CHAPTER 132

Analytical Modeling of Estuarine Circulation

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Abstract

A mathematical model is developed using analytical techniques to determine the longitudinal and vertical distributions of velocities and salinities, averaged over a tidal period, for mixed but partially stratified estuaries. The flow is assumed laterally homogeneous and the estuary width and depth are assumed to be functions of the longitudinal coordinate only. Required inputs to the model include the salt intrusion length, the ocean boundary salinity, the distribution of the depth-averaged salinity and the freshwater discharge.

The governing equations included in the model are the vertical and longitudinal equations of motion, continuity, salt conservation and an equation of state. The key assumption is made that the longitudinal salinity gradient is independent of depth. This decouples these equations and thus permits an analytical solution to be found.

Using data from laboratory flume tests and field surveys the model solutions are used to find correlations for the mean vertical transfer coefficients of mass and momentum with gross characteristics of the estuary. These correlations, plus the results from a one-dimensional numerical model, permit this analytical model to be used as a predictor of the velocity and salinity profiles in estuaries and to relate changes in freshwater discharge to possible changes in the location of shoaling zones.

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Introduction

Physical and mathematical models of estuaries are developed in order to represent the complex circulation of the prototype in a simplified form which can be tested and studied to determine the possible consequences of modifications of controlling factors on the natural circulation. Examples of such changes could include the dredging of a navigation channel, the diversion of freshwater inflow to other basins, or the placement of a diffuser for the heated condenser water of an electric power station. The former might seriously alter the salinity distribution while the latter could obviously influence normal biological cycles. Recourse to various types of models must be made to provide estimates of the impact of such changes.

At the present time, physical models of estuaries are the most important technique for determining the effects of changes in the prototype. Their great expense and slow building and operating times are drawbacks which sophisticated mathematical models may avoid. However, one can expect these physical models to continue to be important tools for estuarine analysis for a long time to come.

Mathematical models include both numerical and analytical models. At the present time, the numerical models available for engineering applications are of either the unsteady one- or two-dimensional type. A one-dimensional model averages all dependent variables over the cross-sectional area, and thus yields changes in mean values with time and along the longitudinal axis. These models can be used to predict water surface elevation, mean currents, and mean salinities. They can also be used with certain reservations to determine the cross-sectional mean concentration of a non-conservative water quality parameter, such as dissolved oxygen or biochemical oxygen demand.

Two-dimensional numerical models usually allow variations along the lateral as well as along the longitudinal axis. In this case, the only averaging is with depth. Again, these models can predict currents, water quality parameters, etc. These models are more complex than the one-dimensional case with regard to the computational techniques required.

This investigation develops a two-dimensional analytical model of estuarine circulation including vertical and longitudinal distributions of velocity and salinity. All equations are averaged over one or more tidal periods. This model can be coupled with a one-dimensional numerical model which is not time-averaged, but is averaged over a cross-section.

The ability to calculate vertical variations of the important flow parameters is often a useful tool for solving estuarine problems. Vertical salinity stratification is a key element in the circulation pattern of an estuary. Models which can predict the effects of changing geometry, freshwater inflows, etc. on this stratification are of great value. The modeling of vertical velocity profiles is another useful model capability. Many problems of shoaling in estuaries can only be properly studied with a knowledge of the vertical distribution of velocity.

If a model similar to the one described above is to have practical application as a predictive tool, all parameters included in the solution technique must be determinable in advance. Thus, an important part of the objectives of this study is to obtain relationships between the various time-averaged coefficients of turbulent diffusion and eddy viscosity included in the model and the gross parameters of estuarine circulation.

Previous Investigations

Pritchard (1952) describes the circulation in the Chesapeake Bay estuarine system, and in particular, in the James River estuary. Data from an extensive program of field surveys are discussed, in which salinities, temperatures and velocities were measured at several depths and stations and averaged over one or more tidal periods. The resulting net circulation and salinity distributions are typical for partially stratified conditions. A basic feature of this net circulation is a reversal in the vertical distribution of the time-averaged horizontal velocity. In the surface region, extending to about middepth, the net flow is towards the ocean, while the bottom region has flow in the opposite direction, towards the river end of the estuary. The depth integral of this velocity is equal to the net discharge of freshwater. Although two regions can be identified for the velocity, the vertical salinity distribution can not be separated into two distinct zones. In partially stratified estuaries, there is a continuous increase in salinity from the surface to the bottom, without a noticeable point of discontinuity.

Pritchard (1952, 1954) also identifies several interesting features of the longitudinal salinity gradient. For all depths, there is an increase in salinity from the freshwater region to the boundary salinity at the ocean end. In addition, over most of the estuary this longitudinal salinity gradient is nearly independent of depth, i.e., vertical position. This latter feature does not hold very near the ocean boundary or where the salinity goes to zero, upstream.

Hansen and Rattray (1965) present an analytical model of estuarine circulation averaged over one or more tidal periods. A simultaneous solution of the equations of mass and momentum conservation, assuming geometric similarity of velocity and salinity profiles and lateral homogeniety is developed. The estuary is divided into three regions inner, central and outer, for which different assumptions about salinity gradients and mixing coefficients are made.

McGregor (1972) develops an analytical model of the net, non-tidal bottom transport velocity for an estuary. This model is similar to other studies in that a longitudinal force balance includes only the pressure gradient and the vertical eddy stress gradient. For the pressure gradient, both a surface slope and density gradient are evaluated from recorded data for the Humber estuary. The solution technique introduces a number of empirical constants for fitting these distributions, as well as an empirical expression for the mean eddy viscosity. By proper fitting of the numerous constants, McGregor is able to match the net bottom velocity zero points with the shoaling zones for the Humber. The analysis is a good illustration of the roles of the surface slope, salinity gradient and river discharge in determining the zones of high rates of shoaling. However, due to the need to fit several constants to previous data, the model is of limited predictive capability.

