

CHAPTER 131

A NUMERICAL MODEL OF THE ST. LAWRENCE RIVER

by

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ABSTRACT

A one-dimensional numerical model of a 340 mile section of the St. Lawrence River has been formulated to study tidal propagation. For a more detailed study of the flow distribution in a localised section of the river a two-dimensional model was used. A half mile square grid was used to schematise an area of approximately 20 miles long by 15 miles wide. This two-dimensional model was embodied within the one-dimensional model to permit a free interaction of flow across the boundaries.

For the one-dimensional case, a comparison of model and prototype results is included for both elevation and velocity. For the two-dimensional model a comparison of flow distribution was made by using field results obtained from photographing ice movement and from drogue movement.

To interpret the results of the two-dimensional model into a simple method of flow visualisation, use was made of animation techniques. A movie film was made that demonstrates both tidal rise and fall and the associated horizontal velocities. Elevation was reproduced by use of varying shades of coloured paper to simulate contours, velocities were represented by simulating drogue movement to produce smoke streaks.

INTRODUCTION

The St. Lawrence River extends from Lake Ontario to the Gulf of St. Lawrence connecting the International Great Lakes to the Atlantic Ocean. The region studied stretches from Father Point at the seaward end to the Port of Montreal, a distance of approximately 340 miles (Fig. 1).

Numerical simulation has been used extensively in studies of many areas of the river. The simulation techniques have included the methods of characteristics and finite differences. The latter method has been applied in both the implicit and explicit modes and for both one and two-dimensional applications. This paper describes a combined one and two-dimensional model employing an explicit finite difference solution.

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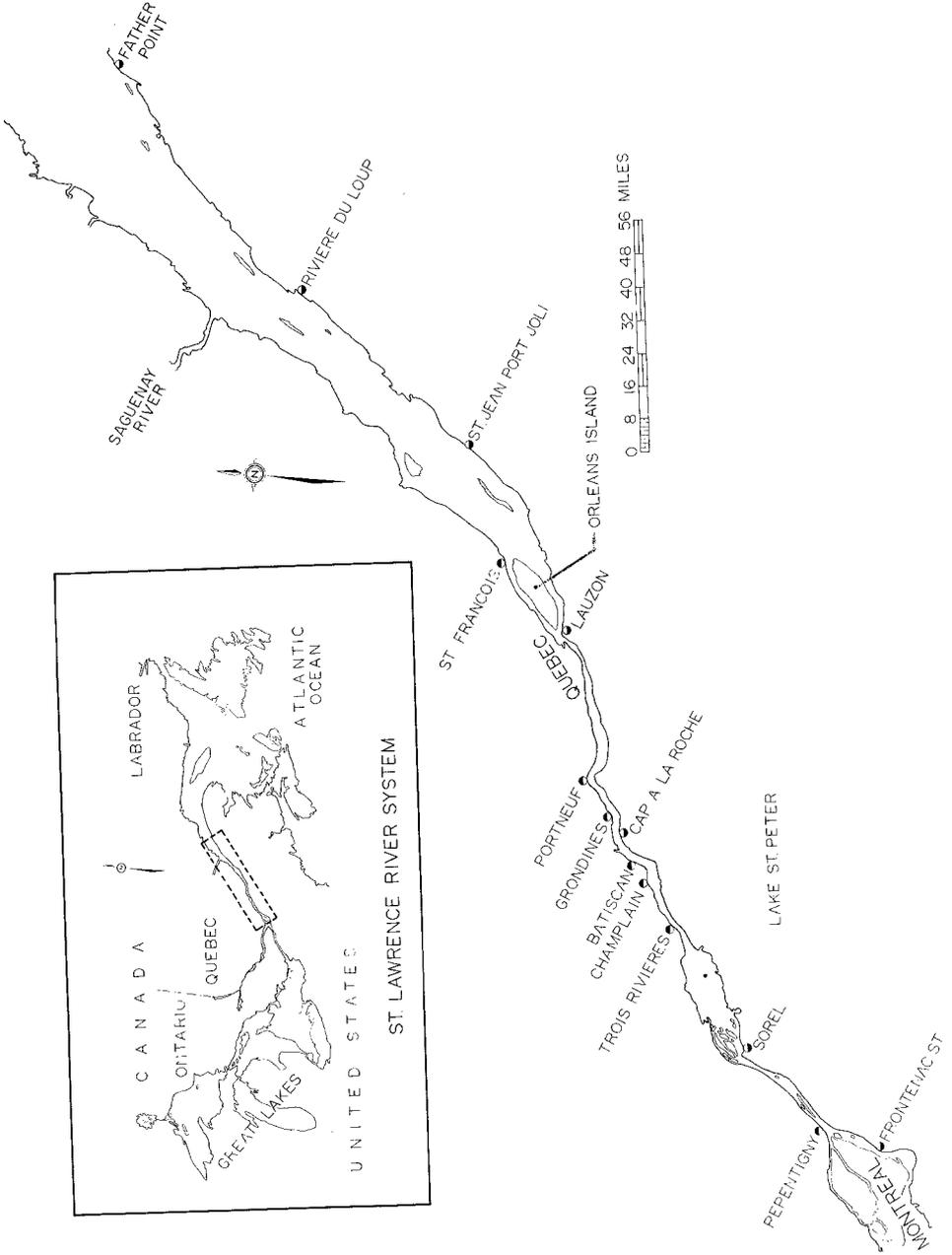


