

CHAPTER 113

SET-UP DUE TO IRREGULAR WAVES

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ABSTRACT

Energy losses in breaking irregular waves are estimated on the assumption that a wave, while breaking, loses only that portion of its height which would be in excess of the breaker height for the given wave period and the mean local depth. This leads to expressions for the magnitude of the radiation stresses as a function of the distance offshore. From this the variations in mean water level and the longshore current velocity are calculated with existing methods. Laboratory measurements of set-up in two-dimensional irregular waves are described. The data appear to some extent to be internally inconsistent; this may be due to enclosed air bubbles.

1 INTRODUCTION

The water motion in and near the surf zone is of prime importance in many coastal engineering problems. Theoretical studies of its dynamics have been made mainly since the last world war. However, it is only since the introduction of the concept of radiation stress that a satisfactory formulation has been obtained. This had led to quantitative predictions concerning the change in mean water level, and the generation of longshore currents, due to regular waves. For application to natural waves these results should be extended so as to include the wind-wave variability. A description of the wave field in terms of a linear spectral model is not suitable in the surf zone because of the strong nonlinearities. In the following a semi-theoretical approach is used which deals with average properties of individual waves in the space-time domain. Only quasi-two-dimensional situations are considered, i.e. straight, parallel depth contours and average flow parameters which do not vary in the longshore direction. The symbols are defined in Appendix 2.

2 REVIEW OF ESTABLISHED RESULTS

2.1. Radiation stress

Consider a situation in which it is possible to define a mean motion and a superimposed relatively rapidly fluctuating motion. The momentum balance for the mean motion then contains terms representing average momentum transfer caused by the fluctuations. These are equivalent to stresses. They are called Reynolds stresses in the case of turbulence.

When the fluctuating velocity field is due to waves they are called radiation stresses, following Longuet-Higgins and Stewart (1960), who formally introduced the concept. (It was developed independently by Dorrestein (1961) for the calculation of wave set-up, and by Lundgren (1963), who called it "wave thrust".) The term radiation stress actually refers to the contribution of the waves to the time average of the vertically integrated horizontal flux of horizontal momentum. In contrast to the Reynolds stresses in turbulence, the radiation stresses can be readily calculated in terms of external flow (wave) parameters. The principal radiation stresses in long-crested, progressive sinusoidal waves are proportional to the mean energy density E , with coefficients of proportionality which are functions of the depth-wavelength ratio only:

$$S_{11} = (2n - \frac{1}{2}) E \quad (1)$$

$$S_{22} = (n - \frac{1}{2}) E \quad (2)$$

in which

$$n = \frac{1}{2} + Kd/\sinh 2 Kd \quad (3)$$

The gradients of the radiation stresses appear as driving forces in the momentum equations for the mean motion. The complete equations can be found elsewhere (Bowen, 1969) and will not be reproduced here.

2.2. Set-up and longshore currents due to regular waves.

Several effects of the waves on the mean motion can be calculated by means of the concept of radiation stress, provided it is possible to estimate the wave parameters. This is most difficult for the wave energy. For regular waves useful results have been obtained on the supposition that energy losses can be neglected outside the surf zone, and by assuming that in the surf zone the wave height decays in a constant proportion to the local mean depth (Longuet-Higgins and Stewart, 1963; Bowen et al, 1968):

$$H(x) = \gamma \{h(x) + \bar{\zeta}(x)\} = \gamma d(x) \quad (4)$$

Set-up. The differential equation for the set-up $\bar{\zeta}$ due to waves of perpendicular incidence reads

$$\frac{d\bar{\zeta}}{dx} = - \frac{1}{\rho g(h+\bar{\zeta})} \frac{dS_{11}}{dx} \quad (5)$$

Integration of this equation, with S_{11} calculated on the basis of the assumed wave height variation described above, leads to a predicted set-down of the mean water level outside the surf zone, and a set-up shoreward of the breakerline, as indicated schematically in Fig. 1. The theoretical predictions have been verified experimentally (Bowen et al, 1968). The coefficient γ is a function of the incident wave steepness and the beach slope.