Theoretical Considerations

The objective of the present study is to develop an analytical model of time-averaged estuarine circulation which will avoid the less tractable features of the previous models described above. The governing equations are similar to Hansen and Rattray's model, which was originally suggested by Pritchard's analysis of the James River estuary. A solution technique which is continuous over the entire length of an estuary is desired and which makes no assumptions about similarity of velocity or salinity profiles. Only the vertical eddy flux of salt and momentum are included, and thus only two eddy coefficients need to be specified.

In order to provide the analytical solution with a predictive capability, empirical correlations for these two parameters with gross characteristics of the flow field are sought, as a fundamental feature of the complete solution.

The model equations describing the circulation and distribution of salinity are the equations of motion, of continuity, of conservation of salt and an equation of state. The model is reduced to the longitudinal and vertical dimensions by assuming lateral homogeneity. The orientation of the coordinate system is with the x-axis positive towards the head of the estuary (upstream) and the y-axis positive downward. An additional simplification is made restricting the width b(x) and the mean water level h(x) to be functions of the longitudinal coordinate only. An inflow of freshwater Q_f occurs at the far upstream end.

For the conditions described, the conservation of momentum for the longitudinal direction can be written

$$\frac{\partial ub}{\partial t} + \frac{\partial u^2 b}{\partial x} + \frac{\partial uvb}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} b \qquad (1)$$

where u is the velocity in longitudinal direction, v is the velocity in vertical direction, t is the time, ρ is the density, p is the pressure, x is the longitudinal direction, y is the vertical direction, and b is the width. This equation is a balance of forces for the estuary at any time in a tidal period, i.e., before time-averaging. The viscous frictional terms and Coriolis forces have been neglected. In addition, the approximation of Boussinesq has been applied to neglect density variations in all but the buoyancy terms. The pressure is for the fluid only, atmospheric pressure being assumed constant.

For the conservation of momentum in the vertical direction, hydrostatic conditions are assumed. Thus, inertial and convective accelerations are neglected. The vertical equation of motion can therefore be written

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} + g \qquad (2)$$

where g is the acceleration of gravity.

The conservation of salt equation, before time-averaging, and neglecting molecular diffusion is

$$\frac{\partial sb}{\partial t} + \frac{\partial usb}{\partial x} + \frac{\partial vsb}{\partial y} = 0 \qquad (3)$$

where s is the salinity and is a function of x, y, and t.

For incompressible flow, the two-dimensional equation of continuity is

$$\frac{\partial ub}{\partial x} + \frac{\partial vb}{\partial y} = 0 \qquad (4)$$

There are three time scales of interest for the model being considered. Turbulent fluctuations of the dependent variables may be assumed to take place within a few minutes. These variables also have a diurnal or semi-diurnal component due to the tidal motion. Finally, slow variations over several tidal periods can result from the changing freshwater inflows and monthly changes in tidal amplitude. The details of the time-averaging of these equations is given by Fisher, et al. (1972).

From the analysis of the James River data, Pritchard (1954, 1956) argues that the dominant terms in the longitudinal equation of motion (1) are the pressure gradient and the vertical eddy flux of momentum, all other terms being of second order. This assumption is included in the present development. For the salt balance (3) the tidal cross-products and horizontal eddy flux are neglected by similar arguments. The reduced equations are further simplified by introducing mean eddy coefficients for the remaining turbulent terms

$$-\frac{\partial \langle \overline{u'v'} \rangle}{\partial y} \equiv \frac{\partial}{\partial y}\left(D_y \frac{\partial U}{\partial y}\right) \tag{5}$$

$$-\frac{\partial \langle \overline{v's'} \rangle}{\partial y} \equiv \frac{\partial}{\partial y}\left(K_y \frac{\partial S}{\partial y}\right) \tag{6}$$

where: —— is average over turbulence, $\langle\ \rangle$ is average over tidal period

where u', v', s' are turbulent components and U and S are averages over a tidal period of the horizontal velocity and salinity, respectively. D_y and K_y are mean eddy coefficients. These definitions for D_y and K_y are convenient with regard to reducing the mathematical complexity of the model. However, they are strictly artificial in that they do not preserve the mechanisms of turbulent mixing, i.e., tidal activity, in their formulation. In particular, Equation 5 relates the net turbulent momentum flux $\langle\overline{u'v'}\rangle$ to the net, non-tidal velocity U. By purely physical arguments this flux should be related to the tidal velocity u_t. This apparent inconsistency is partially resolved since D_y is later correlated with the tidal velocity. The equations are now written

$$0 = -\frac{1}{\rho_m}\frac{\partial P}{\partial x} + \frac{1}{b}\frac{\partial}{\partial y}\left(bD_y \frac{\partial U}{\partial y}\right) \tag{7}$$

$$0 = -\frac{1}{\rho_m}\frac{\partial P}{\partial y} + g \tag{8}$$

$$\frac{\partial Ub}{\partial x} + \frac{\partial Vb}{\partial y} = 0 \tag{9}$$

$$\frac{\partial Sb}{\partial t} + \frac{\partial bUS}{\partial x} + \frac{\partial bVS}{\partial y} = \frac{\partial}{\partial y}\left(bK_y \frac{\partial S}{\partial y}\right) \tag{10}$$

The value and distributions of the mean eddy coefficients are unknown. If a solution to the above set of equations can be shown to match recorded data by proper fitting of D_y and K_y, one must assume that either all the neglected terms are zero, or more probably, that these neglected terms have been absorbed into these coefficients. A comparison of Equations 5 and 6 with the classical definitions of eddy viscosity and eddy diffusivity clearly shows the difference in the meaning of these terms

$$\frac{\partial}{\partial y} (<\varepsilon_y \frac{\partial \tilde{u}}{\partial y}>) \neq \frac{\partial}{\partial y} (D_y \frac{\partial U}{\partial y}) \tag{11}$$

$$\frac{\partial}{\partial y} (<k_y \frac{\partial \tilde{s}}{\partial y}>) \neq \frac{\partial}{\partial y} (K_y \frac{\partial S}{\partial y}) \tag{12}$$

More specifically, D_y and K_y are not simply ε_y and k_y averaged over a tidal period.