Fig. 1. The St. Lawrence River

A detailed investigation was required of a section of the river approximately 20 miles long and varying in breadth from 2 to 15 miles (Fig. 2). The section included a number of channels interspersed by islands, the distribution of flow between these channels was of particular interest. A two-dimensional model with a half-mile square grid was used to represent this area. The boundaries of this model were extended to river sections where the lateral variations in flow were small. The flow conditions at these boundaries could not be specified from available field data, moreover they were subject to modification by engineering works within the area of interest.

This difficulty was resolved by extending the simulation of the river to sections where boundary conditions could be accurately specified and to where they would not be subject to modification. The river lengths between these external boundaries and those of the two-dimensional model were simulated by a one-dimensional numerical model.

The model was calibrated to reproduce tidal elevations throughout the river. For the two-dimensional model the friction coefficient was taken as constant throughout the area. The flow distribution obtained from the model was compared with prototype observations obtained from filming ice break-up and from drogue movements.

While the results of a model study of this nature are of fundamental importance for hydraulic engineering they may only constitute one of several factors which affect the decision making process in a comprehensive engineering scheme. In such instances a distinct limitation of numerical models has been the difficulty of presenting results in a form readily understood by the layman. Whereas for the alternative form of simulation by hydraulic scale models flow visualisation can be so gainfully exploited.

In an attempt to overcome this shortcoming of numerical models recourse was made to animation techniques. A description of the techniques developed in the compilation of the film is included.

MATHEMATICAL MODEL

(a) One-dimensional

The equations of motion may be written as follows:

Motion in the x-direction:

$$\frac{\partial U}{\partial t} + U \cdot \frac{\partial U}{\partial x} + g \cdot \frac{\partial H}{\partial x} + F = 0 \quad (1)$$

Continuity:

$$\frac{\partial Q}{\partial x} + B \frac{\partial H}{\partial t} - \partial \frac{Q_T}{\partial x} = 0 \quad (2)$$

where x - a horizontal axis in the longitudinal direction of the river flow
 t - time
 U - velocity in the x -direction
 g - acceleration of gravity
 H - elevation of the water surface above a horizontal datum
 Q - discharge = $U \cdot A$, A area of cross-section
 B - channel breadth
 Q_T - tributary discharge.

The friction term F was represented in this case by the Chezy formula

$$F = \frac{g \cdot U \cdot |u|}{C^2 \cdot R}$$

C - the Chezy friction coefficient
 R - the length parameter normally taken as the hydraulic radius but in this study the hydraulic mean depth is used.

An explicit finite difference solution was used to solve the above equations. The method of schematisation and the details of the numerical solution closely followed that described by Rossiter and Lennon*. The standard procedures for calibrating the model were adopted. The river was divided into ten sections within which the friction coefficient was considered constant. The optimisation of these ten coefficients was achieved after about five adjustments.

(b) Two-dimensional

The equations of motion may be written as follows:

In the x -direction:

$$\frac{\partial U}{\partial t} + U \cdot \frac{\partial U}{\partial x} + V \cdot \frac{\partial U}{\partial y} + g \cdot \frac{\partial H}{\partial x} + F_x - \Omega \cdot V = 0 \quad (3)$$

In the y -direction:

$$\frac{\partial V}{\partial t} + U \cdot \frac{\partial V}{\partial x} + V \cdot \frac{\partial V}{\partial y} + g \cdot \frac{\partial H}{\partial y} + F_y + \Omega \cdot U = 0 \quad (4)$$

*Rossiter, J.R.; Lennon, G.W.; "Computation of Tidal Conditions in the Thames Estuary by the Initial Value Method". Proc. Inst. Civil Eng., Vol. 31, May, 1965.

