CHAPTER 94

A PROBABILISTIC APPROACH TO DETERMINE WAVE FORCES ON OCEAN PILE STRUCTURES

by

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ABSTRACT

Multiple linear regression analysis is applied to predict horizontal velocities and acceleration of water particles subjected to waves. Furthermore it was used to predict the coefficient of drag for circular piles. The mean value functions of the parameters are calculated and the assumption of their lognormal distribution reasonably well verified. The method used here is free from theoretical assumptions about wave mechanisms and, hence, explains the behavior of experimental results.

Using Monte Carlo simulation these regression relations were then utilized to generate the distribution functions of wave forces. Using the Morison force equation, in this simulation, the distribution function for the drag and the inertial components of the force are determined separately. A linear superposition of those time varying processes was performed to obtain the distribution of the total maximum force. Finally a probabilistic wave height-wave force relationship was developed for the purpose of creating a force distribution function given a random sea state.

1. INTRODUCTION

Circular piles are often used to support various ocean structures. The wave forces to which they are subjected are of utmost importance for the design of such structures.

The actual situation in the ocean presents a confusing and often frustrating problem to the engineer. Our limited knowledge about the ocean waves has made engineering design extremely difficult, resulting in a number of failures in

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the past.

The following factors complicate the design considerably: first, wave characteristics change continuously as the wave travels. Secondly, even the forces exerted by identical waves vary greatly. This variation is caused by fluctuations in the fluid particle velocities and accelerations as well as eddies around piles caused by the rapidly reversing flow. These random phenomena can only be handled using a probabilistic approach.

A cumulative distribution function of wave forces created from field data would give the engineer sufficient information for his design. In reality most of the available data on ocean waves consist of records of wave height, period and length, three easily measured properties. However, useful wave force measurements have been conducted very seldom due to high cost and extraordinary experimental difficulties involved.

Therefore a method is proposed which transforms available data into useful form. In other words, records of wave heights, periods and lengths can be converted in cumulative distribution functions of wave forces. The method utilizes a multiple regression analysis to develop a relationship between those easy to measure wave properties and the parameters used in the Morison[1] force equation. These parameters include water particle velocities and accelerations as well as the coefficients of drag and inertia. Using Monte Carlo simulation these regression relationships are then the basis of the development of the cumulative distribution function of the wave forces. Given this distribution, the engineer can make reliability statements about his design. This information developed above can then be utilized to create a probabilistic wave force-wave height relationship. This relationship enables one to take the randomness of the sea into account. The most likely cumulative distribution function of the wave forces for a particular wave height distribution given a wave period can be determined. Integration over all possible periods for a fully developed sea leads to a cumulative distribution function of the force, which again can be used for reliability statements.

2. DEVELOPMENT OF THE MAXIMUM FORCE DISTRIBUTION FUNCTION

The generally accepted and most widely used formula for a force on a pile is the so-called Morison formula,[1]

$$F = \frac{1}{2} D\rho C_D |u|u + \rho \frac{\pi D^2}{4} C_M \frac{\partial u}{\partial t} \qquad (1)$$

The first part of this equation represents the form drag caused by surface shear. The second part of the equation is the acceleration force on the displaced volume of fluid including the virtual mass effect. Although the particle motion in waves is orbital, the method is based on rectilinear flow. Morison also assumed that the drag force and the inertia force can be treated separately, and that the total force can be obtained by adding the solutions linearly.

The fluid density ρ and the pile diameter D are assumed to be constant. However, based on experimental evidence[2,3,4,5,6] the water-particle velocity u, the acceleration $\partial u/\partial t$, the coefficient of drag, C_D, and the coefficient of mass, C_M, are considered random variables. Their mean value functions and distributions are developed by utilizing available experimental data [2,3,4] and applying the mathematical method of multiple linear regression analysis. The results obtained are the following:[7]

$$u^* = \exp(0.442 + 0.04L^* - 0.231d^* + 0.221S^*) \qquad (2)$$

where $u^* = \frac{uT}{H}$, $L^* = \frac{L}{H}$, $d^* = \frac{d}{H}$, $S^* = \frac{S}{H}$,

and

$$a^* = \exp(2.856 + 0.04L^* - 0.28d^* + 0.127S^*) \qquad (3)$$

where $a^* = \frac{aT^2}{H}$

and

$$C_D = e^{0.484}(Re)^{-0.694}(H^*)^{0.067}(T^*)^{0.115} \qquad (4)$$

where $H^* = \frac{H}{D}$, $T^* = \frac{uT}{D}$

Note that the equations (2), (3) and (4) represent mean value functions. The assumption of the lognormal distribution of the dependent parameters u^*, a^* and C_D was reasonably well verified by means of probability and error plots.[7] The coefficient of mass, C_M, was assumed to be lognormally distributed with a mean value of 2.5 and a standard deviation of 1.2. Equations (2) and (3) represent the maximum water particle velocities and accelerations within a wave.

These equations can be expressed as a function of time

as

$$u(t) = u \cdot \cos\omega t \tag{5}$$

$$a(t) = a \cdot \cos\omega t \tag{6}$$

where $\omega=2\pi/T$, the angular frequency. Substituting equations (5) and (6) in equation (1) the following expression is obtained:

$$F_T(t) = C_D\left(\frac{\rho D}{2}\right)\left(\frac{H}{T}\, u^*\right)^2 \cos\omega t|\cos\omega t| + C_M\left(\frac{\rho\pi D^2}{4}\right)\left(\frac{H}{T^2}\, a^*\right)\sin\omega t \tag{7}$$