The effect of temperature on the relationship between density and salinity is not included in this model. A simple linear empirical expression is used

$$\rho = \rho_o(1 + \alpha s) \tag{13}$$

where ρ_o is a reference density and α is a conversion constant. The range of temperatures encountered in estuaries does not require a more complex expression, in light of other model assumptions.

Additional Assumptions

The governing equations developed in the preceeding sections can not be solved analytically in their present form. Previous investigators have introduced similarity assumptions for the velocity and salinity distributions as well as restrictions on the longitudinal salinity gradient. As stated in a preceeding section, the present investigation seeks to avoid the limitations of a similarity solution. However, as will be developed in the following sections, the longitudinal salinity gradient will be modified to allow an analytical solution to be found.

The Pritchard (1952, 1954) investigation of the James River revealed that for the stations and conditions of the survey, the longitudinal salinity gradient did not vary appreciably with vertical position. Harleman and Ippen (1967) showed a similar pattern for the analysis of data from a laboratory flume. Taken to the extreme, this observed feature suggests that the longitudinal salinity gradient may be assumed independent of its vertical position, i.e.,

$$\frac{\partial S}{\partial x} = \frac{\partial S}{\partial x}(x), \text{ although } S = S(x,y) \tag{14}$$

The longitudinal salinity gradient $\frac{\partial S}{\partial x}$ is replaced in Equations 7-10 with the longitudinal gradient of a depth averaged salinity S_d. Next, a steady-state condition is assumed. In addition, the two mean eddy coefficients D_y and K_y are assumed independent of vertical position. These coefficients have been shown to represent the rather complex effects of time-averaging and of the neglecting of terms considered of smaller order. The vertical dependence of these coefficients is not known, although several investigators have attempted to analyse these terms from experimental and field observations. Thus, D_y and K_y are assumed to be independent of y, and are replaced with effective coefficients for the entire depth of flow, D and K, respectively.

As with most problems of fluid dynamics, it is convenient to develop analytical solutions in a non-dimensional form in order to permit generalized discussions of results. The choice of terms introduced to non-dimensionalize the various dependent and independent variables, although somewhat arbitrary, should recognize the possible difficulties in quantifying these new parameters. The following definitions will be shown to satisfy this condition:

$$\eta \equiv \frac{y}{h} \qquad \xi \equiv \frac{x}{L_i} \qquad \psi \equiv \frac{\Psi}{Q_f} \qquad \theta \equiv \frac{S}{S_o} \qquad \theta_d \equiv \frac{S_d}{S_o} \tag{15}$$

where L_i is the mean intrusion length, defined as the distance from the ocean boundary to a point where the time-averaged, depth averaged salinity is one percent of the ocean salinity. S_o is the ocean salinity, h is the depth of the mean water level and Q_f is the freshwater discharge, as previously noted.

Ψ is a stream function satisfying continuity. These quantities are introduced into the governing equations which have been reduced by assuming the Boussinesq approximation.

$$\left\{\frac{gS_o h^4 b}{L_i D Q_f}\right\} \frac{\partial \theta_d}{\partial \xi} = - \frac{\partial^4 \psi}{\partial \eta^4} \tag{16}$$

$$- \frac{\partial \Psi}{\partial \eta} \frac{\partial \theta_d}{\partial \xi} + \frac{\partial \theta}{\partial \eta} \frac{\partial \psi}{\partial \xi} = \frac{bKL_i}{Q_f h} \frac{\partial^2 \theta}{\partial \eta^2} \tag{17}$$

wherein ψ, θ, θ_d, ξ and η are all dimensionless variables. These equations can be further simplified by defining two coefficients,

$$C_1(\xi) = \frac{g\alpha S_o h^4 b}{L_i D Q_f} \tag{18}$$

$$C_2(\xi) = \frac{KL_i b}{Q_f h} \tag{19}$$

The boundary conditions included in this analysis are that the net velocity gradients at both the surface and the bottom are zero, that the depth-average of the velocity is equal to the freshwater velocity, that the vertical salinity gradient is zero at the surface, and that the depth-average of the salinity is equal to the specified mean value, S_d. In non-dimensional form these conditions are

$$\eta = 0; \quad -\frac{\partial^2\psi}{\partial\eta^2} = 0, \quad \frac{\partial\theta}{\partial\eta} = 0, \quad \psi = 1$$

$$\eta = 1; \quad -\frac{\partial^2\psi}{\partial\eta^2} = 0, \quad \psi = 0 \tag{20}$$

$$\int_0^1 \theta d\eta = Sd/S_o$$

The solution of Equation 16 is found by integration, and after substitution of the flow boundary conditions may be written

$$\psi = \frac{\partial\theta_d}{\partial\xi}\frac{C_1}{24}\{-\eta^4+2\eta^3-\eta\}-\eta+1 \tag{21}$$

This expression for the stream function is similar to the solution of Hansen and Rattray (1965) in form, differing only by the choice of boundary conditions and the restrictions on the longitudinal salinity gradient.

