = 0 \quad \ldots\ldots (7)$$

Experiments conducted at two values of $D(g')^{1/5}/Q^{2/5}$ have indicated [5] that the relationship among the parameters of equation 7 is as given in Figures 4 and 5. These show similar features to Figure 2 and provide alternative design diagrams for scaling procedures.

Prior to experimental work, care was taken to ensure that the two values of $D(g')^{1/5}/Q^{2/5}$ chosen for study would be suitable for application to prototype outfalls.

Tamai [6] has suggested that most cooling water outfalls operate at densimetric Froude numbers of between 2.0 and 14.0 but, due to the size of apparatus available, it was not feasible to obtain the larger of these limits in the laboratory. Efforts were therefore concentrated on low values of the outfall Froude number and $D(g')^{1/5}/Q^{2/5}$ = 1.16 and 0.58 correspond to densimetric Froude numbers of 0.88 and 5.0 respectively.

SCALING PROCEDURE

The design method is basically similar to that described for lock exchange flow but is less dependent on assumed conditions.

For circular outfalls, values of the basic prototype parameters may be obtained directly from prototype data. Either diagram may be used depending on the proposed outfall conditions, and it may be necessary to estimate the final scales from calculations involving both diagrams. However, it will be shown later that the actual value of $D(g')^{1/5}/Q^{2/5}$ is not of great relevance during the scaling procedure.

Consider the Longannet outfall. Basic prototype data are; (2,7) Q = 3,200 ft.³/sec., $\Delta\rho$ = .0015 gm/ml., L = 4,000 ft., and ν = 1.2 x 10^{-5} ft²/sec. (extension L = 4,000 ft. was chosen as the limit of spread for which correct representation was required in the model.)

Then $$\frac{L(g')^{1/5}}{Q^{2/5}} = 86.5 \qquad (8)$$

and $$\frac{Q(g')^{1/3}}{\nu^{5/3}} = 1.85 \times 10^{11} \qquad (9)$$

From Figure 4

$$\frac{T(g')^{3/5}}{Q^{1/5}} = 220 \qquad (10)$$

In a natural scale model, the time and distance parameters must have the same value as in the prototype. Thus the minimum model value of $Q(g')^{1/3}/\nu^{5/3}$ which would avoid viscous influence is

$$Q(g')^{1/3}/\nu^{5/3} = 0.6 \times 10^6$$

and for a natural scale model the scale is given by

$$\frac{1}{X} = \left(\frac{Q_m}{Q_p}\right)^{2/5} = \left(\frac{0.6\times10^6}{1.85\times10^{11}}\right)^{2/5} = 1/157 \qquad (11)$$

A similar process applied to Figure 5 leads to

$$\frac{1}{X} = \left(\frac{2.3\times10^6}{1.85\times10^{11}}\right)^{2/5} = 1/93 \qquad (12)$$

These scales show the beneficial effect of the outfall jetting action. Barr's data for convective flow leads to a natural scale of 1/14. For an outfall with low jetting action (Figure 5) the limiting scale is 1/93, and with a greater forced element of flow (Figure 4) the scale is 1/157.

EFFECT OF DISTORTION

The effect of distortion is to reduce the required model value of $L(g')^{1/5}/Q^{2/5}$, thus allowing smaller $Q(g')^{1/3}/\nu^{5/3}$ without entering the viscous zone.

$$\text{ie} \quad \frac{L(g')^{1/5}}{Q^{2/5}}m = \frac{L(g')^{1/5}}{Q^{2/5}}p \cdot \frac{1}{e^{3/5}} \qquad \ldots\ldots (13)$$

$$\frac{T(g')^{3/5}}{Q^{1/5}}m = \frac{T(g')^{3/5}}{Q^{1/5}}p \cdot \frac{1}{e^{4/5}} \qquad \ldots\ldots (14)$$

$$\text{and} \quad \frac{Q(g')^{1/3}}{\nu^{5/3}}m = \frac{Q(g')^{1/3}}{\nu^{5/3}}p \cdot \frac{e^{3/2}}{x^{5/2}} \qquad \ldots\ldots (15)$$

For a horizontal scale of 1/400 and a distortion of 2, and taking the prototype values given in equations 8 - 10;

$$\frac{L(g')^{1/5}}{Q^{2/5}}m = 56.2 \qquad \ldots\ldots (16)$$

$$\frac{T(g')^{3/5}}{Q^{1/5}}m = 126 \qquad \ldots\ldots (17)$$

$$\frac{Q(g')^{1/3}}{\nu^{5/3}}m = 0.16\times10^{6} \qquad \ldots\ldots (18)$$

Figure 4 shows that these points lie just outside the viscous zone and thus, in a model, no viscous influence would be observed throughout a spread of 4,000 model feet.