Continuity:

$$\frac{\partial H}{\partial t} + \frac{\partial}{\partial x} (U \cdot (H + D)) + \frac{\partial}{\partial y} (V \cdot (H + D)) = 0 \quad (5)$$

where x, y - two orthogonal horizontal axes

U - velocity in the x -direction

V - velocity in the y -direction

H - elevation of the water surface above a fixed horizontal plane

D - depth of the bed below the same fixed plane.

F_x and F_y the friction terms in the x - and y -directions respectively were represented by the Chezy formula

$$F_x = \frac{g \cdot U \cdot |(U^2 + V^2)^{\frac{1}{2}}|}{C^2 (H + D)}; \quad F_y = \frac{g \cdot V \cdot |(U^2 + V^2)^{\frac{1}{2}}|}{C^2 (H + D)}$$

$\Omega \cdot V$ and $\Omega \cdot U$ are the horizontal components of the Coriolis force. $\Omega = 2\omega \sin \phi$, where ω is the angular velocity of the earth's rotation. ϕ is the latitude of the location.

The equations were solved using an explicite finite difference solution of the type described by Reid and Bodine*. The convective terms in Eqns. (3) and (4) were omitted to avoid instabilities and also to save computational effort. Their omission is unlikely to be of significance in a problem of this nature. There was not sufficient data to warrant a spatial variation in the value of the Chezy friction coefficient. It was taken as a constant equal to that used in the corresponding area in an earlier purely one-dimensional model of the river.

(c) Boundary of the One and Two-dimensional Model

At the junctions of the one and two-dimensional schemes the following techniques were used.

- (i) The first water level position in the one-dimensional section is located at a distance $\Delta s/2$ from the limit of the two-dimensional scheme where Δs is the constant grid size used in the two-dimensional scheme.
- (ii) For calculating the velocities at the boundary of the two-dimensional scheme it is assumed that the water level at the external grid point is equal to that given at the corresponding position in the one-dimensional model.

*Reid, R.O.; Bodine, B.R.; "Numerical Model for Storm Surges in Galveston Bay". Proc. Am. Soc. Civ. Eng., Vol. 94, No. WW1, Feb., 1968.

- (iii) For the calculation of this water level referred to in (i) and (ii) above, the following procedure is adopted.

The equation of continuity (2) is written as

$$b \cdot \frac{\partial h}{\partial t} + \frac{Q_1 - Q_2}{\partial x} = 0 \quad (6)$$

where Q_2 refers to the discharge flowing from the two-dimensional model

$$\text{and} \quad Q_2 = \sum_{i=1}^{i=N} U_i (H_i + D_i) * \Delta s$$

N is the number of lateral sections at the limit of the two-dimensional scheme. A complete description of the mathematical model is given by Prandle and Crookshank*.

MODEL RESULTS

A comparison of computed and recorded water levels over an eight day period is shown in Figs. 3 and 4 for four locations along the river. A velocity comparison at St. Francois is shown in Fig. 4.

To compare flow distribution in the two-dimensional model two techniques were used. The first, shown in Fig. 5, compares the predicted flow distribution with field measurements obtained from aerial photographs of ice movement. The field data applies to surface velocities and the ice movement is subject to the effect of wind, ice jams and inertial forces associated with its physical properties. However, the results show reasonable agreement and suggest that this technique for obtaining field results can be used to advantage. Further, the possibility of using the model to predict ice movement is an obvious corollary.

The second comparison of flow distribution was made by the use of drogues. Two drogues were released in the river simultaneously, one with a centre of drag at ten feet below the surface and the other at twenty feet below the surface. A comparison with the predicted path for such tests is shown in Fig. 6. Again, the drogues are subject to wind stress at the surface and do not represent depth-averaged velocities. However the comparison is reasonable and it is hoped to

*Prandle, D.; Crookshank, N.L.; "Numerical Model Studies of the St. Lawrence River". N.R.C. Report No. MH109, August, 1972, National Research Council, Ottawa, Canada.