The solution of the salt balance Equation (17) is found by multiplying by an integration factor. The resultant equation for the salinity is

$$\theta(\xi,\eta) = \int f(\xi,\eta)d\eta+\theta_d-\int_0^1\int f(\xi,\eta)d\eta d\eta \tag{22}$$

where

$$f(\xi,\eta) = \exp\left(\int \frac{B}{C_2}d\eta\right)\int \frac{A}{C_2}\exp\left(\int -\frac{B}{C_2}d\eta\right)d\eta \tag{23}$$

$$+ \; b_1(\xi)\exp\left(\int \frac{B}{C_2}d\eta\right)$$

and

$$A(\xi,\eta) = -\frac{\partial\psi}{\partial\eta}\frac{\partial\theta_d}{\partial\xi} \qquad B(\xi,\eta) = \frac{\partial\psi}{\partial\xi}$$

The depth average of the salinity, averaged over a tidal period, S_d and its longitudinal gradient $\frac{\partial S_d}{\partial x}$ must both be specified in the solutions. For the purpose of evaluating the model from recorded data, these parameters can be simply backfigured from the measurements. However, in order for the analytical model to have a predictive capability, these terms must be predictable themselves. There have been numerous semi-empirical fits for this one-dimensional salinity distribution, Harleman and Ippen (1961), McGregor (1972) and others. However, a recently developed numerical model by Thatcher and Harleman (1972) permits one to compute a one-dimensional unsteady salinity distribution. This approach results in a general, non-empirical analysis for this input parameter. The intrusion length can also be evaluated by their technique.

The freshwater inflow and ocean boundary salinity are considered to be fundamental quantities, as are the depth and width distributions.

The remaining two quantities needed to evaluate the analytical solution are the eddy coefficients, K and D. Nothing can be said about these terms prior to their evaluation from recorded data. The procedure for their determination is to fit the analytical solutions for velocity and salinity with flume and field data and to pick the best fit values for K and D by trial and error. Since the stream function is dependent only on D, this procedure is not too cumbersome, even though the salinity is dependent on both D and K. This process of back-calculating D and K from recorded data has been repeated for several data sets. The resulting distributions of these coefficients was then correlated with parameters characteristic of the flow conditions.

Evaluation of Solution

The analytical solution for velocity and salinity distribution developed above was evaluated with laboratory data from the Vicksburg salinity flume, the Delft Hydraulic Laboratory salinity flume and the James River field study. This combined set of data covers a wide range of flow conditions and degrees of salinity stratification. For each case studied, a best-fit value for the two mean eddy coefficients was found at each longitudinal station.

The laboratory flume of the Corps of Engineers, U.S. Army, Vicksburg Waterways Experiment Station (WES), is described in detail in a WES report (1955). The flume is a lucite channel 327 ft. long, 0.75 ft. wide and 1.5 ft. in total depth. At the ocean end there is a tidal reservoir which can maintain a constant salinity and a periodic surface level. The opposite end has a freshwater reservoir. Roughness is achieved by 1/4 inch strips attached to the side walls on 2 inch centers. Different estuarine conditions are modeled by varying the freshwater inflow, the tidal amplitude and the basin salinity. Figures 1 and 2 compare the analytical solution with data from WES test 16. The solid lines are for the bottom boundary condition discussed above. The broken lines illustrate the analytical solution using a bottom condition of zero horizontal velocity. From these figures, and similar analysis for other flume tests, the condition of zero gradient was selected as the preferable condition for the model. Figure 1 shows how the analytical solution, using the longitudinal salinity gradient determined from the data, can follow the changes in the velocity distribution along the length of the flume. This characteristic of the solution enables the model to identify the limit of the upstream flow of saline water, the null point identified with shoaling zones in estuarine channels.

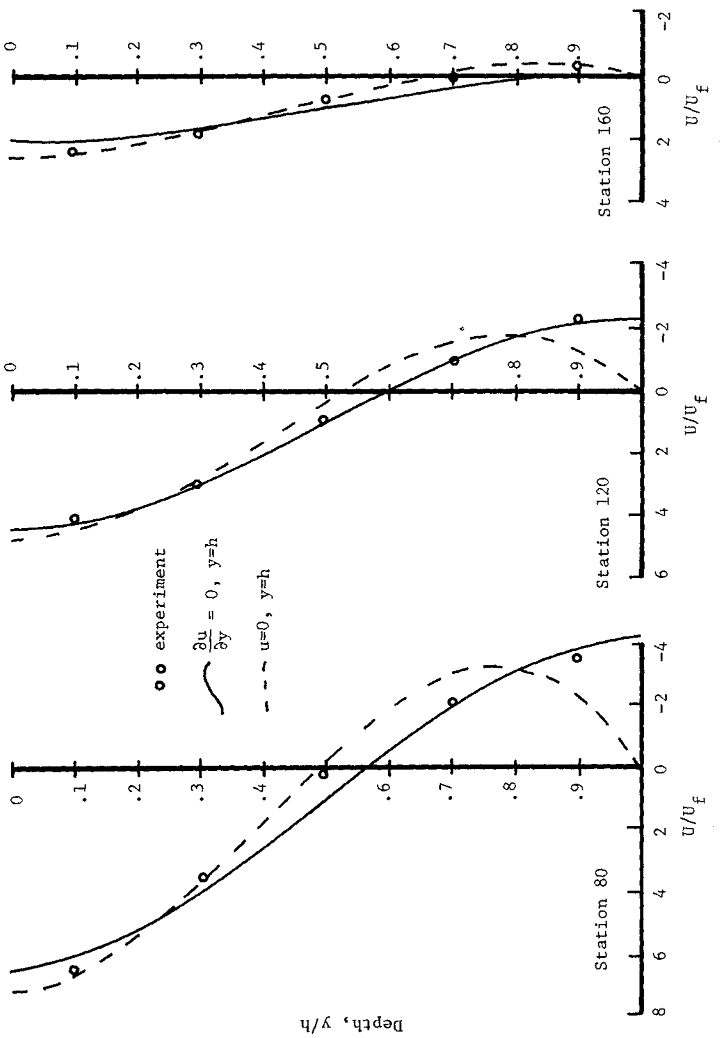

Figure 1 Horizontal velocity profiles, WES 16, stations 80, 120 and 160

U_f = 0.2 ft.sec., S_o = 29.2 ppt, tidal period = 600 sec., tidal amp. = .05 ft.