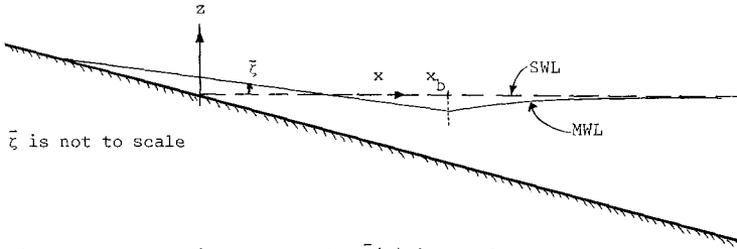


Figure 1 - Theoretical result for $\bar{\zeta}(x)$ in regular waves.

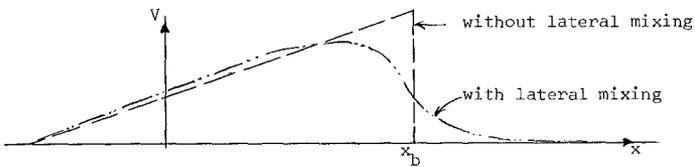


Figure 2 - Theoretical result for $V(x)$ due to regular waves.

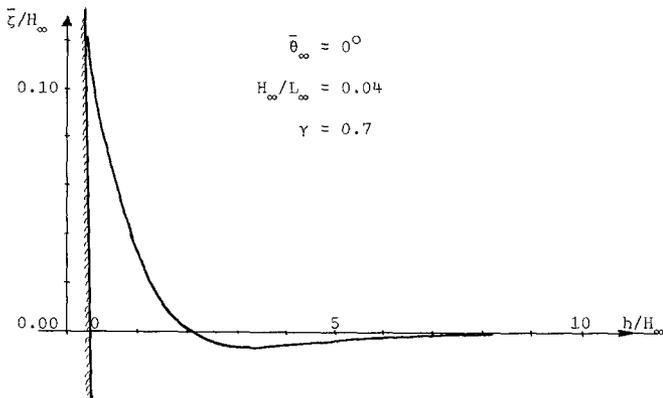


Figure 3 - Calculated set-up in irregular waves.

Longshore current velocities. (Bowen, 1969; Thornton, 1969; Longuet-Higgins, 1970). Obliquely incident waves exert a longshore thrust on the water mass in the surf zone, driving the longshore current. The net driving force per unit horizontal area has been shown to be proportional to the local rate of energy dissipation per unit area. It changes discontinuously from zero outside the breakerzone to a maximum immediately shoreward of the breakerline, due to the assumed wave height variation. In steady uniform flow this driving force is balanced by bottom friction and lateral friction due to molecular or turbulent viscosity. These should be expressed in terms of wave parameters and the longshore current velocity in order to calculate the distribution of the latter in the direction perpendicular to the shore. The computed velocity profile is discontinuous at the breakerline if lateral turbulent momentum exchange is neglected. Inclusion of some form of lateral mixing smoothes the velocity profile, but does not alter it drastically in the surf zone on gently sloping beaches, except of course near the breakerline (Fig. 2).