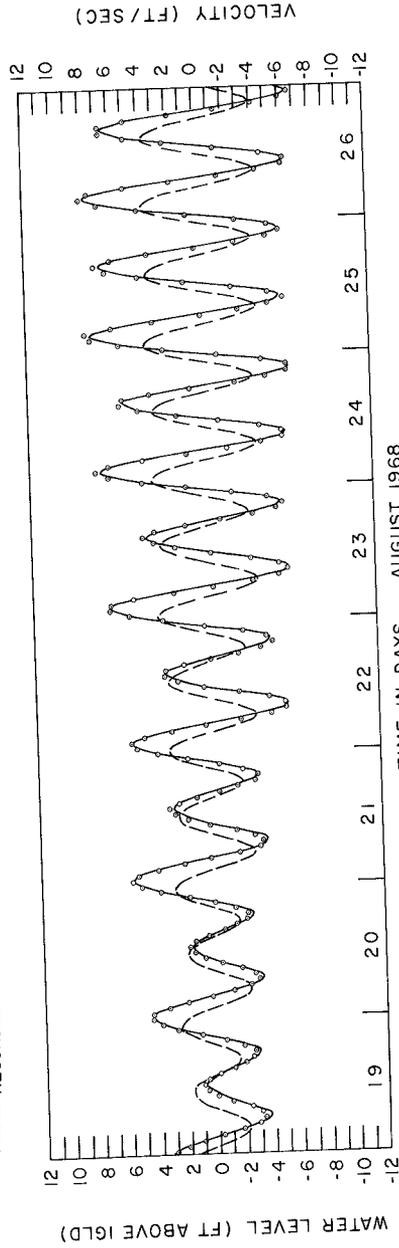
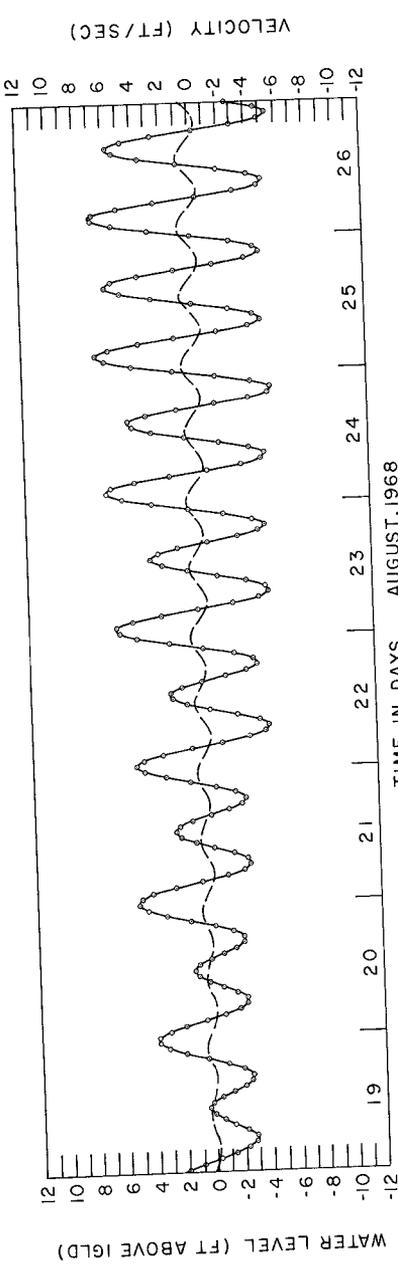


Fig. 3. Water Levels and Velocities; Computed and Recorded

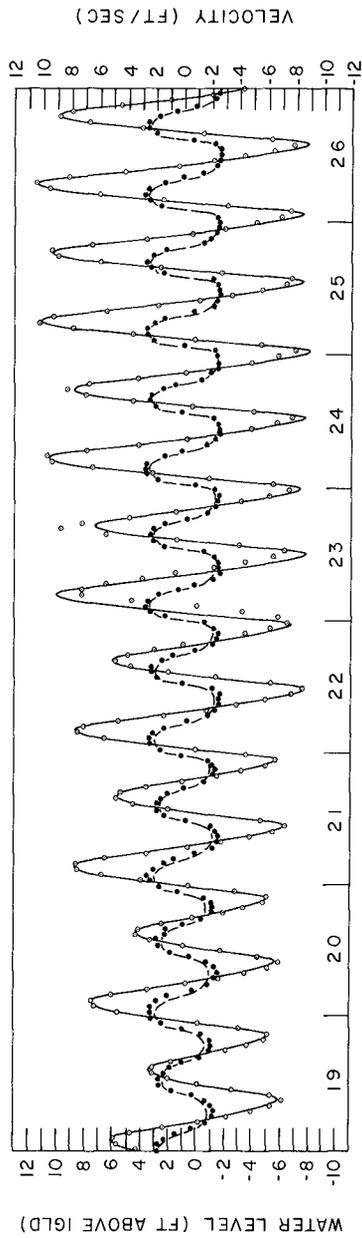
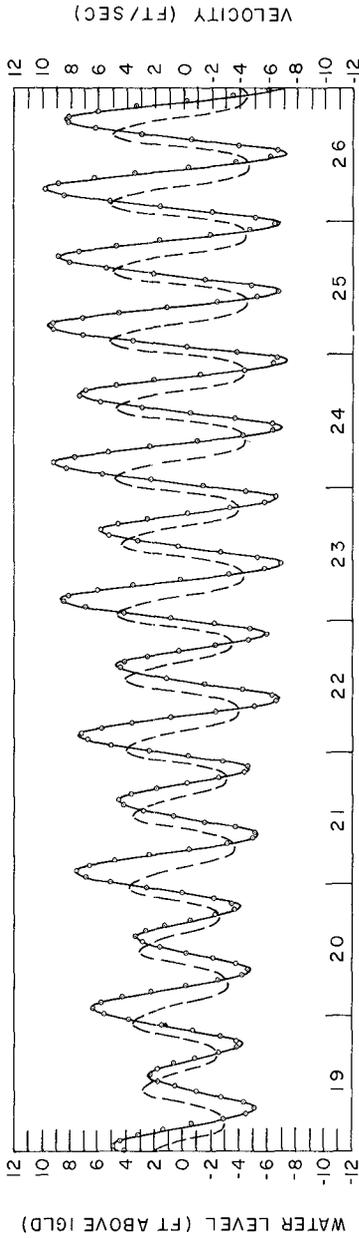