Since the drag component is 90° out of phase with the inertia component, the maximum drag force occurs when the wave crest or trough passes the pile. The maximum inertia force occurs when the wave passes the still water level. The maximum total force exerted by a wave is derived by setting the first derivative of equation (7) to zero. This results in the following equation:

$$F_{T_{max}} = C_D\left(\frac{\rho D}{2}\right)\left(\frac{H}{T}\, u^*\right)^2 \left\{1 + \frac{1}{4}\left[\frac{C_M\left(\frac{\rho\pi D^2}{4}\right)\left(\frac{H}{T^2}\, a^*\right)}{C_D\left(\frac{\rho D}{2}\right)\left(\frac{H}{T}\, u^*\right)^2}\right]^2\right\} \tag{8}$$

The expression $F_{T_{max}}$ in equation (8) is a function of the previously determined random variables u, a, C_D, C_M and is therefore also a random variable. Equation (8) can be written:

$$f_{F_{T_{max}}}(f_{T_{max}}) = f_{C_D}(C_D)\left(\frac{\rho D}{2}\right)\left(\frac{H}{T}\, f_{u^*}(u^*)\right)^2 \cdot \left\{1 + \frac{1}{4}\left[\frac{f_{C_M}(C_M)\left(\frac{\rho\pi D^2}{4}\right)\left(\frac{H}{T^2}\, f_{a^*}(a^*)\right)}{f_{C_D}(C_D)\left(\frac{\rho D}{2}\right)\left(\frac{H}{T}\, f_{u^*}(u^*)\right)^2}\right]^2\right\} \tag{9}$$

The maximum total force probability density function $f_{F_{T_{max}}}(f_{T_{max}})$

can be obtained rigorously by derivation of the distribution as a function of the random variables u, a, C_D and C_M which are assumed to be lognormally distributed. Although the probabilistic portion of the derivation of the distribution is a rather direct problem, the calculus needed to evaluate the resulting integrals seems to be intractable. Fortunately, an approximate method for creating derived distributions, the Monte Carlo Method,[8] furnishes a distribution of sufficient accuracy.

3. DEVELOPMENT OF A PROBABILISTIC WAVE FORCE - WAVE HEIGHT RELATIONSHIP FOR AN OCEAN SURFACE

In the previous section it was shown how to obtain a maximum wave force probability density function for a single wave.

The actual situation in the ocean presents a complex and confusing problem. The local wind generated waves are superimposed on the large ocean waves whose origin may be thousands of miles away. A deterministic approach to ascertain the wave heights of ocean waves is impossible because of the present inadequate understanding of complicated wave systems. However, by observing the random nature of the ocean waves, it is plausible that some statistical regularity can be found.

3.1 Statistical Distribution of Wave Heights

Wiegel et al.[4] measured the forces exerted by ocean waves on vertical circular cylindrical piles near Davenport, California, in water ranging from 45 feet to 50 feet deep. The relation of H versus T of the measured data is plotted in Figure 1. The data show a considerable amount of scatter, indicating the presence of waves with various combinations of H and T in the ocean. To gain insight into this apparent randomness, the data are grouped into several ranges of the period T as follows:

Period Group		Average Period Representing the Range (sec.)
1	$6 \leq T < 10$	$T_1 = 8$
2	$10 \leq T < 12$	$T_2 = 11$
3	$12 \leq T < 14$	$T_3 = 13$
4	$14 \leq T < 16$	$T_4 = 15$
5	$16 \leq T < 20$	$T_5 = 18$

