CHAPTER 123

AN EVALUATION OF MIXING IN THE TAY ESTUARY

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ABSTRACT

The authors consider the derivation of an expression for salinity distribution in an estuary, from an instantaneous mass balance equation for a solute in a continuum. The expression is compatible with existing field instrumentation and limited economic resources. Details are given of a prototype survey which included continuous monitoring of the salinity distribution of the Firth of Tay, an estuary subject to a wide range of fluvial and marine influences. The survey results enabled an apparent dispersion coefficient to be evaluated and an estimate to be made of the net mixing.

INTRODUCTION

An important feature of estuarine management and research is a knowledge of the temporal and spatial distribution of salinity with respect to fluvial and marine influences.

At present there exists an incomplete understanding of the solute transport associated with turbulent flow in the presence of density gradients, that is characteristic of many partially and apparently well mixed estuaries. Study of this phenomenon is hindered by the difficulty of directly monitoring turbulent fluctuations of fluid velocity and solute concentration, and the wide range of the temporal and spatial variation of these variables for given influences external to the estuarine system. A further difficulty is that in many estuaries, the components of these marine and fluvial influences exhibit wide variations in both magnitude and time scale.

A mathematical representation of solute distribution may be formulated from the fundamental principle of mass balance. Its practical application in the absence of a priori relationships for the variables describing mass transport, can only be achieved by the application of statistical averaging, which leads to the assumption of empirical functions that require evaluation through the comparison of theory with prototype data.

The results predicted with such a model must be treated with caution unless the dependence of the empirical functions is well understood for the relevant input conditions to the system. The expense and difficulty of collecting prototype data requires the use of the simplest form of model compatible with an adequate return of information.

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2153
In this paper, a description is given of the use of a one dimensional form of the solute mass balance equation in order to obtain a mathematical representation of salinity distribution in the Tay Estuary. Reference is made to the proper reduction of dimensionality and time averaging of the instantaneous three dimensional solute mass balance equation. The conditions of significant tidal variation of both channel cross sectional area and solute concentration are recognized. An apparent coefficient of dispersion and a net mixing term are defined, evaluated and correlated to fluvial discharge and distance along the estuary.

THE TAY ESTUARY

The estuary links a catchment having an area of approximately 7,500 km² through 50 km of tidal waters to the North Sea. The River Tay and its main tributary, the River Earn, have mean discharges of 160 cumecs and 20 cumecs (1) respectively. The combined monthly mean discharge shows an annual variation ranging from 60 cumecs to 250 cumecs (calculated from 15 years data).

The mean tidal ranges of spring and neap tides are 4.7 m and 2.3 m at the bar (2). The amplitude of the tidal wave is similar at the mouth of the estuary and at Dundee, and thence it undergoes a gradual reduction until the relevant tidal limit is reached.

The cross sectional area of flow at mean water level is fairly constant as far as Dundee and then decreases approximately linearly with longitudinal distance to Newburgh where the channel becomes of the more uniform nature characteristic of a fluvial regime. Between Newburgh and Balmerino, a main channel, bordered by tidal flats, closely follows the southern shore. It then assumes a more central position through the moving sand bank complex upstream of Broughty Ferry, whereafter it reaches the mouth of the estuary in a well defined channel bordered by shoals and tidal flats. A maximum depth of 30 m occurs near Broughty Ferry, but generally depths range from 20 m to 2 m relative to low slack water levels. A detailed description of the bathymetry and sedimentological features of the estuary is given elsewhere (3).

The salinity intrusion extends to the Hewburgh region during low river flows. Generally the difference between salinity near the free surface and near the bed ranges from less than 0.1 ppt to the order of 2 - 3 ppt. For particular conditions of wind and tide, or near slack water, considerably greater differences may occur.