Applying the same analysis to Figure 5 shows that, in the case of less jet action ($D(g')^{1/5}/Q^{1/5} = 1.16$) the model spread would become viscous in the latter part of the design extension. However, with a distortion of 4.0, both diagrams indicate freedom from viscous influence and thus, provided the outfall densimetric Froude number is greater than 0.88, a horizontal scale of 1/400 and a distortion of 4 would give a model spread, equivalent to 4,000 ft. prototype, free from viscous influence.

An additional constraint is that the correlation between the length and time parameters in the model must be identical to that obtained in the prototype. Figure 6 illustrates the experimentally determined correlation for non-viscous spread and shows that $D(g')^{1/5}/Q^{2/5}$ must have the same value in model and prototype. The outfall must thus be designed on a standard Froudian basis. Distortion effects are also given in Figure 6 which shows that any distortion alters the correlation in such a way as to reduce the required rate of spread in the model below that which will be achieved. The overall effect is, thus, twofold. Distortion can eliminate the gross errors of viscous influence, but at the same time reduces the possibility of achieving the correct rate of spread.

It is noticeable that the discrepancy increases as the distortion is increased, and it is therefore important, from the point of view of spread, to keep the design distortion to a minimum. Errors are greater at the higher value of $D(g')^{1/5}/Q^{2/5}$ than at the lower value, and this indicates that distortion causes greater misrepresentation of spread when the jetting action is low than would be the case for an outfall with a considerable element of forced flow. For $D(g')^{1/5}/Q^{2/5} = 0.58$ the errors are probably negligible.

These errors are not indicated by two dimensional data because in lock exchange flow there is little diminution of tip velocity as the spread moves away from the source. Thus a non-viscous spread diagram for lock exchange flow results in a straight line, and equations 3 and 4 show that exaggeration would simply move the prototype point (e = 1) along the line towards the origin.

RELEVANCE OF THE NON DIMENSIONAL GROUP $D(g')^{1/5}/Q^{2/5}$

As has been stated, the model outfall must be designed to have the same value of $D(g')^{1/5}/Q^{2/5}$ as the prototype. However, its actual value is not of great relevance for the purpose of choosing scales. The variation between the scales chosen on the basis of Figures 4 and 5 is not extreme. One diagram gives a natural scale of 1/93, the other 1/157. With a horizontal scale of 1/400, the calculated exaggerations are 2.0 and 4.0. Thus, it would not be difficult to estimate the required scales for an outfall having a value of $D(g')^{1/5}/Q^{2/5}$ between, or near, those chosen for the design diagrams. In any case, it is probable that most models involving outfall studies would require a greater exaggeration than 4.0 simply to obtain turbulent conditions, and this would ensure the formation of non-viscous spread for any densimetric Froude number greater than 0.88.

CONCLUSIONS

The new approach follows the lines laid down by Barr using two dimensional data [3] but has three distinct advantages.

1. There is no need to imagine an analogous lock exchange flow, and values of the relevant parameters can be calculated directly from prototype data.

2. The new approach makes use of three dimensional data instead of two dimensional data. Calculations have shown that exaggeration causes errors which are not indicated by Barr's congruency diagram but, for an outfall with $D(g')^{1/5}/Q^{2/5}$ less than 0.58, the magnitude of the discrepancy is relatively small.

3. The method includes consideration of the forced element of the flow and provides a more accurate result. There is less tendency towards over-design.

A comparison with Barr's method has shown that the Longannet model was probably distorted more than was necessary to obtain correct reproduction of the spread phenomenon. This resulted from scales being chosen from data pertaining to a purely convective flow. However, it is not suggested that the Longannet model was over-distorted. Other conditions, such as estuarine circulation and turbulence, are relevant, and what must be done is to ensure that the design distortion is not less than that required to provide conditions under which the model will correctly reproduce the prototype spread. Greater distortion than the minimum builds in a factor of safety against viscous influence but introduces errors, which may be important, and an effort should, therefore, be made to keep the design distortion to a minimum.