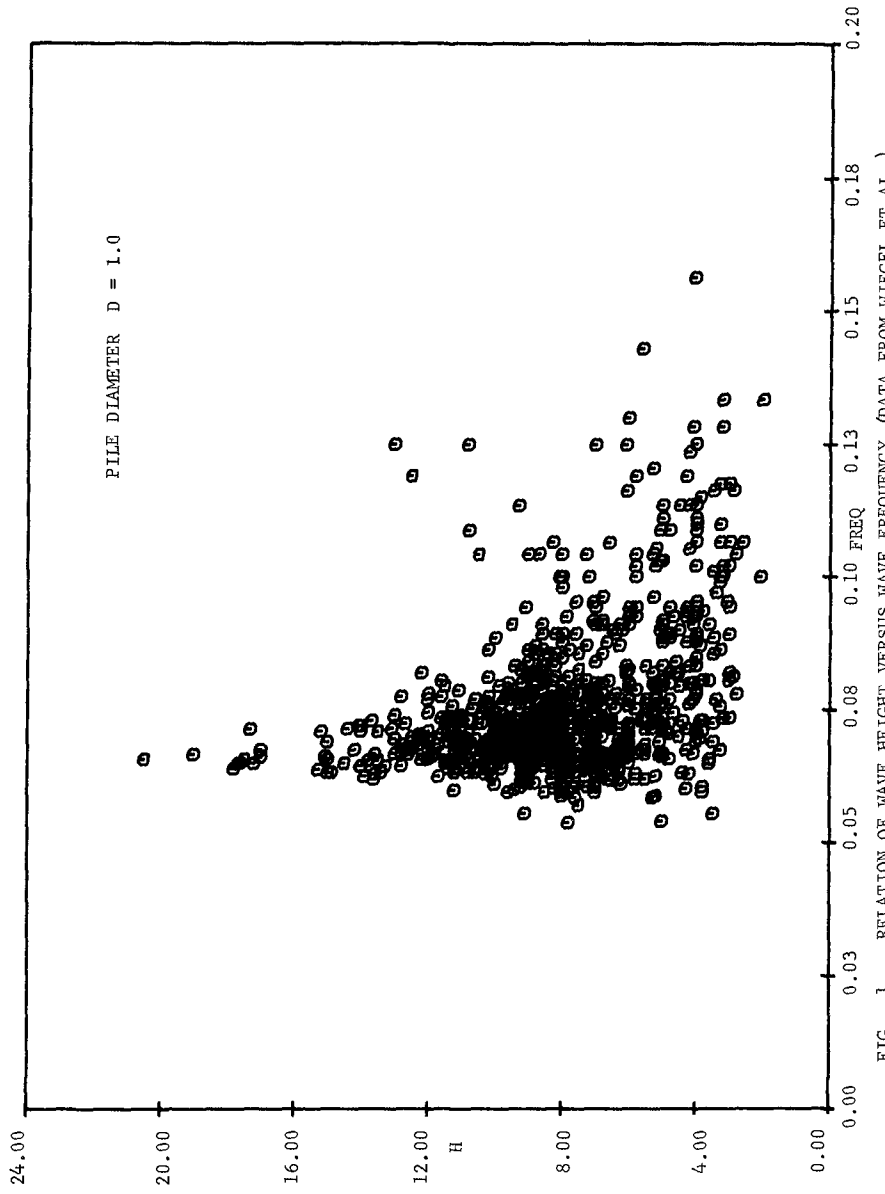

FIG. 1 RELATION OF WAVE HEIGHT VERSUS WAVE FREQUENCY (DATA FROM WIEGEL ET AL.)

Histograms are then constructed for each period. By normalizing the histogram and dividing the resulting probability mass function by the size of the H interval, the frequency function of H for each period group is obtained. These frequency functions are shown in Figure 2(a)-(e). It is interesting to see that the distribution of H resembles the Raleigh distribution whose PDF is given by

$$f_H(h) = \begin{cases} (2h/c^2)\exp(-h^2/c^2), & \text{for } h \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{10}$$

where h represents a particular realization of the random wave height, H, and c^2 is the mean square of H.

The idea that the wave heights follow the Raleigh distribution was introduced by Longuet-Higgins[9] in his theory for waves having narrow band spectra.

In the derivation of equation (10), Longuet-Higgins assumes that the ocean waves involved were nondirectional and infinitesimal so that simple linear wave theory would apply. Borgman[10] argued that although for large waves the data are not exactly distributed according to the Raleigh distribution, the Raleigh curve appears similar enough to the data to justify its use in most engineering approximations. However, for this problem it is only of importance that there is some statistical regularity in the distribution of H. Figures 2(a)-(e) indicate that such regularity exists.

3.2 Functional Relationship Between H and F

The analytical tools which are employed to find the functional dependence of F on H are the regression analysis and the Monte Carlo simulation. From Section 2, it can be seen that the functional relation between H and F is a probabilistic one. Equation (9) shows how to obtain the maximum total force probability density function. For this equation every parameter is given in Wiegel's data[4] except the wave length L. However, L can be approximated by utilizing the linear wave theory[5]

$$L = \frac{gT^2}{2\pi} \tanh\left(\frac{2\pi d}{L}\right) \tag{11}$$

Equation (11) is an implicit formula for L which may be solved

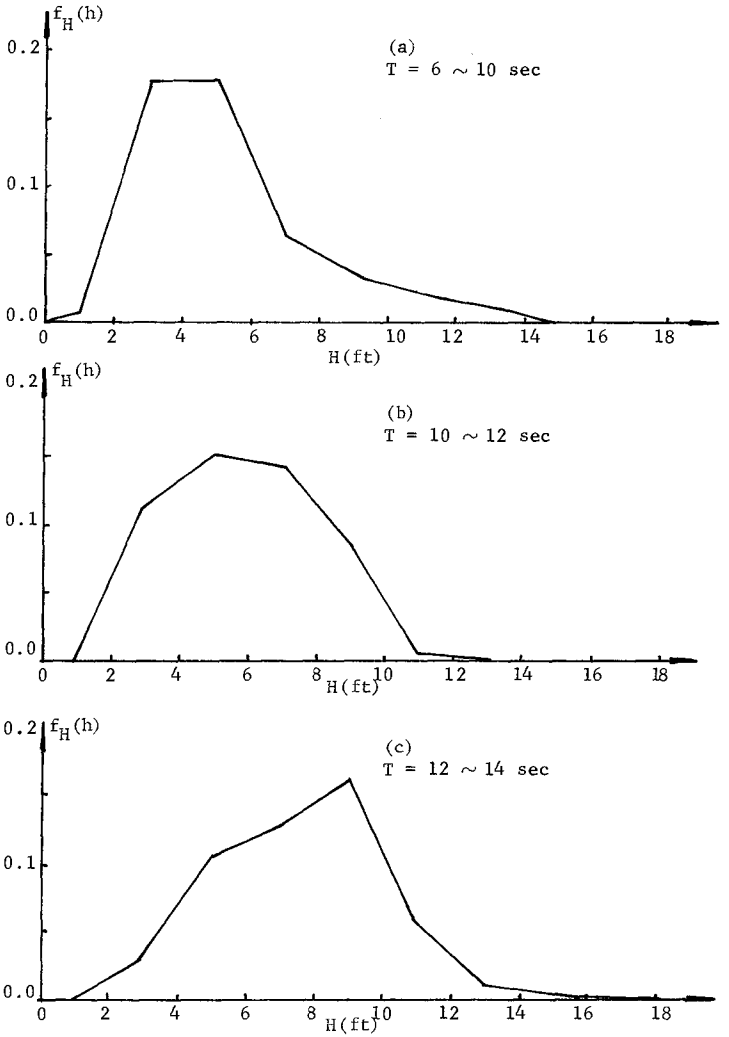

FIG. 2. WAVE HEIGHT FREQUENCY FUNCTIONS
(a) FOR T = 6 ~ 10, (b) T = 10 ~ 12,
(c) T = 12 ~ 14 sec. [continued]