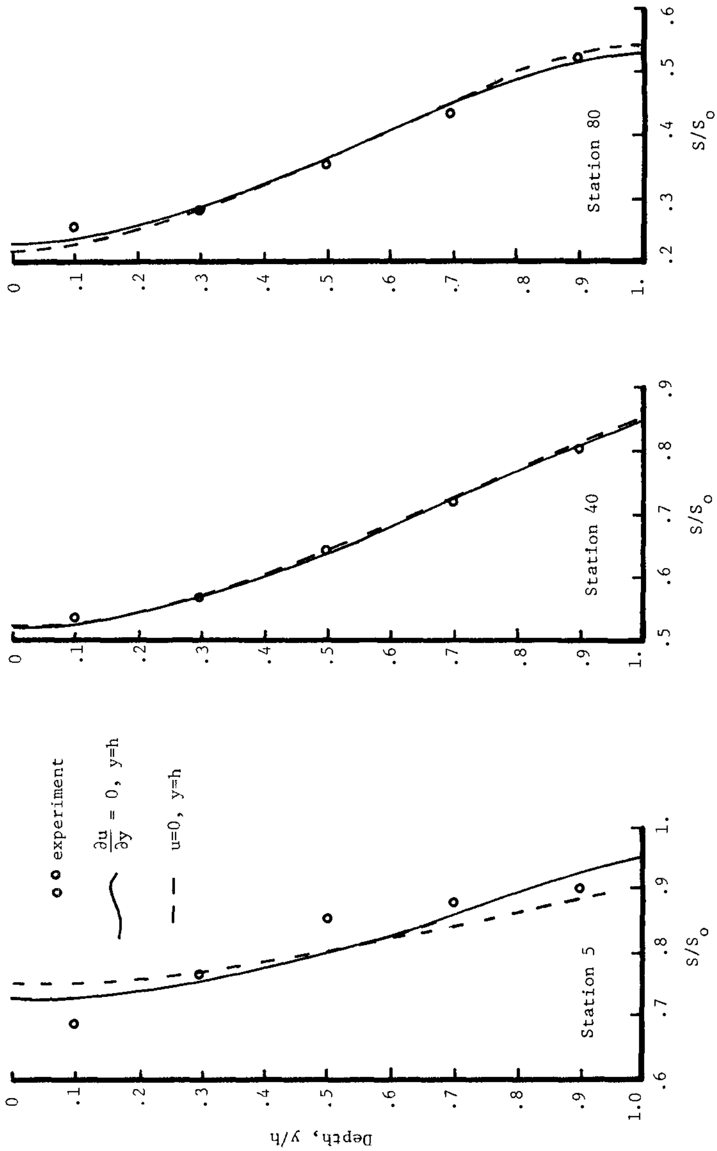

Figure 2 Salinity profiles, WES 16, stations 5, 40 and 80

At the Delft Hydraulics Laboratory an experimental investigation of salinity intrusion in estuaries similar to the Vicksburg studies has been carried out. The details of flume design and measurement technique are reported in Delft (1970). The flume is 546 ft. long, 2 ft. wide and 0.7 ft. deep (msl). For the Delft tests the bottom roughness was achieved by vertical bars .5 x .5 cm^2 in cross-section attached to the flume bottom. By changing the number of bars the roughness could be varied for different runs.

Four Delft tests were analysed with the analytical model. All the tests were for steady-state conditions and the longitudinal salinity distribution was back-figured from the recorded data as was done for the WES tests. Figure 3 illustrates the comparison between the analytical solution and flume data for test 116 at a central portion of the salinity regime.

A final example of the analytic solution is given in Figure 4 for the James River estuary. The difference between computed and actual velocities over most of the depth is probably due to several factors, including the uncertainty of time-averaged field measurements, and more importantly, the simplifying assumption of constant width with depth for the analytic solution. The salinity profiles for this same station show better agreement than the velocities. However, there appears to be a sharp vertical gradient near middepth for the field data which is not observed for the analytical solution. This difference may be a result of the same factors cited before for the velocity profile.

In general, the analytical model, although clearly capable of reproducing flume conditions more exactly, does not appear to break down for the prototype conditions and scales exemplified by the James River estuary.

Evaluation of Time-Averaged Coefficients

The evaluation of the mean eddy coefficients K and D was carried out for ten separate tests, including three field studies. These coefficients showed a varying degree of longitudinal dependence, as discussed by Fisher, et al. (1972). The longitudinal variations of the mean eddy coefficients suggested that although D and K are functions of x, this dependence is of secondary importance. By introducing the additional assumption that these mean eddy coefficients may be replaced with effective constant values for the entire longitudinal distance of the salinity regime, correlations of these coefficients are greatly simplified.

The significance of being able to use constant values for D and K, i.e., $\bar{D}$ and $\bar{K}$, is that only two unknown parameters need now be specified in order to apply the analytical model to a given set of estuarine conditions, i.e., freshwater discharge, ocean salinity, depth, etc. All other model parameters can be readily determined with the possible exception of the longitudinal salinity distribution. This latter input can be computed with the aid of a one-dimensional numerical model, as previously discussed. The determination of $\bar{K}$ and $\bar{D}$ for input to the model is made by using an empirical correlation of these constant coefficients with the gross characteristics of the estuarine system.