The scaling procedure may be criticised on the basis that it assumes dilution, velocity structure and depths of spread to be correctly simulated, provided rate of spread is accurately reproduced. Since the main aim of the design method is to ensure freedom from viscous effects, it is considered that this assumption is not unreasonable, but obviously there is a need for further data regarding the structure of three dimensional spread.

Application of the data to circular, closed conduit outfalls is straightforward, but it is more difficult to relate the data to channel outfalls. It may be that Figures 4 and 5 should be considered as interim design diagrams until information is available for open channel outfalls, but, until such time, they represent a more satisfactory design basis than is otherwise available.

In applying this scaling procedure it should be remembered that this applies to only one aspect of the spread phenomena. The other aspects which must be considered have different scaling requirements and, to ensure a satisfactory model, it is necessary to choose a scale which will meet all requirements or, since this is often impossible, to determine the best possible compromise.

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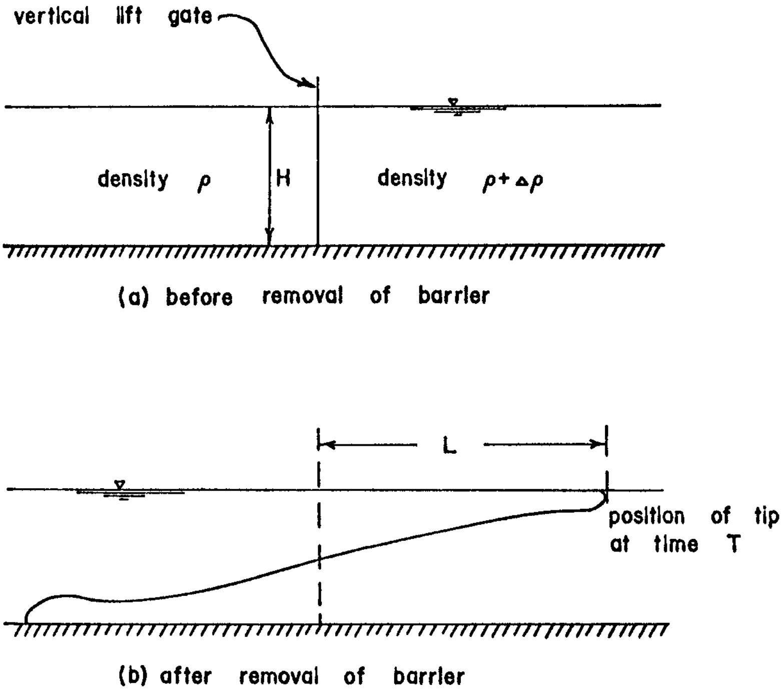
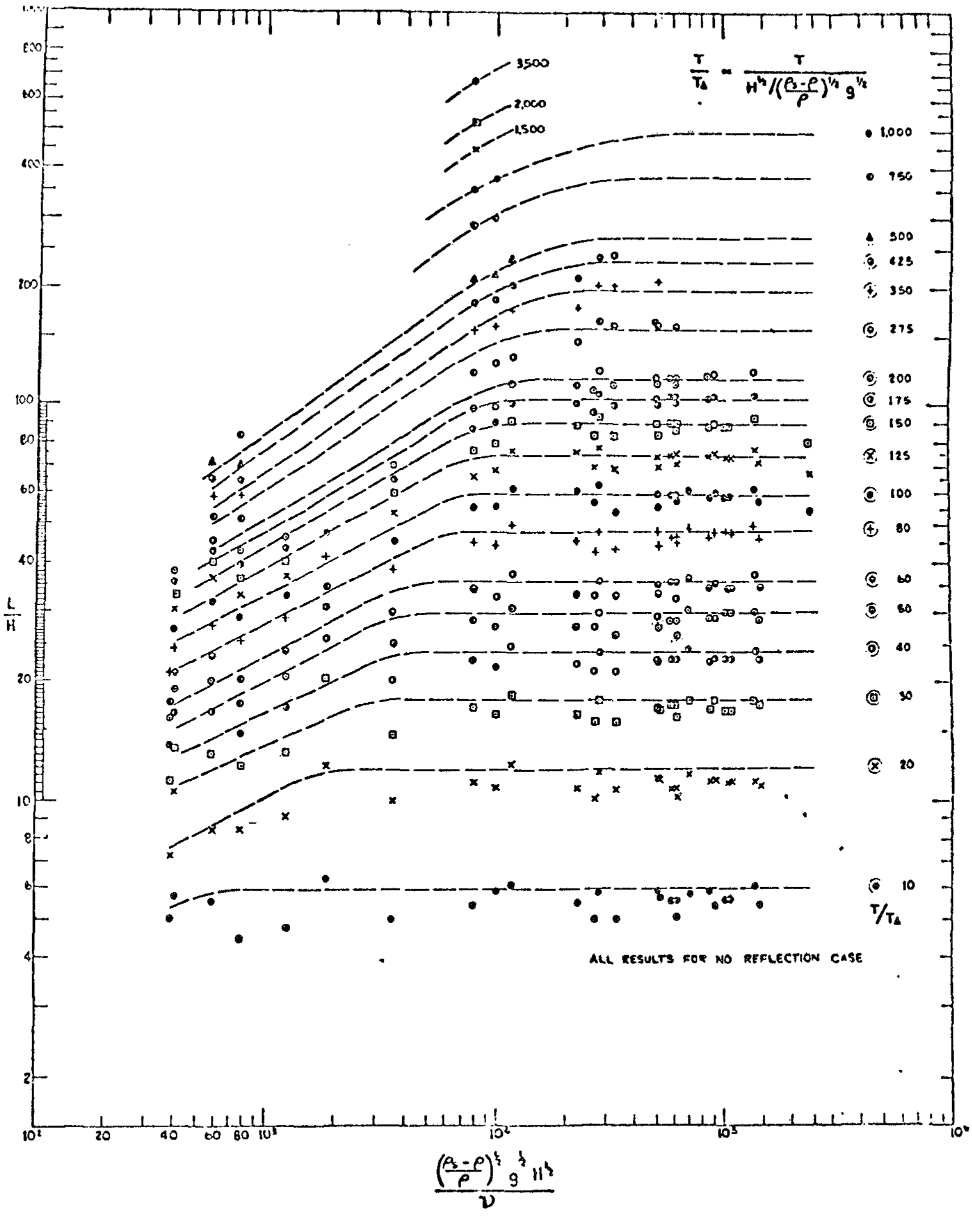