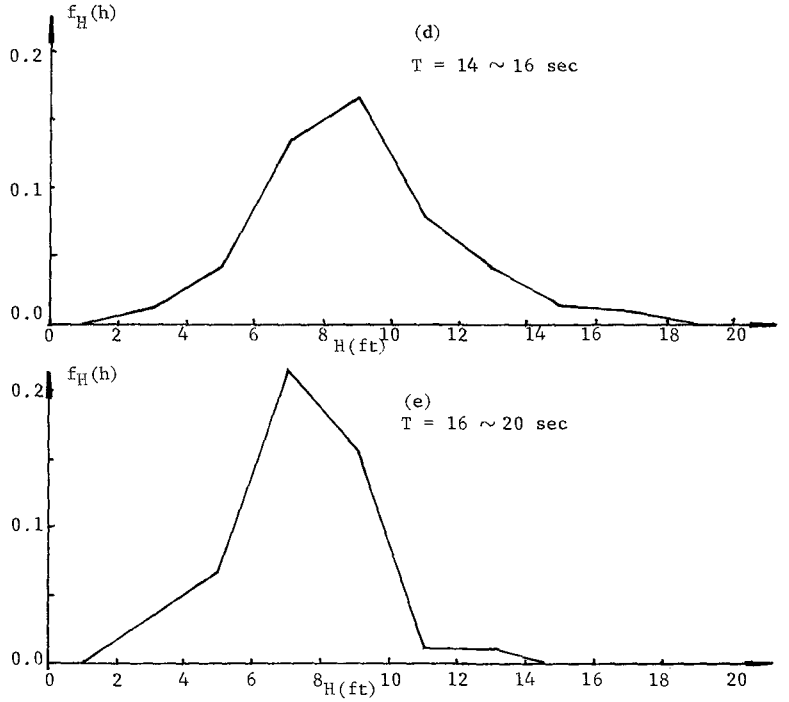

FIG. 2. WAVE HEIGHT FREQUENCY FUNCTIONS
(d) FOR T = 14 ~ 16, (e) T = 16 ~ 20 sec.

by using the Newton-Raphson method.

To develop the mean value functions of the wave forces, Monte Carlo simulations must then be performed for constant wave periods T and various wave heights H. This process results in a family of mean value functions, shown in Figure 3.

Figures (4)-(8) show the coaxial diagrams which demonstrate the relationships between H, T and $F_{T_{max}}$. Part A of the graphs contains the CDF for the wave height, determined from field data. Part B relates the wave height to wave force for a given value of T.

Since the relationship between H and T is probabilistic, each T curve is a mean value function. Part C is a CDF for the wave force which is derived from parts A and B of the graphs as illustrated in the diagram. This process can be easily performed since the relationship between $F_{T_{max}}$ and H is a monotonically increasing function. Because the height-force relationship is probabilistic, the CDF of the wave force is a graph of the most likely force to be expected. Thus the method of coaxial relationships shown here can be used to generate a CDF for wave forces for a particular period and a given wave height distribution.

3.3 Application of the Method Proposed in Design

In order to apply this method in design more extensive data are needed on wave height distributions. If a pile is to be designed to last several years then the statistical frequency of unusual high waves must be known. The data series used to determine the CDF of H would consist of wave data for large storms or a tabulation of those waves exceeding a certain wave height in a given record. For example, in the Gulf Coast, hurricane data would probably be used in design. It may also be necessary to consider the probability of hurricane or storm occurrence in the analysis.[11]

4. CONCLUSIONS

This analysis has demonstrated a method for determining the cumulative force distribution of piles caused by a given wave height distribution. To create a force distribution the Morison force equation was used to develop the relationship between wave force ans several wave properties using experimental data. In utilizing the Morison equation, the horizontal water particle velocity and acceleration as well as the coefficients of drag and mass were considered random variables. The mean value functions and variances of these random variables were developed using multiple linear regression analysis and