2.3. Discussion.

In the theories referred to in the preceding paragraph the variation of mean wave energy density with distance offshore plays a predominant role. For regular waves useful approximations were obtained by distinguishing two regions with different regimes, separated by a well-defined breakerline with fixed position. However, this method fails for irregular waves because no point can be defined inshore of which all the waves are breaking while offshore from it no waves would break. Instead, at each point only a certain percentage of the waves passing it are breaking or broken, while this percentage in general varies gradually with distance offshore. Associated herewith is a gradual variation of average values of other wave parameters, such as energy density, energy flux, momentum flux, etc. The problem is how these can be estimated in terms of the characteristics of the incident irregular waves. The attempt to find a solution to this problem includes the choice of a mathematical model for the description of irregular waves. Two such models are in general use: the linear spectral model on the one hand and the wave-by-wave description on the other. The former operates in the frequency domain, and is suitable for relatively low waves. The latter operates in the space-time domain and utilizes theoretical and empirical probability distributions characterizing individual waves. The applicability of this method is not restricted to low waves. For this reason it can be used with advantage in situations where nonlinearities are important. The procedure then is to use relationships for periodic nonlinear waves as a basis for the calculation of the corresponding probabilities in irregular waves. This approach has been applied to calculate distributions of wave forces on piles (Borgman, 1965; Pierson and Holmes, 1965), of run-up on slopes (Saville, 1962; Battjes, 1971) and of breakers on a beach (Collins, 1970). Although this method can make no claim to rigor, it is probably more rational for the present problem than the linear spectral model because of the essential role of wave breaking, which is a highly nonlinear phenomenon occurring to individual waves (crests) in physical space, and not to individual spectral components. For this reason it was decided to utilize the wave-by-wave description of the irregular waves. The adaptation of this method which was used is described in the following paragraph.

3 ENERGY LOSSES IN BREAKING IRREGULAR WAVES

An application of the wave-by-wave description to the problem of irregular breaking waves has been given by Collins (1970). Energy losses are incorporated in the model by assuming that the height of a wave after initial breaking decays in constant proportion to the mean local depth. Two different breaking criteria are used. They are taken from theoretical and experimental results for regular waves; their validity is restricted to shallow-water conditions.

The model used herein is based on a breaking criterion which should be approximately valid in shallow water as well as in deep water, while the decay of the wave height after breaking is assumed to be such that a height is maintained equal to the local breaker height. In other words, only the energy corresponding to the height in excess of the local breaker height is assumed to be dissipated. It is expected that this a more realistic model for the waves which break in not-very-shallow water than that used by Collins.

The breaking criterion for regular waves which has been adopted for use is based on Miche's formula for the limiting steepness of stable periodic waves in water of constant depth:

$$\left(\frac{H}{L}\right)_{\max} = 0.14 \tanh \frac{2\pi d}{L}, \quad (6)$$

which in shallow water approximates to

$$\left(\frac{H}{d}\right)_{\max} = 0.28\pi \approx 0.88 \quad (7)$$

Since we are here dealing with deforming waves in water of variable depth, Miche's formula cannot be expected to apply. It is known empirically that the H/d ratio for waves breaking on a beach in shallow water varies mildly with the initial wave steepness and the beach slope. (It ranges approximately between 0.6 and 1.2.) In view of this the following breaking criterion was chosen:

$$\frac{H_b}{L} = 0.14 \tanh \left(\frac{\gamma}{0.88} \frac{2\pi d}{L} \right) \quad (8)$$

which in shallow water approximates to

$$H_b = \gamma d \quad (9)$$

The value of γ varies with beach slope and, to a smaller extent, with the steepness of the incident waves. In applications to irregular waves it will be treated as a constant for given beach slope and mean wave steepness. The wavelength L in Eq. 8 is calculated from the wave period T and the mean water depth d by means of the classical formula for sinusoidal gravity waves in water of constant depth. Eq. 8 then defines a breaker height H_b for any given depth and wave period. The effects of period variability on H_b are not considered in the following; if necessary they can be taken into account numerically without changing the essence of the approach.

For calculating the mean energy density at some point it is assumed that those wave heights which in absence of wave breaking would exceed H_b are reduced by breaking to the value H_b . Let the (fictitious) wave heights \tilde{H} in absence of breaking be Rayleigh-distributed with rms value H_o . For given incident waves, H_o can be calculated at any point with existing methods, taking shoaling, refraction and bottom friction into account. The fictitious wave height distribution would then be known, and given by

$$\begin{aligned}\tilde{F}(h) = \Pr(\tilde{H} \leq h) &= 0 && \text{for } h < 0 \\ &= 1 - \exp(-h^2/H_o^2) && \text{for } h \geq 0\end{aligned}\quad (10)$$

where

$$H_o^2 = E\{\tilde{H}^2\} \quad (11)$$

This distribution will be clipped at $h = H_b$ in order to obtain an approximation of the actual wave height distribution:

$$\begin{aligned}F(h) = \Pr(H \leq h) &= 0 && \text{for } h < 0 \\ &= 1 - \exp(-h^2/H_o^2) && \text{for } 0 \leq h \leq H_b \\ &= 1 && \text{for } h > H_b\end{aligned}\quad (12)$$