ONE DIMENSIONAL REPRESENTATION OF SOLUTE DISTRIBUTION

A solute mass balance equation based on fundamental physical principles may be derived for turbulent flow in estuaries. Assuming the variation in fluid density to be small compared with that of the other variables, and that the effects of molecular diffusion may be neglected, then the equation for a/
conservative solute may be written as
\[
\frac{\partial C}{\partial t} + \frac{\partial}{\partial x_i} (U_i C) = 0 \tag{1}
\]
where \(i = 1, 2 \text{ or } 3\), refers to a cartesian coordinate system

- \(C\) = Solute concentration
- \(U_i\) = Component of fluid velocity
- \(t\) = Time

The reduction of dimensionality of equation (1) has been shown by Pritchard (4), with the "turbulent" terms introduced by a temporal average over a small time interval, to give
\[
\frac{\partial}{\partial t} (C A) + \frac{\partial}{\partial x} (A U C) + \frac{\partial}{\partial x} (\langle A' C' \rangle \Delta t + U' C' A) = 0 \tag{2}
\]
where \(x\) = Distance from mouth of estuary

- \(A\) = Lateral cross sectional area of channel
- \(\Delta t\) = A small time interval
- \(f_d\) = \(\langle f \rangle = \frac{1}{\Delta t}\int f \, dt + f'\), \(f = f_d + f'\)

Further temporal averaging over a tidal period has been considered by Okubo (5) and modified by Williams and West (6) for the case where there exists a significant tidal variation of cross sectional area and of the spatial mean value of solute concentration. Assuming steady state conditions gives
\[
\frac{\partial}{\partial x} (C_{AT} Q_T) + \frac{\partial}{\partial x} (\langle A (U' C') + \langle U' C' \rangle + U' C' + A' C' U \rangle) = 0 \tag{3}
\]
where \(f_{AT}\) = \(\frac{1}{T} \int f_A \, dt\)

- \(f_A\) = Function dependent on \(t\)
- \(T\) = Tidal period
- \(Q\) = Fluvial discharge

An inspection of the terms in equation (3) is useful at this stage. A value of the term \(Q_{AT}\) may be obtained by standard river gauging techniques and a good estimate of \(C_{AT}\) for salinity at any section may usually be calculated from data acquired using a small fast craft and in situ salinity measurement procedures.

To the authors knowledge, the useful measurement of the turbulent perturbations \(U_i\) and \(C_i\) in estuarine flow is impractical at present. The measurement of turbulent mean velocity components \((\bar{U}_i)\) is technically feasible, but gathering detailed data is generally economically prohibitive in most natural tidal channels having an irregular cross section.

Thus in practice it is convenient to rewrite equation (3) using a/
coefficient $D_{\text{AT}}$ to give
\[
\frac{\partial}{\partial x} \left( C_{\text{AT}} Q_{\text{T}} \right) - \frac{\partial}{\partial x} \left( \frac{A_{\text{T}} D_{\text{AT}}}{\partial x} \frac{\partial c_{\text{AT}}}{\partial x} \right) = 0 \quad (4)
\]

where $D_{\text{AT}}$ is defined as
\[
D_{\text{AT}} = \frac{1}{\partial_{x} c_{\text{AT}}} \left( \langle A(u'_{\text{AT}} c'_{\text{A}} + \langle u'_{\text{AT}} c'_{\text{A}} \rangle \rangle_{\text{T}} + A'_{\text{T}} c'_{\text{AT}} U_{\text{AT}} \rangle_{\text{T}} \right)
\]

The precise physical meaning of $D_{\text{AT}}$ is not easily understood. However, its use introduces uniformity and simplicity into the algebra and permits practical results to be achieved. The intuitive use of a turbulent diffusion coefficient ($D_{\text{AT}}$) has been discussed by many authors (for example Okubo (7)) but further progress to a useful end, analytically or numerically of necessity requires the condition.
\[
D_{ij} = 0 \quad i \neq j
\]

where $D$ refers to elements of the turbulent diffusion and $i, j = 1, 2$ or 3

This assumption is difficult to justify in the light of the present understanding of estuarine flow, but is often accepted in the process of model development. On this basis the term $D_{\text{AT}}$ is proposed. It is a function of mean $(A, T)$ concentration gradient, the spatial and temporal variation of fluid velocity and solute concentration, and the temporal variation of cross sectional area. Herein the term $D_{\text{AT}}$ is loosely termed an apparent dispersion coefficient.

It should be noted that it is necessary to include a convective term with the non-convective terms.

