CHAPTER 91

PRESSURE UPON VERTICAL WALL FROM STANDING WAVES

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When surge waves approach a vertical wall a standing wave is formed ahead of the latter. This is the only case when the interaction between waves and structure result in a stable mode of motion with distinct kinematic characteristics. Such motion can be described by equations of hydromechanics without the introduction of any hydraulic coefficients; a comparison of various theoretical solutions with experimental data can serve as an additional criterion for evaluating the accuracy of this or that solution.

The first theoretical solution for wave pressure acting upon a vertical wall under the effect of standing waves at a finite depth has been published by Sainflou in 1928 (1).

By correlating motion equations for surge waves derived by Gerstner as early as 1802 and Flamant's equations for standing waves on an infinite depth, Sainflou derived for the case of standing waves on a finite depth the following relations:

$$x = x_o - 2\,r \cos 6t \sin kx_o$$
$$y = y_o + 2\,r_1 \cos 6t \cos kx_o + 2\,krr_1 \cos^2 6t \qquad (1),$$

In Eq. (1) :

$$r = \frac{h}{2}\,\frac{ch\,k(H + y_o)}{sh\,kH}; \quad r_1 = \frac{h}{2}\,\frac{sh\,k(H + y_o)}{sh\,kH};$$

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x_o and y_o - ordinates of a particle at rest;
h, λ and τ - respectively height, length and period of a wave;

H - depth; $k = \frac{2\pi}{\lambda}$; $\sigma = \frac{2\pi}{\tau}$;

abscissae axis coincides with still water level; ordinate axis coincides with wall surface and is directed upwards.

When deriving the pressure relation Sainflou utilized the hydrodynamic equilibrium equation in terms of Lagrange variables:

$$\frac{1}{\rho}\frac{\partial p}{\partial x_o} = -\frac{\partial^2 x}{\partial t^2}\frac{\partial x}{\partial x_o} - \left(g + \frac{\partial^2 y}{\partial t^2}\right)\frac{\partial y}{\partial x_o},$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y_o} = -\frac{\partial^2 x}{\partial t^2}\frac{\partial x}{\partial y_o} - \left(g + \frac{\partial^2 y}{\partial t^2}\right)\frac{\partial y}{\partial y_o}. \qquad (2)$$

After substituting all the partial derivatives - taking into account the second order terms in relation to wave height we have:

$$\frac{1}{\rho}\frac{\partial p}{\partial x_o} = 2(gkr_1 - \sigma^2 r)\cos\sigma t \sin kx_o + 2k\sigma^2(r^2 - r_1^2)\cos^2\sigma t \sin 2kx_o \qquad (3)$$

$$\frac{1}{\rho}\frac{\partial p}{\partial y_o} = -g - 2gkr\cos\sigma t\cos kx_o - 2gk^2(r^2 + r_1^2)\cos^2\sigma t + 2\sigma^2 r_1\cos\sigma t\cos kx_o + 4k\sigma^2 rr_1\cos^2\sigma t + 4k\sigma^2 rr_1\cos 2\sigma t \qquad (4)$$

The continuity equation is satisfied on condition

$$k(r^2 - r_1^2) \approx 0 \qquad (5)$$

By omitting in integrating (3) and (4) all terms with second and higher order factors in relation to wave height, Sainflou arrives to the following relation for pressure upon a wall (with $x_o = 0$):

$$\frac{p}{\gamma} = - y_o - 2h \frac{sh\ ky_o}{sh\ 2kH} \cos 6t \qquad (6)$$

By substituting in (6) cos 6t = 1 we derive pressures on approach of wave crest; substitutinn of cos 6t=-1 gives respectively the pressure at trough approach.

For plotting pressure diagrams we preset 5 or 7 values of ordinates of resting particles y_o, then calculate the pressure using Eq.(6), and apply it to points, the positions of which are derived from the equation:

$$y = y_o + 2r_1 \cos 6t + 2rr_1 \cos^2 6t \qquad (7)$$

From the moment of its first publication relation (6) was generally used for practical calculations all over the world. Only in the fifties the works of Miche (2) demonstrated that in some cases the method implies considerable errors. According to Sainflou (6) maximum excess wave pressure upon a wall always tekes place on the approach of the crest (cos 6t = 1), and pressure value is positive for all points acrosss the height (Fig.1a). But practically Miche was the first to demonstrate (1) that at considerable depth ahead of the wall maximum pressure can occur not on crest approach, but in some intermediate moment; during the approach of the crest even negative pressures can possibly occur near the bottom (Fig. 1b). Calculation methods were developed, which take into account the second order terms in relation to wave height. Rundgren's (3) and Kuznetsov's (4) methods are among the most widely known.