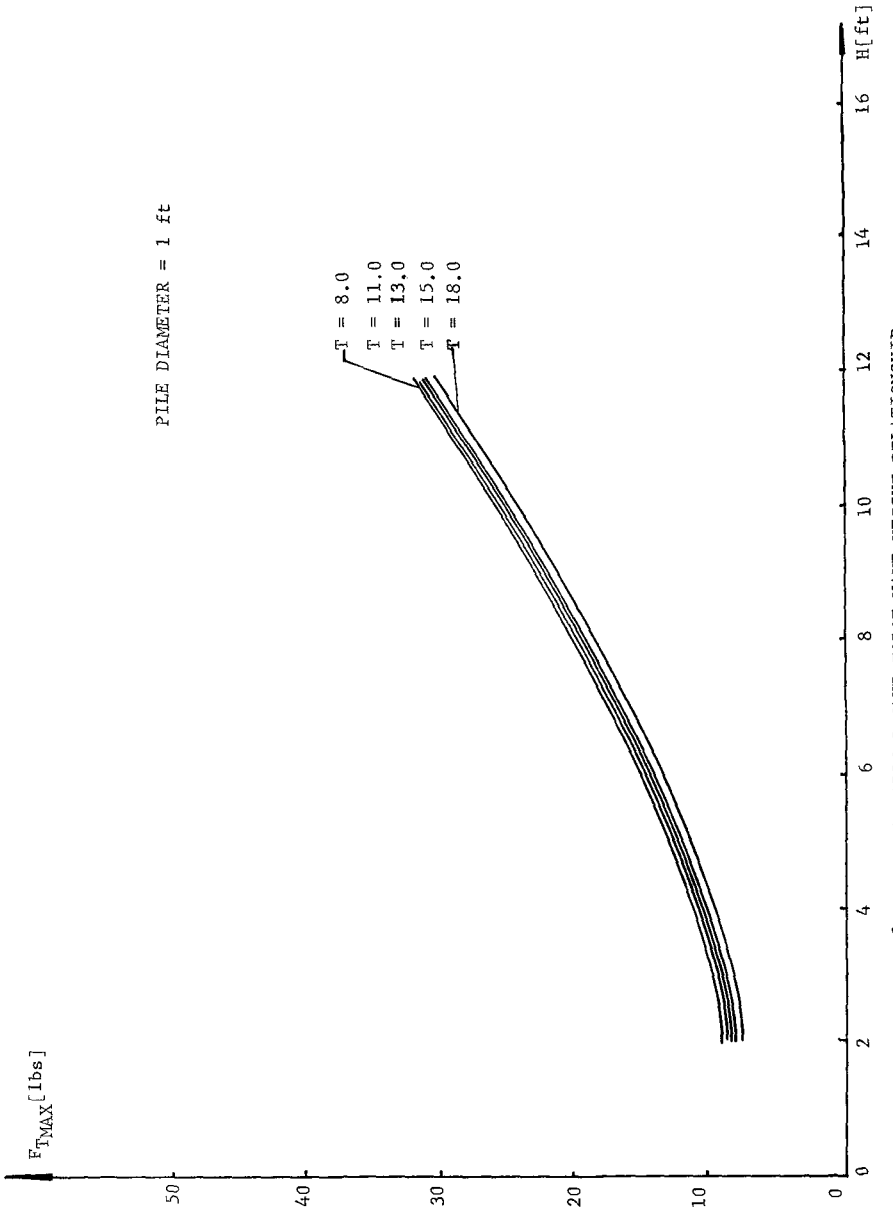

FIG. 3. PROBABILISTIC WAVE FORCE-WAVE HEIGHT RELATIONSHIP.

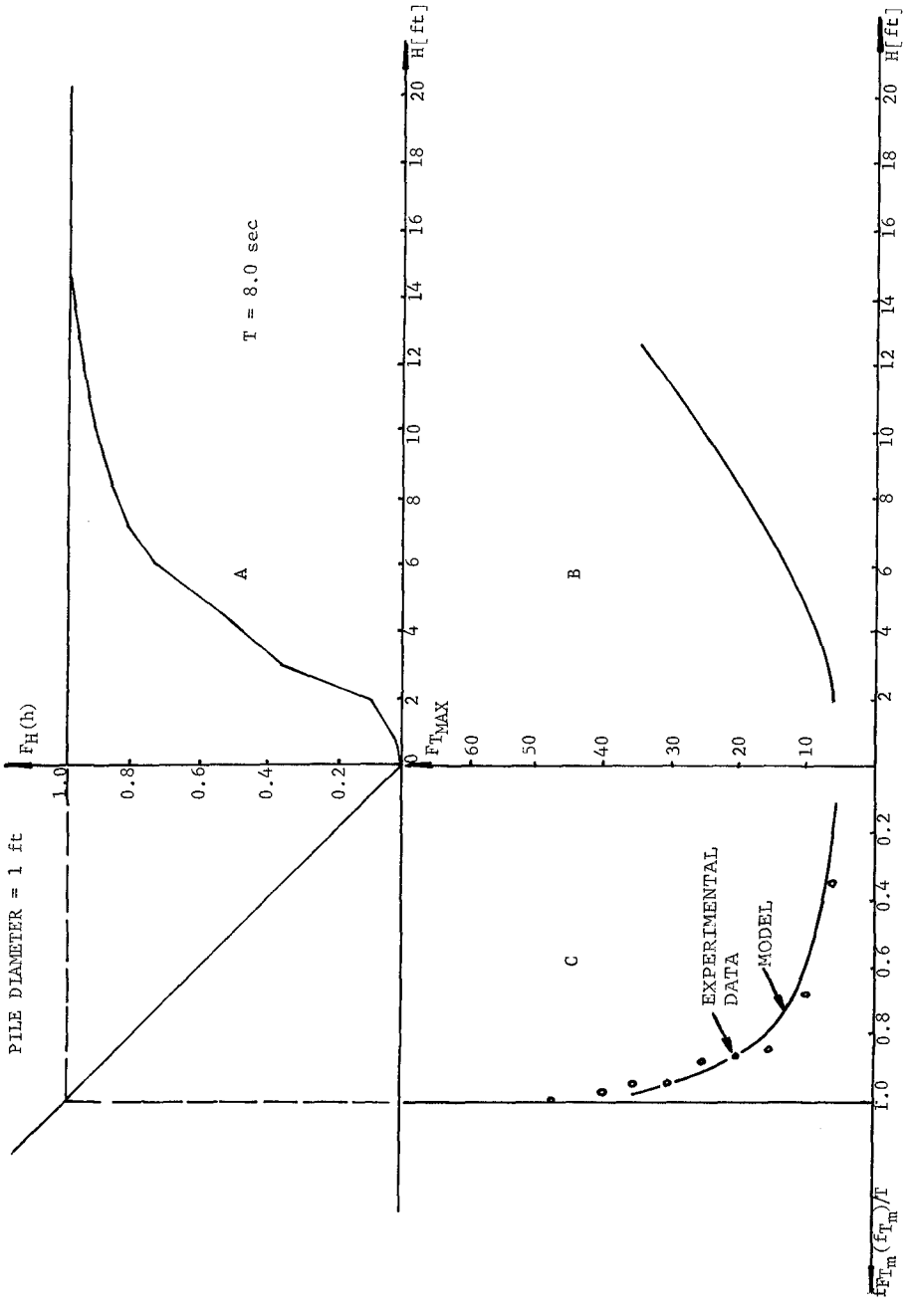

FIG. 4. CO-AXIAL DIAGRAM FOR RELATIONSHIP BETWEEN H, T, AND F_{MAX} FOR T = 8 SEC.