**DATA ACQUISITION AND ANALYSIS**

Turbulent mean values of salinity at a point are likely to be the combined result of marine and fluvial inputs to the estuarine system over a period of time of the order of weeks. To establish the magnitude and time scale of these effects the continuous monitoring of salinity, with respect to tidal range and fluvial discharge, was arranged at a point approximately equidistant between the mouth of the estuary and the upstream limit of salinity intrusion. The tidal excursion in the Tay is such that at a point so placed, the mean $(T)$ salinity is a function of conditions over 75% of the intrusion length.

Water sampling instrumentation and a water level recorder were installed at Newport Pier. The site had the advantages of being accessible in all anticipated weather conditions and was for most of a tidal cycle subject to a strong tidal current. Hourly water samples having a volume of 600 - 800 ml were
collected and analysed for salinity using a salinity/temperature bridge. The automatic sampler (North Hants. Eng. Co. MK IV) consists essentially of twenty-four evacuated bottles with clockwork actuated valves. To be independent of mains electricity on an exposed site was considered to be important. The suction head of 7 m at low tide made the use of a battery powered sampler unattractive.

The water level recorder produced quarter hourly readings on water resistance computer compatible punched tape. Economic and site conditions limited the stilling well size to 100 mm and float diameter to 80 mm. Calibration (8) indicated an accuracy of ±13 mm though during quiet surface wave conditions the effect of interference between float and counterweight can be observed in figure (3).

River discharge data was supplied by the Dept. of Agriculture, Fisheries and Food for Scotland as mean daily discharges. Hydrographic data was taken from charts prepared by the Dundee Harbour Trust and by the Admiralty.

The salinity record was examined with respect to salinity in the main channel at that section and for the effects of tidal range and river flow.

Confirmation that the autosampler data reflected conditions elsewhere on the cross section was sought by comparing the autosampler data with spatial mean values of vertical profiles measured in the main channel, where the lateral salinity variation was known to be small. The results shown in fig (2) are typical of the three periods monitored. The lag of the point values on the flood is more likely to be the effect of the main flood current tending to flow to the north of the ebb current (10) rather than vertical salinity gradients.

The tidal dependence of the salinity time series was demonstrated through the use of the least squares criteria to approximate to the data a function of the form (9)

$$y = a_0 + a_1 \sin \omega t + a_2 \cos \omega t$$

where

$$\omega = \frac{2\pi}{T}$$

$$\frac{a_0, a_1, a_2}{\sqrt{a_1^2 + a_2^2}} = \text{constants}$$

$$\text{amplitude}$$

The period was varied from 11.90 hr to 13.00 hr in steps of 0.01 hr. This revealed for both the water level and salinity data the existence of large values of amplitude for the semi-diurnal (12.42 hr) and 1/4 day (12.00 hr) components.

Insight into the salinity river flow relationship was achieved by filtering a major part of the tidal effects by approximating equation (6) to various intervals of data and taking a_0 as parametric representation of salinity in that interval. An interval of 2h hr was found to be a satisfactory compromise between loss of detail of the record and of the smoothing of the semi-diurnal tidal/
effects. During periods of steady fluvial discharge when the tidal effects on mean (T) salinity might be expected to be apparent, the changes in value of $a_0$ suggest a steady trend towards an equilibrium value for that flow. However, the tidal range was found to be related to the salinity range, to a first approximation by a linear function.

The values of $a_0$ were plotted against an arithmetic average of the mean daily fluvial flow for the preceding 7 days. Study of the records of several floods indicated that 7 days was approximately the average time between peak discharge at the Ballathie (R. Tay) gauging station and the corresponding minimum value of mean (24 hr) salinity at Newport Pier.

A linear relationship could be approximated to the salinity river flow data. Such a relationship must of necessity be of an approximate nature because of the dynamic features of the system and of the variability in the maximum flow, volume and duration of fluvial floods. Data for effectively steady state conditions is given in figure (4).