The mean square value of H , which is approximately proportional to the mean wave energy density, can be calculated from this distribution function using the definition

$$H_{rms}^2 = E\{H^2\} = \int_{-\infty}^{\infty} h^2 dF(h) \quad (13)$$

which gives

$$H_{rms}^2 = \tilde{F}(H_b) H_o^2 \quad (14)$$

or

$$H_{rms}^2 = (1 - \tilde{Q}_b) H_o^2, \quad (15)$$

in which

$$\tilde{Q}_b = 1 - \tilde{F}(H_b) = \Pr(\tilde{H} > H_b) = \exp(-H_b^2/H_o^2) \quad (16)$$

is the fraction of the waves that break at the point with breakerheight H_b . From a comparison of Eq. 11 and Eq. 15 it follows that clipping the upper fraction \tilde{Q}_b of the fictitious wave height distribution gives a relative reduction of mean square value equal to \tilde{Q}_b . Only the Rayleigh distribution has this property.

It can happen that in very shallow water $H_b^2/H_o^2 \ll 1$. In that case Eq. 14, upon substitution of Eq. 10, reduces approximately to

$$H_{rms}^2 = H_b^2 \quad (17)$$

It may be seen that H_{rms} in this limiting case equals the value which it would have for regular waves with height H_b . This is to be expected because the approximation involved in effect consists of considering almost all the wave heights to be equal to H_b , while neglecting the contributions to $E\{H^2\}$ from wave heights less than H_b . Note that the condition $H_b^2/H_o^2 \ll 1$ may not be fulfilled anywhere if gradual energy dissipation by processes other than breaking is important, as can happen on very gentle slopes.

The condition $H_b^2/H_o^2 \ll 1$ usually implies $d/L \ll 1$, in which case Eq. 9 is a valid approximation of Eq. 8. With this substitution Eq. 17 becomes

$$H_{rms}^2 = \gamma^2 d^2, \quad (18)$$

so that the energy variation in irregular breaking waves according to this model approaches that which is usually assumed for regular waves (Eq. 4) as the relative water depth decreases.

Regarding the validity of the approximations given above, it should be remembered that only the final result (H_{rms}) is used in the subsequent calculations of the radiation stresses and their effects on the mean motion. The actual wave height distribution in the surf zone is undoubtedly smoother than the clipped distribution given by Eq. 12, which at $h=H_b$ increases discontinuously from $1-\hat{Q}_b$ to 1. However, with a suitable choice of γ the actual cumulative probability of the wave heights may be overestimated for $h < H_b$ and underestimated for $h > H_b$, which should result in relatively smaller errors in the calculated mean square wave height than in the probability function.

4. CALCULATED SET-UP AND LONGSHORE CURRENT VELOCITIES

The expressions for H_{rms} obtained in the preceding paragraph have been used in calculations of set-down and set-up and of longshore current velocities. Some results will be given here, with omission of computational details.