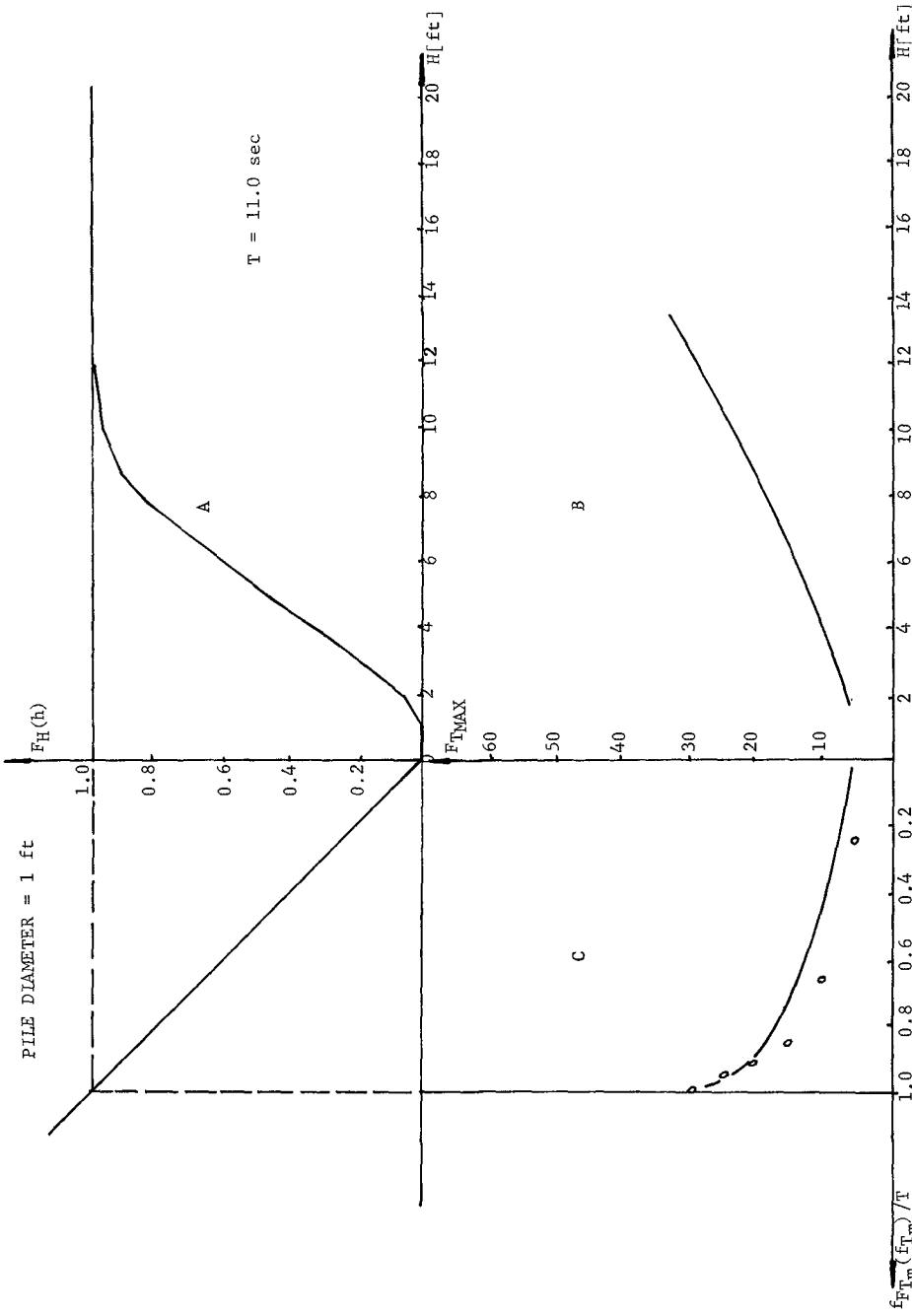

FIG. 5. CO-AXIAL DIAGRAM FOR RELATIONSHIP BETWEEN H, T, AND F_{MAX} FOR T = 11 SEC.