Having established that salinity data might be detected and characterised, salinity readings were taken at five stations in the main channel (approx 1 km apart) for several tidal ranges and river flows, and examined for a similar relationship. At each station vertical profiles of salinity distribution were measured at intervals of about 20 mins for complete flood and ebb tides (i.e. between slack water). Field measurements showed that cross sectional area mean values of salinity could often be estimated to better than 1 ppt from channel station data. An estimate of the spatial mean was obtained by assuming a linear variation between measured values and then taking a spatial average. The spatial mean values of salinity were then approximated to a sixth order polynomial and the mean (A, T) salinity evaluated from this function by temporal averaging.

At Pool, Broughty Ferry, Newport and Flisk, the relationship between mean (A, T) salinity and mean (7 day) fluvial discharge closely approximated to a linear function. The regression lines are shown in figure (4). The lack of variation in river flow on the occasions that the station at Balmerino was monitored serves to indicate reproducibility of results for those conditions at that station. As the slopes of the mean salinity/fluvial discharge functions are similar at Newport and at Flisk, a like value of slope was assumed for the function at Balmerino.

Plotting of the mean (A, T) longitudinal salinity distribution for various flow conditions, figure (5), enabled the abstraction of the data necessary in order to obtain estimates of the value of the apparent dispersion coefficient.

The variation of the apparent dispersion coefficient with mean (7 day) river flow and distance along the estuary is shown in figures (6) and (7). The coefficient is sensitive to the effects of the variation of river flow. At the
three upstream sampling stations, the observed data indicates that the coefficient passes through a maximum value as the river flow increases. A marked spatial variation is exhibited near the mouth of the estuary, elsewhere there is a comparatively gradual linear variation.

It is difficult to gain physical information from the apparent dispersion coefficient. An indication of the magnitude of the net effects of the physical processes over a tidal cycle may be obtained from the net advective salt flux during a tidal cycle, here defined as the net mixing. The net mixing is a function of turbulent effects as well as spatial and longer term temporal variations. A good estimate can be obtained from the term $Q_T C_{AT}$ if the term $A'_T C'_{AT} U_{AT}$ may be assumed to be comparatively small, as is generally the case for the Tay Estuary.

The linear relationship between the mean $(A, T)$ salinity and mean $(7 \text{ day})$ river flow,

$$C_{AT} = b_0 + b_1 Q_T$$

where $b_0, b_1 = \text{Constants at a point x}$ leads to a second order relationship for the net mixing $F$,

$$F = Q_T C_{AT} = b_0 Q_T + b_1 Q_T^2 \gamma$$

Equation (8) indicates that the net mixing is also sensitive to the effects of mean river flow variation and that a maximum value is passed through as mean river flow increases (Figure (9)). The existence of this maxima are not confirmed for all the stations as some occur outwith the limits of conditions observed in the field. The field data indicates a nearly linear increase in mixing along the estuary (Figure (8)).

CONCLUSION

The one dimensional solute mass transport equation in a tidally averaged form is a useful preliminary approach to the study of mixing in an estuary. While unsatisfactory from a physical point of view, the introduction of an apparent dispersion coefficient provides, along with other functions, an algebraic representation of the effects of processes that lead to what is here called the net mixing. The yield of physical information is limited to an estimate of the net effects of the physical processes of solute transport over a tidal cycle. In the Tay Estuary the apparent dispersion coefficient and net mixing are highly dependent upon distance along the estuary and mean river flow.

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Fig. (2) Comparison of autosampler data with channel mean salinity data at Newport
FIG 3  MAX, MEAN & MIN SALINITY (NEWPORT), WATER LEVEL (NEWPORT) & RIVER DISCHARGE (TAY & EARN) VARIATION WITH TIME.
Fig. (4) Variation of mean (A,T) salinity with mean (7 day) fluvial discharge.
Fig. (5) Longitudinal mean (A,T) salinity distribution as a function of mean (7 day) river flow.
Fig. (6) Variation of apparent dispersion coefficient $D_{xAT}$ with mean (7 day) river flow $Q_7$.
Fig. (7) Apparent dispersion coefficient $D_{xAT}$ as a function of distance $x$ from estuary mouth.
Fig. (8) Net mixing (of salt) $F$ as a function of distance $x$ from estuary mouth.
Fig. (9) Variation of net mixing (of salt) $F$ with mean (7 day) river flow $Q_T$.