The radiation stresses were calculated from Eqs. 1, 2, and 3, where E was taken equal to $\rho g H_{rms}^2 / 8$, and n was determined from the mean wave period and the local depth. Energy dissipation by bottom friction was neglected. Thus, only shoaling and, where applicable, refraction have been taken into account in the determination of the fictitious rms wave height H_o . The waves were assumed to have a rather narrow distribution of energy with respect to frequency and direction. Set-up. Eq. 5 for the set-up of waves with perpendicular incidence was integrated numerically for $\gamma = 0.7$ and an incident wave steepness $H_o/L_o = 0.04$, where H_o is the rms wave height in deep water, and L_o the deep water wavelength calculated from the mean wave period. The resulting set-down and set-up given in Fig. 3. The vertical scale is exaggerated. Longshore current velocity. The formulation given by Thornton (1969) and Longuet-Higgins (1970) was used as a basis for the calculation of the longshore current velocity profile. The same linearization of the longshore bottom shear component was employed. However, no shallow-water approximations were used. Lateral momentum exchange due to turbulence was neglected. Some results are given in Fig. 4, in which the normalized local mean longshore current velocity is plotted against the relative depth, with the mean angle of incidence at deep water as a parameter. The normalizing factor is proportional to the ratio between the mean depth gradient and the bottom friction factor $c_f (= \tau_b / \rho u_b^2)$, and to the rms orbital velocity in deep water. The calculated normalized velocity reaches its greatest value for an angle of incidence near 60° . The current is confined to a rather narrow zone for very oblique incidence, due to the pronounced effects of refraction which then occur.

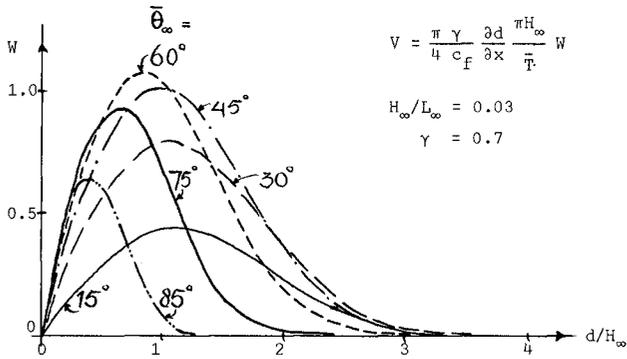


Figure 4 - Calculated longshore current velocities due to irregular waves.

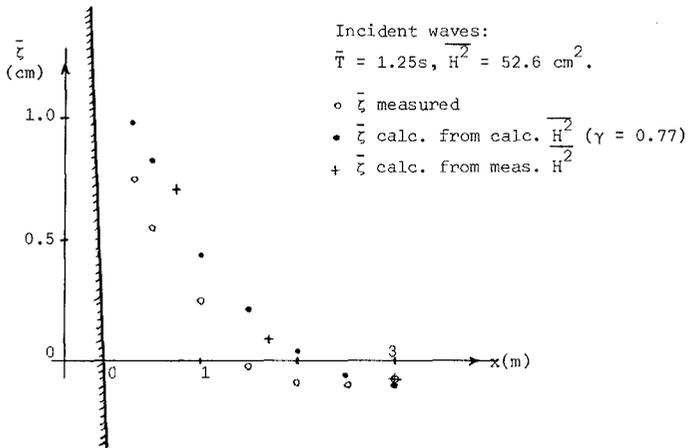


Figure 5 - Measured and calculated set-up.

It should be noted that in the figures 3 and 4 the independent wave parameters are characteristics of the incident waves in deep water. This is in contrast with known formulas for the longshore current velocity due to regular waves, most of which are expressed in terms of wave characteristics at breaking.

It has already been mentioned that the effects of variability of wave period and direction were not taken into account in the preceding calculations. It has been shown elsewhere (Battjes, 1972) that in particular the angular distribution, if not narrow, can strongly affect the radiation shear stress, which provides the driving force for the longshore current. Treating deep-water wind-driven waves (as opposed to swell) as if they were long-crested would result in over-estimating the total longshore thrust by more than 100%. This effect should therefore be accounted for if longshore currents are to be calculated from deep-water waves with a rather broad angular spreading of the energy.

5. LABORATORY MEASUREMENTS

A few measurements were carried out in a wave flume of the Delft Hydraulics Laboratory in order to check the validity of the approach outlined in the paragraphs 3 and 4. The results are briefly described in this paragraph. They will be more fully reported in a forthcoming publication, after the completion of additional tests.