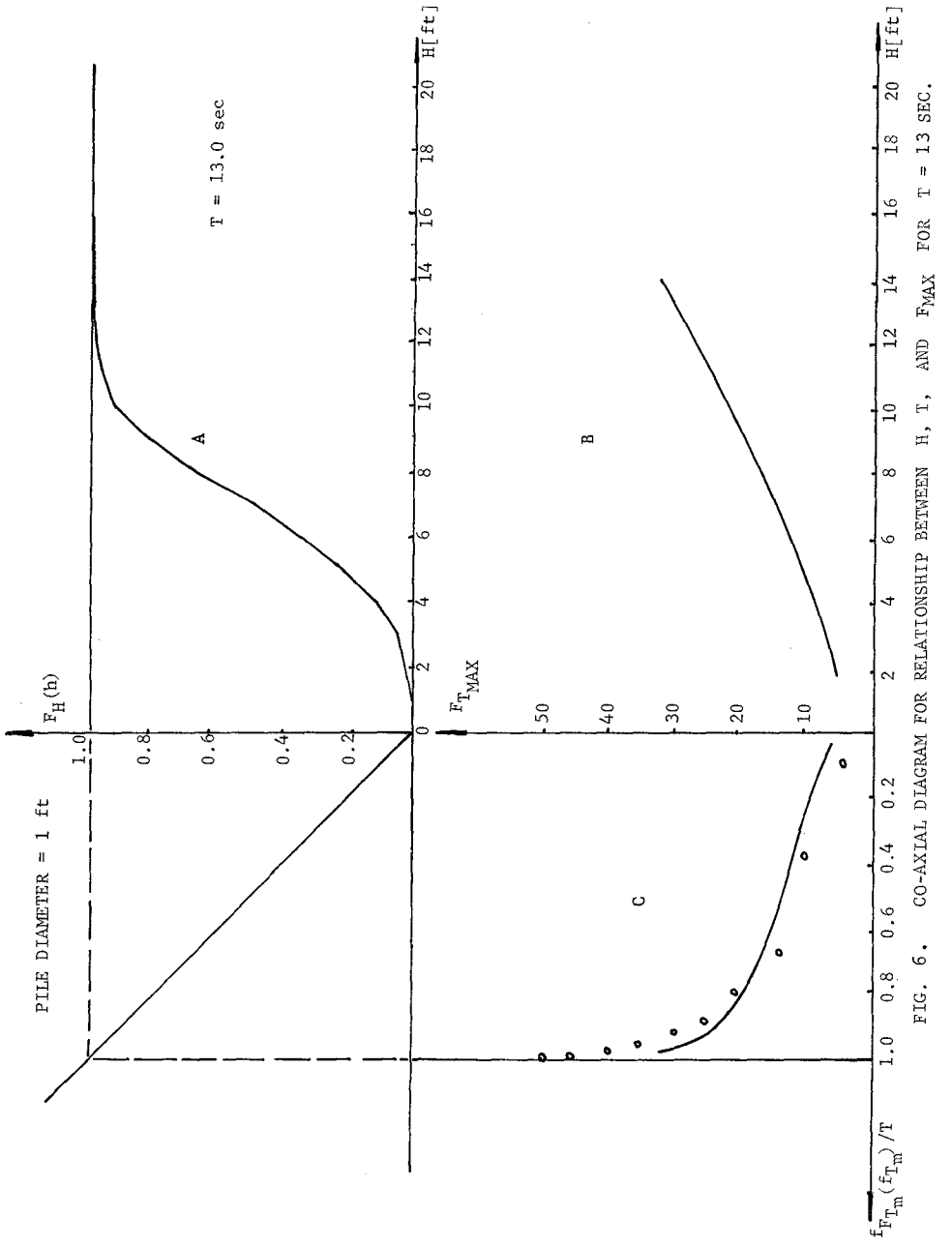

FIG. 6. CO-AXIAL DIAGRAM FOR RELATIONSHIP BETWEEN H, T, AND F_{MAX} FOR T = 13 SEC.

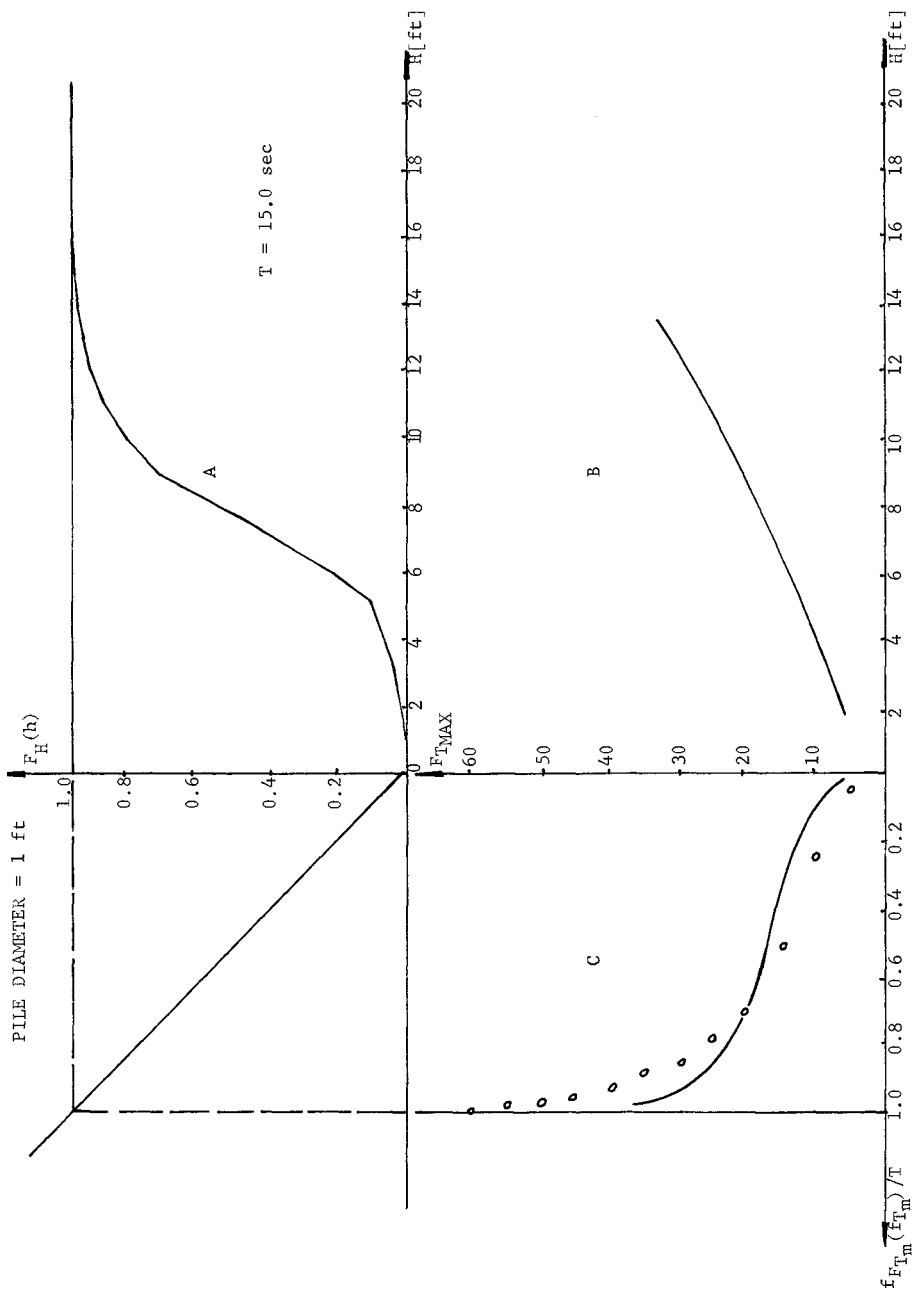

FIG. 7. CO-AXIAL DIAGRAM FOR RELATIONSHIP BETWEEN H, T, AND F_{MAX} FOR T = 15 SEC.