Following the arguments presented above for using constant values of $D(\xi) \approx \bar{D}$ and $K(\xi) \approx \bar{K}$, the dimensionless form of the governing equations suggests that a possible pair of useful parameters for correlating $\bar{K}$ and $\bar{D}$ is

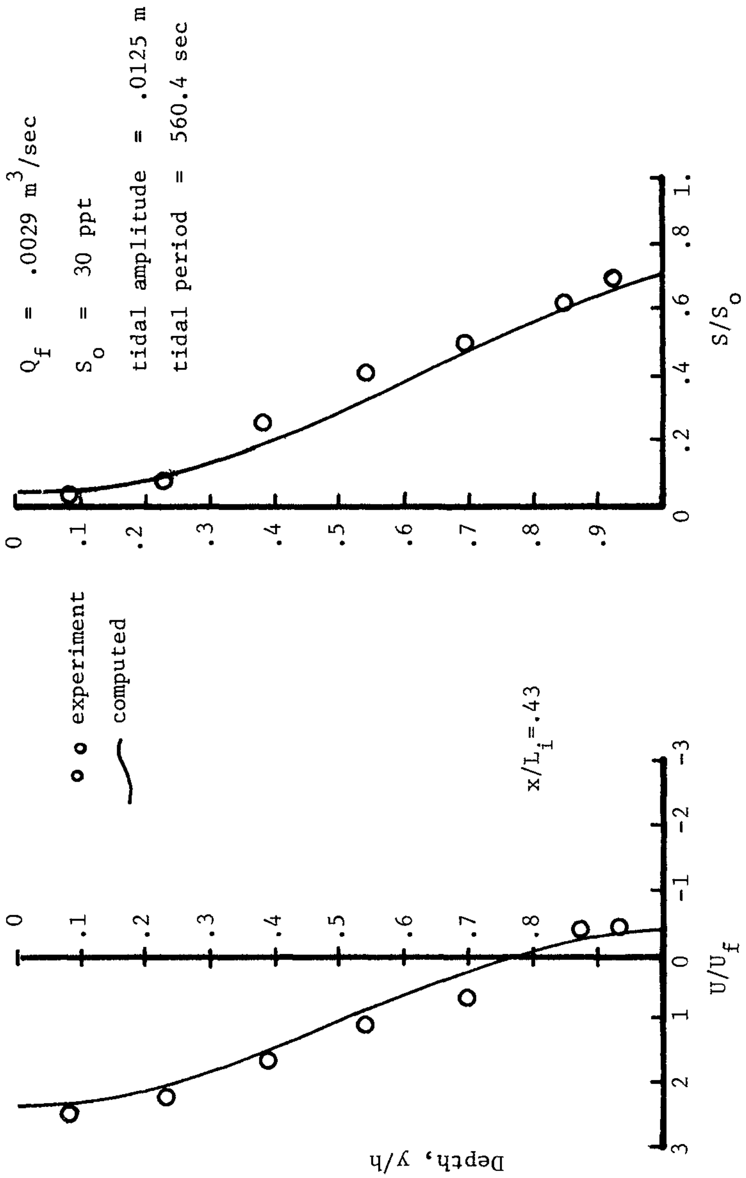

Figure 3 Horizontal velocity and salinity profiles, Delft 116

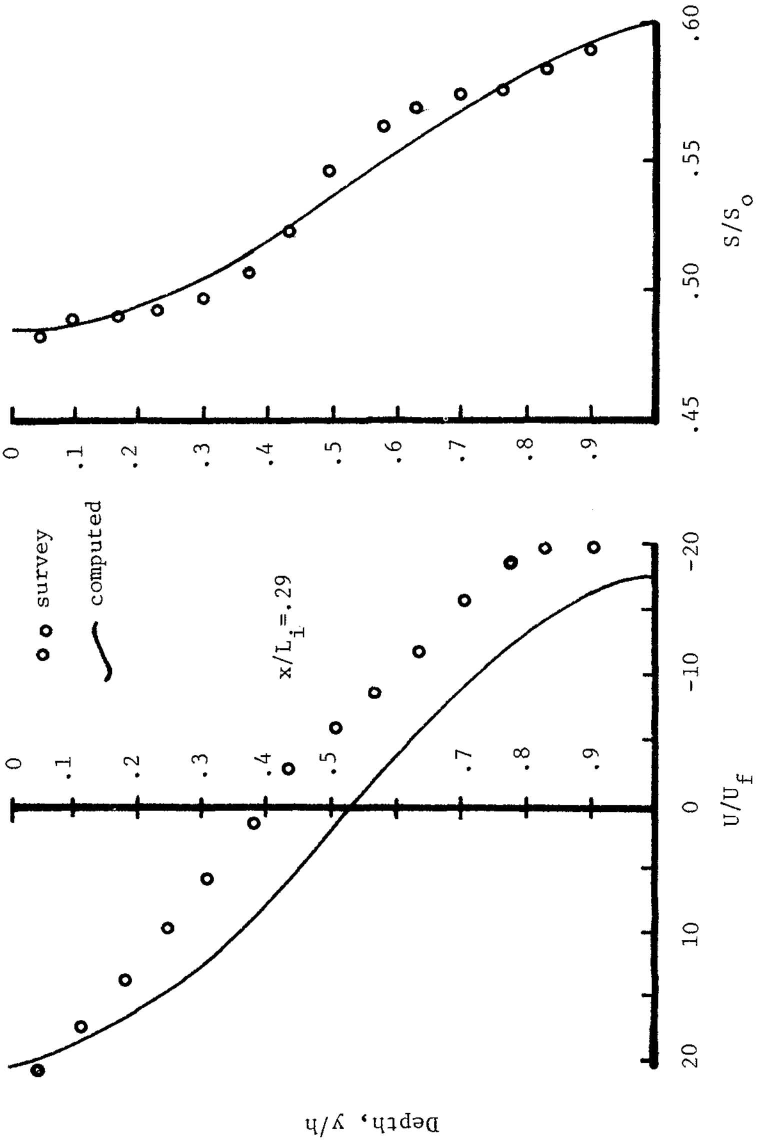

Figure 4 Horizontal velocity and salinity profiles, James River, 26 June - 7 July