Fig. 4. Water Levels and Velocities; Computed and Recorded

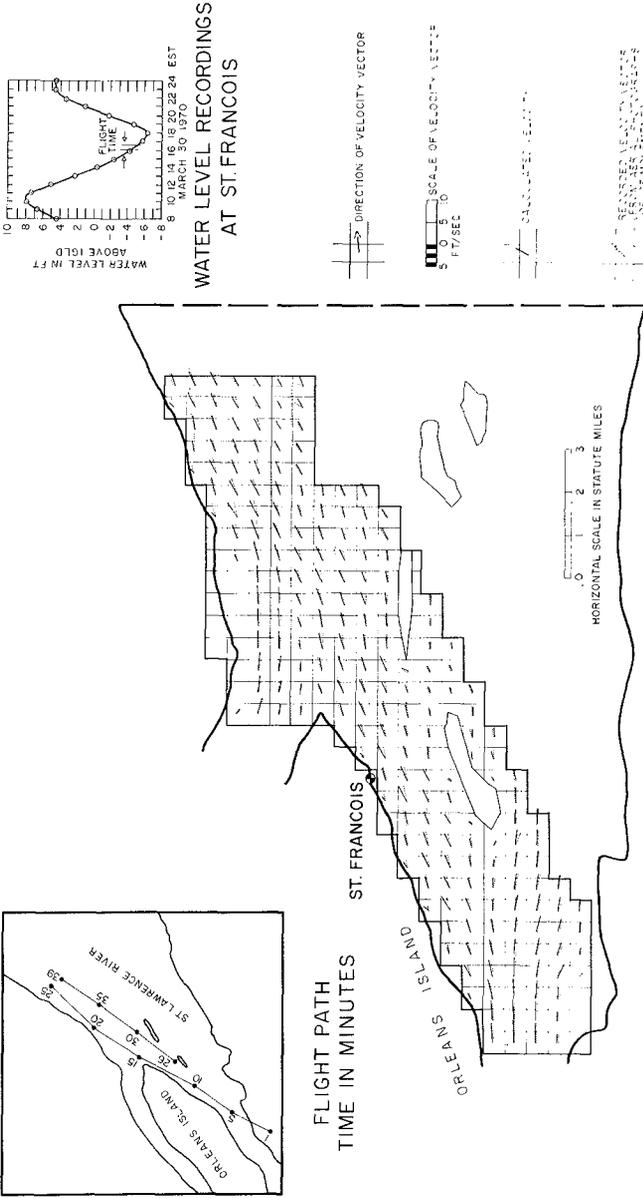


Fig. 5. Velocity Distribution From Ice Movement vs Computed Distribution

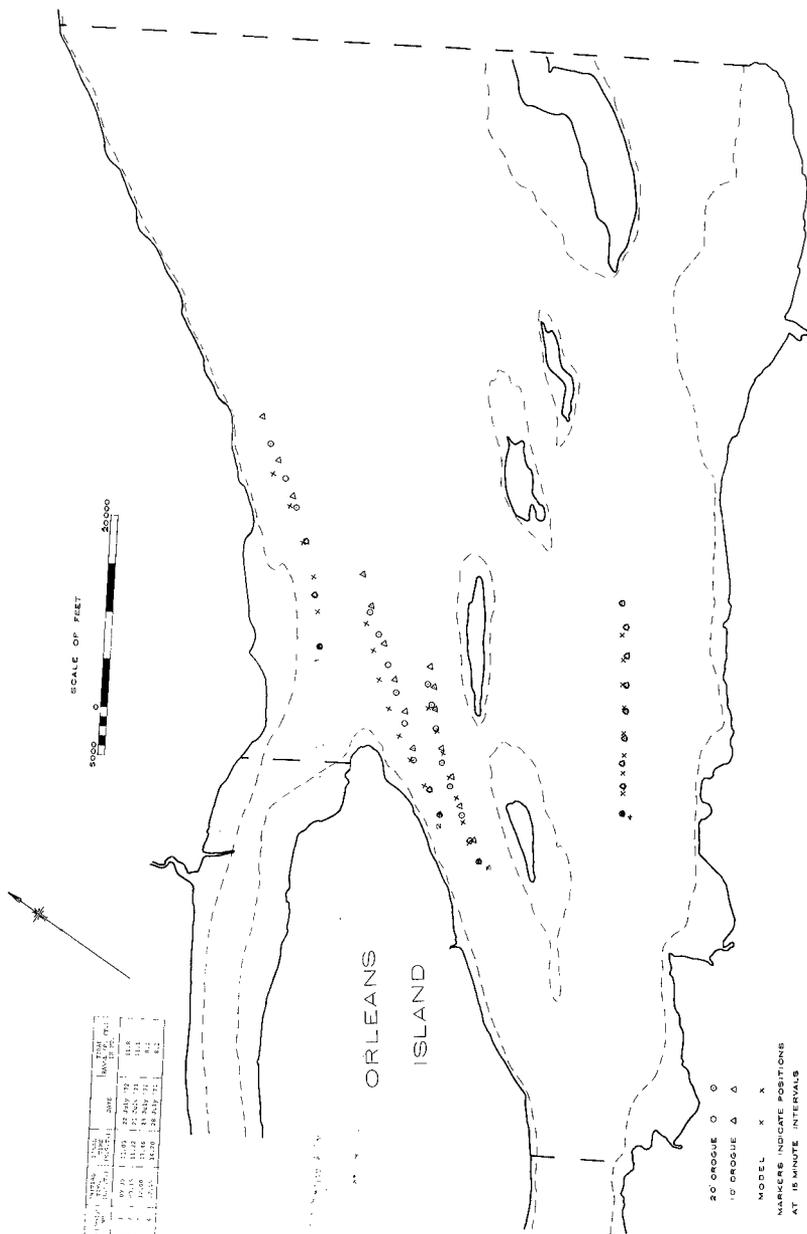


Fig. 6. Drogue Movements; Field and Model Results

extend the field tests to include measurements over a longer period of time. A comparable set of field measurements would be useful in calibrating the model and in studying dispersion or sediment movement.

To illustrate the instantaneous flow pattern computed by the model the form of presentation shown in Fig. 7 is used. By summing the vector velocities over a tidal cycle the net vector or drift velocity can be obtained. These values can be displayed in the same manner as Fig. 7; with due consideration for the limitations of the model this form of output can be extremely useful in many studies, particularly sediment transport.

The advantage in terms of flow visualisation of hydraulic scale models over numerical models is well recognised. This advantage of the scale model is fundamental. However, it is worthwhile to fully exploit the various methods of displaying the results of numerical models. An example of a particularly useful form of output is shown in Fig. 8 where the elevation of the water surface above chart datum at all points along the river is plotted for two complete days. This type of plot displays all the results (of level calculations) of the one-dimensional numerical model on a single figure and thus fully exploits the spatial resolution available from the model.

As an example of the use of this type of output the routing of a vessel entering the Port of Quebec is cited. The example considers the case of a vessel with a draught of 13 ft more than the minimum channel depth. The area where the available depth is limited is bounded by the two vertical lines. Assuming a vessel speed of 10 knots (relative to land for simplicity) the safest course is indicated in Fig. 8.

FLOW VISUALISATION BY ANIMATION*

The two parameters to be displayed are water level and velocity.

Water Level

Water level was simulated by the use of coloured contours. By examining the schemes used in topographical and hydrographical maps it was decided that the use of blue and grey colours would give the ten shades required and at the same time be consistent with standard colouring schemes used to represent water bodies.