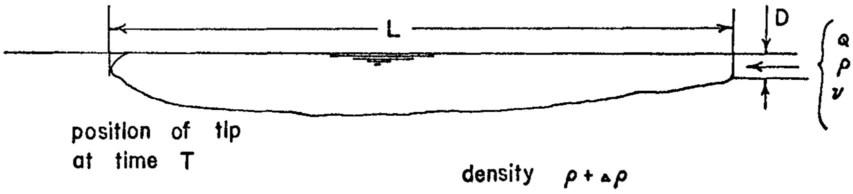
DIAGRAMMATIC ILLUSTRATION OF LOCK EXCHANGE FLOW

FIGURE 1



BARR'S CONGRUENCY DIAGRAM FOR OVERFLOW SPREAD

FIGURE 2



THREE DIMENSIONAL SPREAD

FIGURE 3

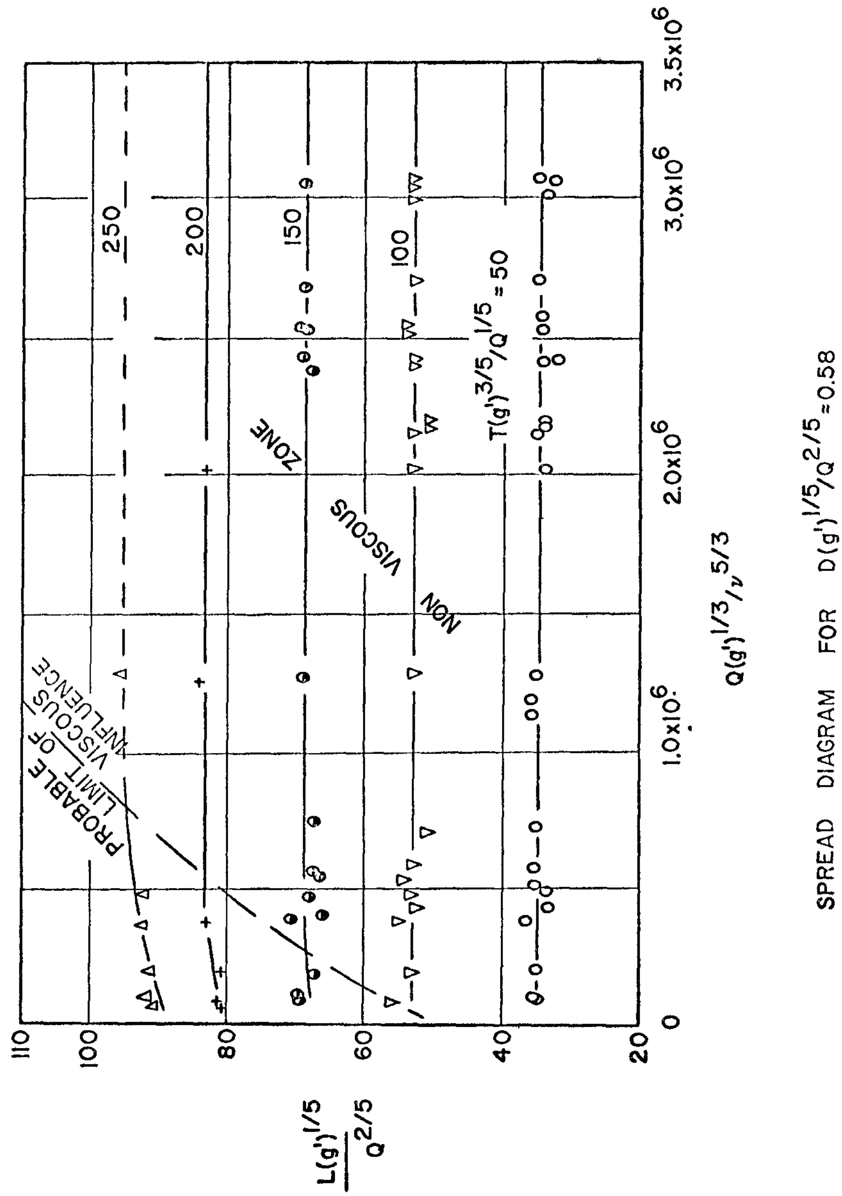

SPREAD DIAGRAM FOR D(g')^1/5/Q^2/5 = 0.58
L(g')^1/5 / Q^2/5
Q(g')^1/3/ν^5/3
T(g')^3/5/Q^1/5 = 50
100
150
200
250
PROBABLE LIMIT OF VISCOUS INFLUENCE
NON VISCOUS ZONE
0
1.0x10^6
2.0x10^6
3.0x10^6
3.5x10^6
20
40
60
80
100
110