$$\overline{C}_1 = \frac{g\alpha S_o h_o^4 b_o}{L_i Q_{fo} \overline{D}} \tag{24}$$

and

$$\overline{C}_2 = \frac{\overline{K} L_i b_o}{Q_{fo} h_o} \tag{25}$$

where the zero subscript, e.g., b_o, h_o, refers to the downstream limit or ocean boundary of the estuary. All terms in these new terms are assumed constant over the longitudinal and vertical dimensions, and the only unknown parameters are $\overline{K}$ and $\overline{D}$.

The values of $\overline{K}$ and $\overline{D}$ should be a function of the degree of mixing of the flow field which is in turn a function of the tidal activity. In recognition of this dynamic relationship of the physical system being modeled, $\overline{C}_1$ and $\overline{C}_2$ have been correlated with a characteristic non-time-averaged tidal velocity. To be consistent with the definitions above, this velocity is specified as the maximum entrance flood velocity u_o, non-dimensionalized by the freshwater velocity at this same boundary $\frac{Q_{fo}}{h_o b_o}$,

$$\overline{C}_3 = \frac{Q_{fo}/b_o h_o}{u_o} = \frac{U_{fo}}{u_o} \tag{26}$$

Figure 5 and 6 show the correlation of $\overline{C}_1$ with $\overline{C}_3$ and $\overline{C}_2$ with $\overline{C}_3$.

It is significant in these figures that both laboratory flume tests and prototype field surveys follow the same correlations. In addition, the range of degrees of stratification include the highly stratified Delft tests 121 and 122 as well as the nearly well mixed middle reaches of the James River estuary. Thus, this empirical approach to evaluating the effective coefficients of mean eddy flux, $\overline{D}$ and $\overline{K}$ is apparently applicable to naturally occuring estuarine conditions.

The Savannah Estuary - An Analytical Investigation of Estuarine Shoaling

The shoaling problems of the Savannah Estuary have been carefully reviewed by Simmons (1965) and Harleman and Ippen (1969). Both hydraulic models and field investigations have shown a relationship between the longitudinal location of maximum shoaling and a null point as indicated in Figure 6. From this figure it is seen that immediately downstream of Savannah Harbor, between stations 120 and 130, a zone of very high shoaling is located by comparison with the rest of the estuary. In addition, for the model data shown in Figure 6, with a freshwater flow equal to 7,000 cfs, the null point also occurs between these two stations.