*"Tidal Propagation in the St. Lawrence River". N.R.C.
Film No. 31, 16 mm audio, 8 minutes.

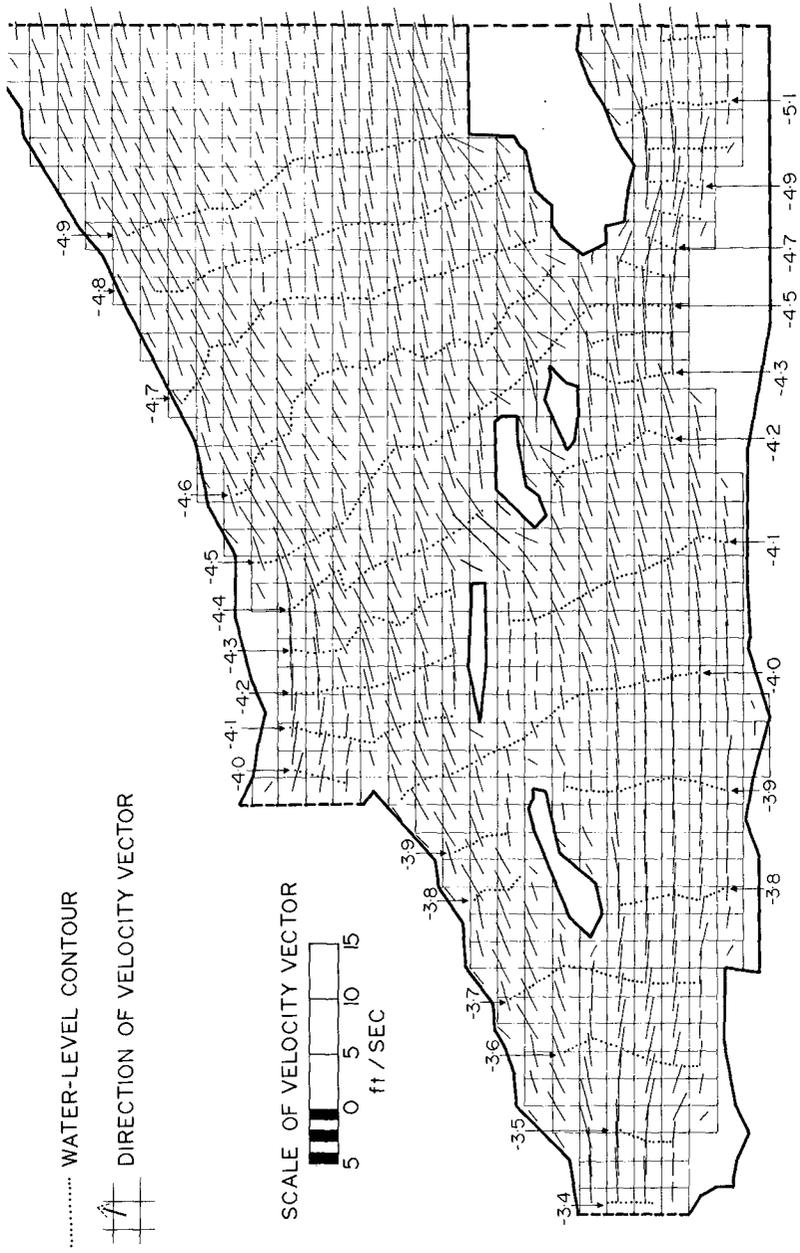
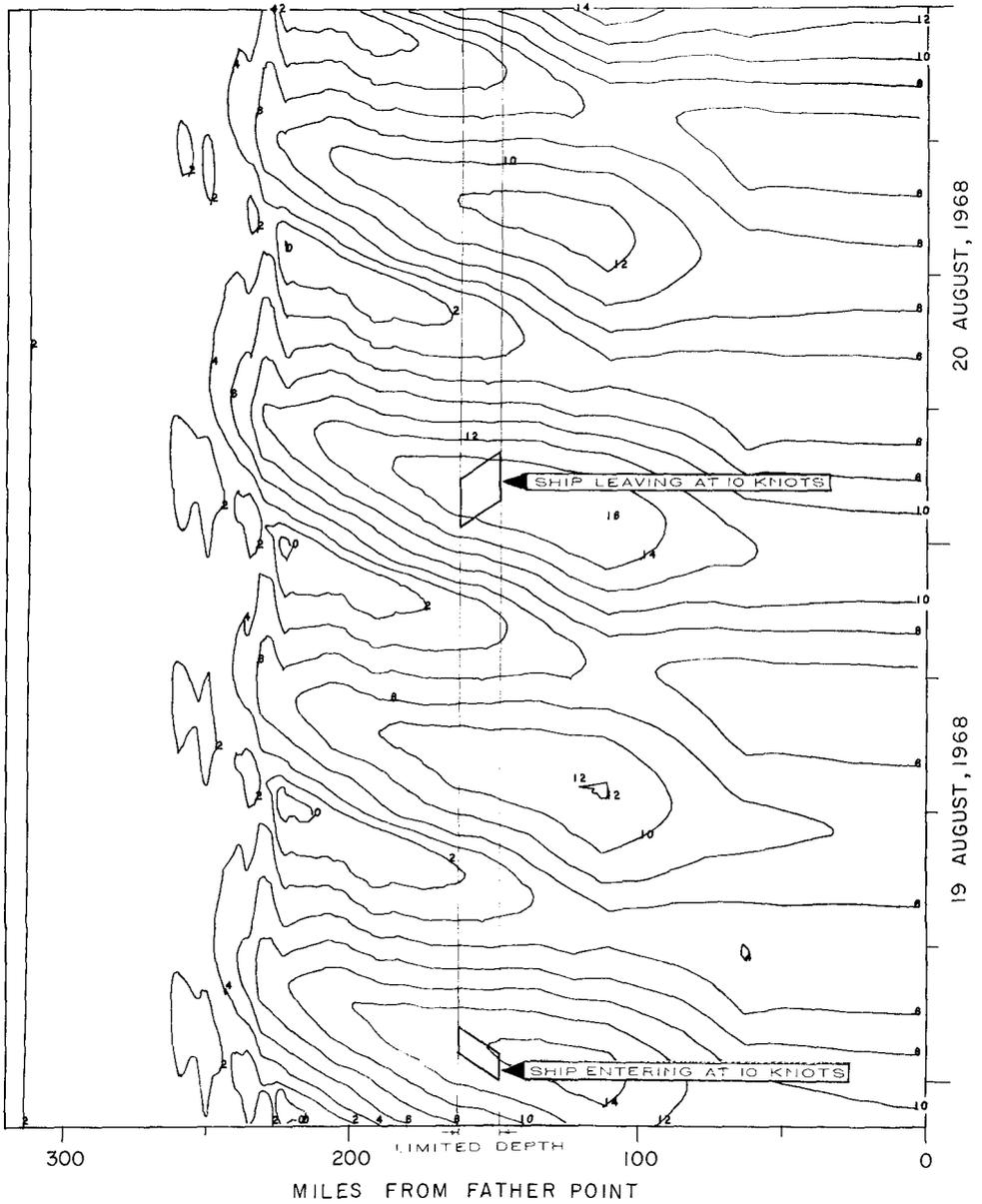