5.1. Experimental arrangement

The wave flume is 100 m long and 2 m wide; the water in the constant depth portion of the flume was about 0.55 m deep. A hydraulically driven wave board capable of generating irregular waves is located at one end of the flume. At the other end a 1:20 straight plywood slope was installed. For measurements of the change in mean water level, particularly the set-up, 7 pressure taps (I.D. 4 mm) were provided, flush with the slope. The taps were connected by plastic tubes to a 15 cm I.D. vertical cylindrical well, where a vibrating-point gauge sensed the water surface elevation to an accuracy of approximately 0.1 mm. The amplified signals were recorded on paper. The water level was adjusted in such a manner that the pressure taps, including the uppermost one, would be submerged all the time during a run, or very nearly so, so that a valid reading of the mean water level could be obtained.

Four resistance type wave gauges were installed. The gauges in a mean depth less than 20 cm were inserted through the plywood slope in order to maintain the minimum submergence necessary for a linear gauge response. As a consequence of this arrangement these gauges could not easily be moved. The signal from each gauge was fed into analog equipment for the on-line determination of wave height histograms (based on 1000 wave heights between zero-crossings) and power spectra. The signals could also be recorded.

A rms incident wave height of 7 cm to 8 cm was used in all runs. It was kept more or less constant so that the pressure taps and the shallow-water wave gauges would be in the zone of breaking without being moved between runs. The mean wave period was approximately 1.2s and 2.0s. The width of the energy spectrum of the waves was varied as much as possible with the available equipment.

5.2. Experimental results

It appeared from the measurements that the applied variation of the width of the energy spectrum hardly affected the wave energy decay and the set-up. Its influence will not be further discussed.

Wave height. A comparison of measured and calculated mean square wave heights is given in table 1. The values of γ on which the calculations were based are also given in the table; they were chosen so as to optimize the overall agreement between measured and computed values. It may be seen that γ for the longer (less steep) waves is greater than that for the shorter (steeper) waves.

wave gauge no.	mean depth d (cm)	$\bar{T}=1.25s; \quad \gamma = 0.77$				$\bar{T}=2.0s; \quad \gamma = 0.88$			
		$\overline{H^2} \text{ (cm}^2\text{)}$				$\overline{H^2} \text{ (cm}^2\text{)}$			
		meas	calc	meas	calc	meas	calc	meas	calc
1	55	52.6	→ 52.6	56.1	→ 56.1	52.7	→ 52.7	63.7	→ 63.7
2	36	-	51.0	46.5	55.0	-	58.2	58.8	69.4
3	16	47.4	50.2	47.8	51.9	70.8	71.3	77.9	80.7
4	8.8	32.6	29.5	36.3	31.2	42.6	45.0	48.8	45.4
5	5.1	12.5	11.5	-	11.6	20.3	18.7	-	19.0

Table 1

This is in agreement with what is known for regular waves. Inspection of table 1 shows that the variation of H^2 with distance offshore is satisfactorily described by Eqs. 15 and 16.

Set-up. An example of measured and computed set-up is given in Fig. 5. (The set-up curves which are not shown are essentially the same as those in fig. 5, and will not be presented individually). The set-up was calculated on the basis of Eq. 5 and the assumptions outlined in the paragraphs 3 and 4. Its value was taken to be zero at the toe of the slope. The set-up (set-down) appears to be fairly well predicted by the theory at $x \approx 3$ m ($h \approx 15$ cm), which is the most seaward measuring point, where only a small fraction of the waves is breaking. However, shoreward of this point, from $x \approx 3$ m to $x \approx 1.5$ m, the computed and measured values diverge, the computed values showing a rise towards the shore which is almost absent in the measurements. The maximum difference is about 2.5 mm. Shoreward of $x \approx 1$ m ($h \approx 5$ cm) both sets of points follow very nearly the same trend.