FIGURE 4

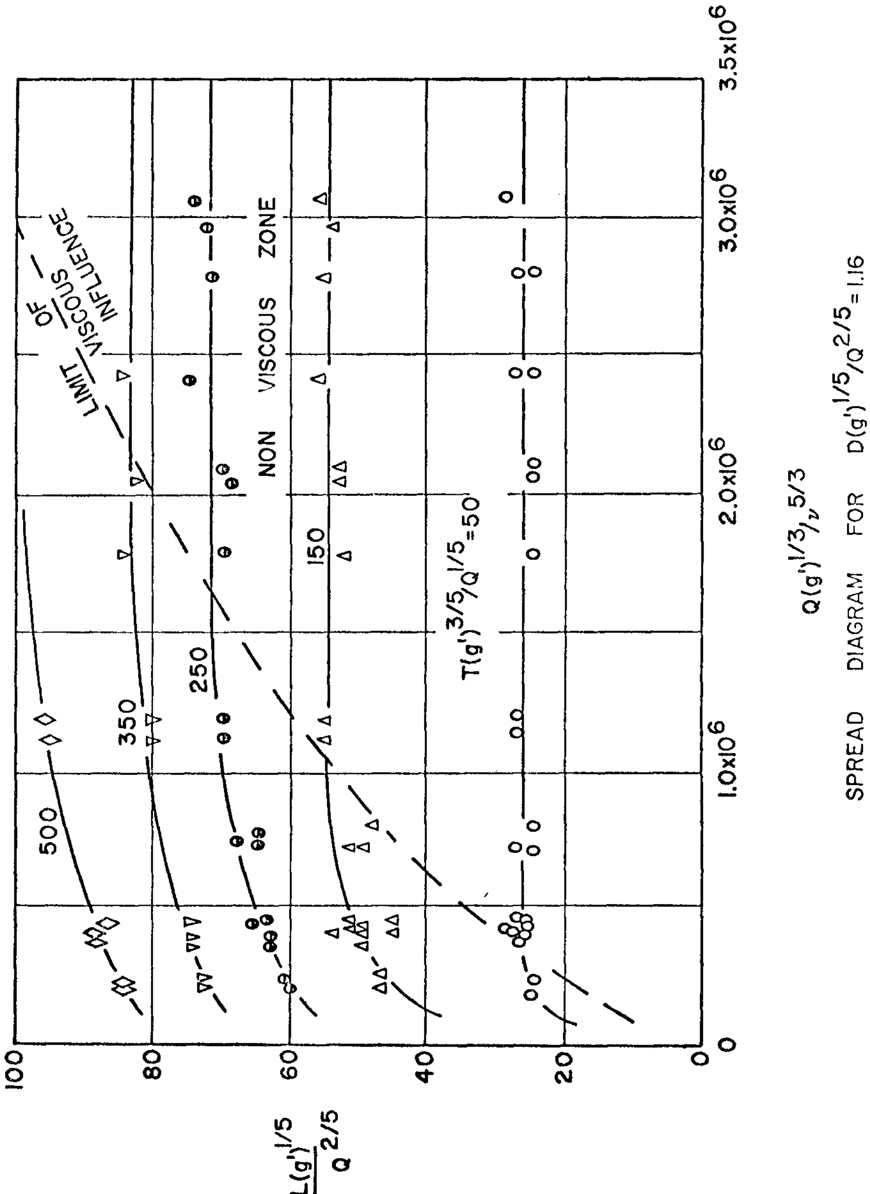
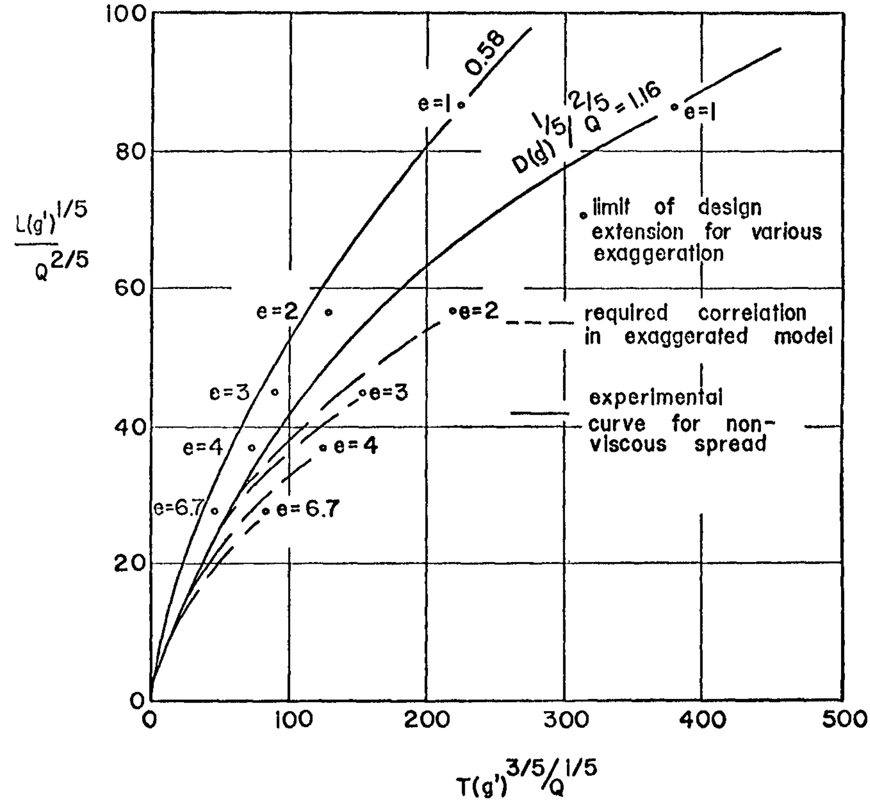

LIMIT OF VISCOUS INFLUENCE
NON VISCOUS ZONE
500
350
250
150
$T(g')^{3/5}/Q^{1/5}=50$
$\frac{L(g')^{1/5}}{Q^{2/5}}$
100
80
60
40
20
0
0
1.0×10^6
2.0×10^6
3.0×10^6
3.5×10^6
$Q(g')^{1/3}/\nu^{5/3}$
SPREAD DIAGRAM FOR $D(g')^{1/5}/Q^{2/5}=1.16$

FIGURE 5



NON VISCOUS SPREAD

FIGURE 6