Fig. 7. Instantaneous Flow Pattern (Model Results)

FIGURES INDICATE TIDAL ELEVATION IN FEET ABOVE CHART DATUM



Having defined the interval of the water level contours, the datum should be set such that the level difference between high water and the highest contour is equal to that between the lowest level and the lowest contour. Failure to recognise this requirement unfortunately resulted, in this case, to an erroneous impression that "high water" persists for longer than "low water".

The physical procedure adopted in the compilation of each frame is shown in the film. The process is obviously tedious and has no particular merit. The objective of the programme was simply to investigate possible methods for flow visualisation. The ultimate aim is to display results using coloured television sets.

Velocity

Since the computational algorithm is based on a Eulerian scheme it is convenient to use an equivalent system for displaying velocities. This has been attempted by allowing the vectors shown in Fig. 7 to rotate and change in magnitude over a tidal cycle. However this did not present a satisfactory pictorial representation.

In hydraulic scale models useful methods for flow visualisation include dye injection and floats. An analogous technique was used for the numerical model by introducing marker particles at fixed intervals of time at the boundaries. These indicate instantaneous velocity and trajectories. The disadvantage of this Lagrangian form of representation is the loss of complete spatial resolution.

The techniques used in obtaining each frame are of interest and are included in the introduction to the film. The possibility of producing this sort of trace by using a cathode ray tube is immediately evident.

CONCLUSIONS

By combining a two-dimensional model with a one-dimensional model an accurate simulation of a particular short reach of a river was achieved. This technique has widespread applications subject to the limitations of the two-dimensional model.

The problem of flow visualisation associated with numerical models has been examined. For the one-dimensional model the use of contours to represent water elevation plotted against time and distance along the river has been found to be extremely useful. For the two-dimensional model animation techniques were used.