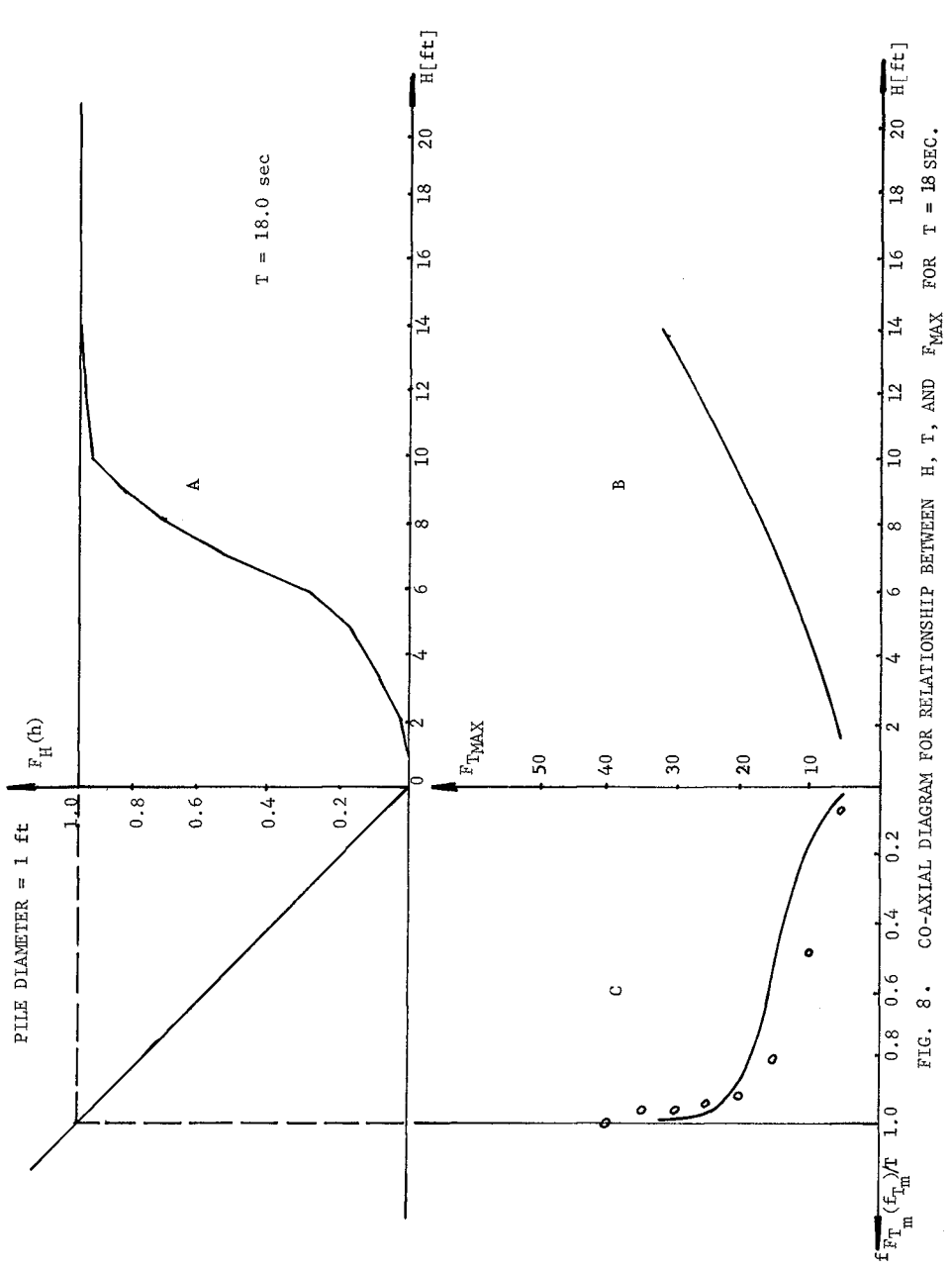

FIG. 8. CO-AXIAL DIAGRAM FOR RELATIONSHIP BETWEEN H, T, AND F_{MAX} FOR T = 18 SEC.

assumed to be lognormally distributed.

Unfortunately, the prediction equations developed are valid only for the range of the original data. The valid range of these prediction equations could be expanded considerably if data were available in other ranges. Currently, experimental data for water particle velocity, and acceleration are seriously lacking. Data are also needed for long-crested, low amplitude waves with large periods and for low amplitude waves with large water depths.

Although prior knowledge of the relationships between the wave particle velocity acceleration and other wave properties is known from linear and other wave theories, an extrapolation beyond the data cannot be justified.

In the development of the regression equation of the drag coefficient considerably more data were available. The big advantage of this equation is that it makes possible the development of a probability density function which is related to particular measured wave properties.

Experimental data used for the determination of the drag coefficient were taken at the still water level. If experimental data were available for different values of S (water particle depth) then a study of the variation of the drag coefficient for different mean water particle positions could be performed. This additional information would give much more insight into the problem.

The Monte Carlo simulation technique was employed to derive the maximum wave force probability density function using the regression equations previously developed.

Finally, a method was proposed which makes the creation of cumulative force distributions for given wave height distributions possible. For this purpose a probabilistic wave height-force relationship was developed. This method also takes the probability of occurrence of extreme waves into account. The agreement between the experimental data and the model is quite acceptable.

NOMENCLATURE

a ..... pile diameter

c^2 ..... mean square of H

C_D ..... coefficient of drag

C_M ..... coefficient of mass

d ..... water depth

D ..... pile diameter

$f_{F_{T_{max}}}(f_{T_{max}})$ ..... maximum wave force probability density function

F_T ..... total wave force

h ..... particular realization of H in Raleigh distribution

H ..... wave height

L ..... wave length

S ..... mean water particle depth

T ..... wave period

u ..... water particle velocity

ρ ..... fluid density

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