The discussion of the results described above naturally centers on the area of strongest disagreement between theory and measurements. If the disagreement were used as an argument to reject the theory then the question arises: which parts of the theory? It was found to describe the rms wave height variation fairly well. In order to eliminate any uncertainties which nevertheless might be present in the theoretically calculated wave heights, the set-up between three wave gauges was computed using measured rather than calculated wave heights. The result is given in Fig. 5. As expected, the disagreement has hardly diminished. The conclusion is that the measurements are in error, and/or that the theory used in relating

wave characteristics and set-up is in error, particularly Eq. 5. The latter possibility is regarded as very unlikely because Eq. 5 has been confirmed in previous tests with regular waves (Bowen et al, 1968). The wave height measurements in the present tests are considered reliable, due to frequent calibrations, which moreover showed good linearity and no measurable drift. Another possibility might be that the set-up measurements are in error. The recording system of the water level variation inside the stilling wells was frequently calibrated and is not suspect. Air bubbles were sometimes present in the tubes connecting the wells to the pressure taps; this was concluded from the recorded signals which in those cases did not go back to the original still water value after a run. All such measurements were discarded and the air bubbles removed. Thus it is believed that the recordings of the vibrating-point gauges give a good measure of the change in the mean static head at the location of the pressure taps due to the waves. It has been shown by Dorrestein (1961) for water of constant density that this change is equal to the change in mean depth if the waves are statistically stationary in time and in the horizontal coordinates. The latter condition is not exactly met in the present tests but an analysis such as given by Dorrestein shows that errors arising from horizontal inhomogeneities cannot account for the observed discrepancy. Lacking further information, it is believed that the inconsistencies may have originated in the neglect of air entrapment during breaking. Air content with concentration C would cause a systematic underestimation of the true mean water level by an amount approximately given by

$$\Delta = \int_0^d \overline{C(z)} dz$$

The concentration \overline{C} varies with the intensity and the frequency of breaking. The difference Δ should be almost zero in relatively deep water where breaking occurs very infrequently, and increase in the shoreward direction as more and more waves are breaking. In very shallow water it may diminish again with the decreasing depth d . The air concentration which would be necessary to account for the observed discrepancies is of the order of a few percent, averaged over the depth and the time. This does not appear to be impossible. However, without additional supporting data the explanation given above should be considered as purely conjectural. It is planned to carry out additional tests from which perhaps more definitive conclusions can be drawn.

APPENDIX 1 - REFERENCES

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APPENDIX 2 - LIST OF SYMBOLS

c_f	bottom shear stress coefficient
C_f^t	volume concentration of air in water
d	mean depth ($= h + \bar{\zeta}$)
E	mean wave energy per unit area
$E \{.\}$	expectation of quantity in brackets
F	cumulative probability of H
\bar{F}	cumulative probability of \bar{H}
g	gravitational acceleration
h	still-water depth
h	realization of H or \bar{H} (in Eqs. 10 through 13 only)
H	wave height (total range in water surface elevation between two successive downcrossings of mean level)
\bar{H}	fictitious wave height in absence of breaking
H_0	rms value of H
H_{∞}	rms value of incident wave height in deep water
k	$2 \pi/L$
L	wavelength
L_{∞}	deep-water value of L calculated from \bar{T} of incident waves
n	coefficient (eq. 3)
Q	probability of exceedance
S_{11}	largest principal radiation stress
S_{22}	smallest principal radiation stress
T	wave period (between successive downcrossings of mean level)
u_b	orbital velocity near the bottom acc. to potential theory
v_b	longshore current velocity, averaged over depth and time
W	normalized value of V (see fig. 4)
x	horizontal coordinate, positive seawards
z	vertical coordinate, positive upwards
γ	coefficient in breaker criterion
Δ	error in set-up measurements
ζ	elevation of water surface above still-water level
θ_{∞}	angle between direction of wave propagation in deep water and normal to the depth contours
τ_B	bottom shear stress

subscript "b" refers to "breaker"

an overbar denotes time average or arithmetic average