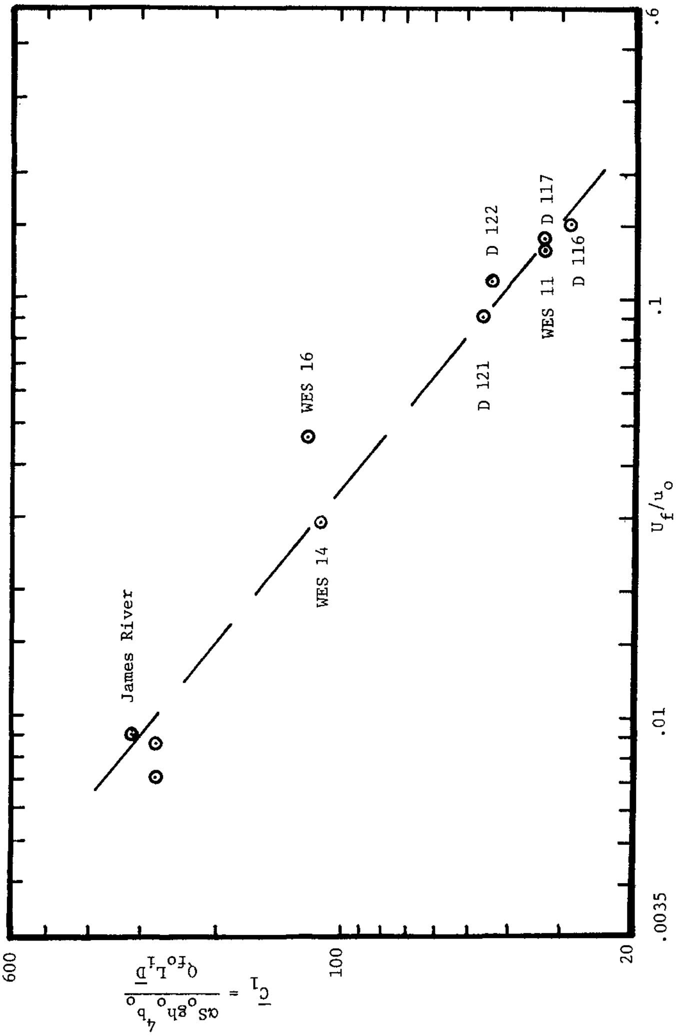

Figure 5 Correlation of $\overline{D}$

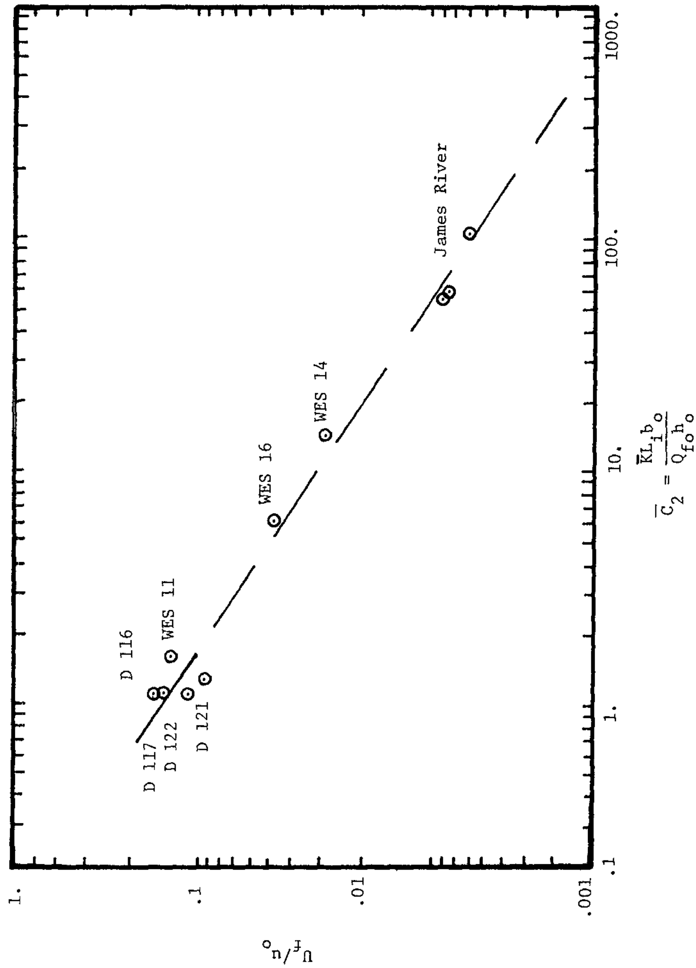

Figure 6 Correlation of $\overline{K}$

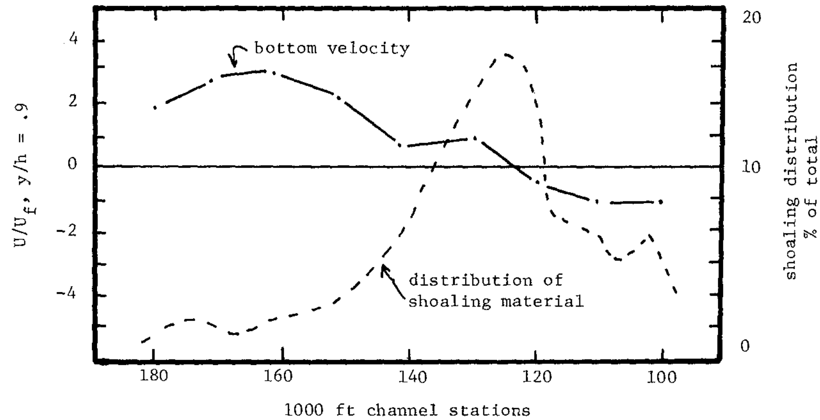

Figure 6 Longitudinal location of maximum shoaling in relation to null point for Savannah estuary, Q_f = 7000 cfs (from Harleman and Ippen 1969)

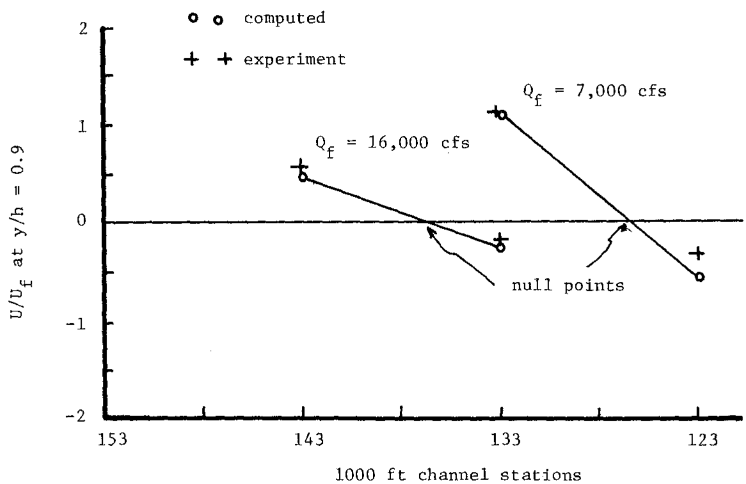

Figure 7 Null point location for Savannah estuary

In their report, Harleman and Ippen present the time- and depth-averaged longitudinal salinity distributions from the model for freshwater flows of 7,000 cfs and 16,000 cfs (their Figure 13). With these curves, and the correlation for eddy coefficients presented above, it is possible to apply the analytical model to this estuary and thus further investigate the null point dependence on freshwater flow rates.

Figure 7 illustrates the analytical results for the null point for the two freshwater flows. The connected circles are the computed values and the crosses are the hydraulic model data, as reported by Harleman and Ippen. The fairly close agreement between computed and experimental values indicates that the Savannah estuary prototype scales and conditions do not seriously violate the assumptions of the analytical model.

In Figure 7 it is seen that the null point shifts downstream about 1,000 feet when the freshwater discharge is increased to 16,000 cfs. Qualitative results of this nature illustrate the usefulness of the analytical model in the analysis of the many factors which determine the circulation patterns in estuaries. When used in conjunction with a numerical model, as discussed above, or a hydraulic model, as in the present illustration, this analytical model should prove to be a valuable aid to engineering analysis.

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