Rundgren's paper, published in 1958, completes the investigations that were started by Miche (2) and Biesel (5). The solution procedure is as follows: basic motion characteristics are found as polynomials and are expanded by the smaller parameter powers in relation to wave

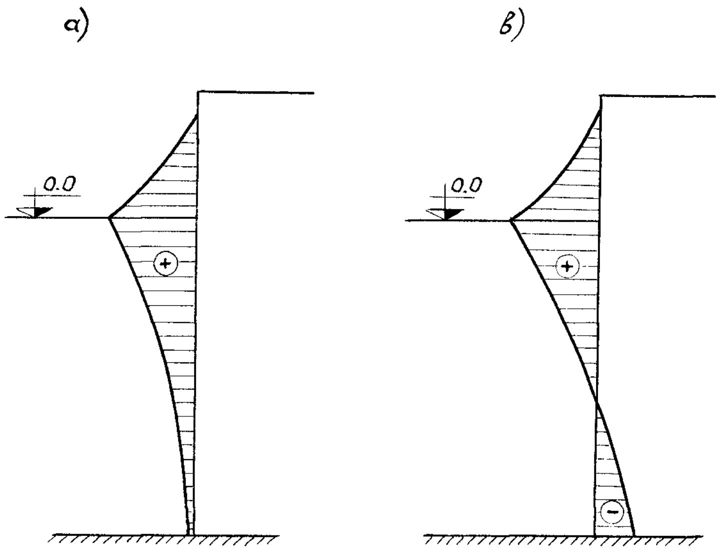

Fig.1. Character of Pressure upon Vertical Wall Diagrams.
a - Sainflou, b - second approximation formulae.

height (retaining second order terms). Final relationships give a good agreement with experimental results; at relatively low heights there however appear considerable errors. This is explained by the appearance of a surplus term in the Cauchy integral, this term becoming markedly increased as relative depth H/λ is decreased. Therefore a limit for utilization of calculation formular $H/\lambda = 0.132$ is set; if relative depth would be greater, then Miche-Biesel - Rundgren equation would give inevitably wrong results. Therefore even though this solution is rather widely used (see e.g. Kamel's paper (6) published in 1971) a search for a new and more accurate calculation method would be very desirable.

Kuznetsov's solution (4) has many important theoretical errors, which are analyzed in (7). Owing to a correction achieved by introducing empirical coefficients for small depths, this solution is in good agreement with experimental results, but for relatively large depths total pressure can be found to be two or more times the true value.

We feel that the cause of inadequacy of all the presently known methods lies in the fact that neither of them takes into account the specific character of wave motion. When studying fluid motion, hydrodynamics neglects particle deformations. It brings no errors into final equations for all types of motion except those for wave motion, since the deformations are of random character. But in wave motion particle deformations are periodical and undirected for significant areas (8). Therefore it is the deformations that undoubtedly affect the motion kinematic, and any accurate solution would be impossible if we do not take them into account. Author's attempts of taking into account particle deformations when deriving equations for surge waves and standing waves revealed great mathematical difficulties awaiting the investigator on this way.

Search for an approximate solution brought the author to the conclusion, that calculation formulae yielding practically acceptable results for all the range of rated depths can be derived from Eq.(1). And indeed, many of the investigators who carried out laboratory experiments on standing waves found that wave profile derived from Eq.(1) gives the best agreement with experimental data for all the depth range. This leads to a suggestion that the discrepancy between experimental pressure diagrams and those calculated from (6) is caused by the approximation in its derivation.

Turning now back to Eqs. (3) and (4) it should be noted that they can be integrated without omitting the second order terms. If we assume a limiting condition for surface p = 0 with $y_o = 0$, then - proceeding from assumption (5) - we derive from (3):

$$g = \frac{\sigma^2}{k} \operatorname{cth} kH \qquad (8)$$

Integrating (4) on substitution of (8) and dividing the result by "g" we get:

$$\frac{p}{\gamma} = - y_o - 2r_1 \cos \sigma t \cos kx_o - 2krr_1 \cos^2 \sigma t +$$
$$+ 2r \operatorname{th} kH \cos \sigma t \cos kx_o + k(r^2 + r_1^2) \operatorname{th} kH \cos^2 \sigma t +$$
$$+ k(r^2 + r_1^2) \operatorname{th} kH \cos 2\sigma t + F(t) \qquad (9)$$

To simplify the final expressions it would be reasonable to transform the fifth term having in view the relationship (5):

$$k(r^2 + r_1^2) \operatorname{th} KH \cos^2 \sigma t = 2kr^2 \operatorname{th} kH \cos^2 \sigma t \qquad (10)$$

(A chek proved that it gives an error in the final result which lies within 1 to 2%).Function F(t) will be found from limiting conditions on surface: p = 0 if $y_o = 0$.

Taking into account that an surface $r = r_1$ cth kH, and substituting (10) we have:

$$F(t) = -\frac{kh^2}{2}\,\mathrm{cth}\,2kH\cos 2\sigma t \tag{11}$$

Assumin $\cos kx_o = 1$ and substituting (10) into (9) we obtain the final expression for pressure in any point:

$$\frac{p}{\gamma} = -y_o - 2h\frac{\mathrm{sh}\,ky_o}{\mathrm{sh}\,2kH}\cos\sigma t - kh^2\frac{\mathrm{sh}\,ky_o\,\mathrm{ch}\,k(H+y_o)}{\mathrm{sh}\,2kH\,\mathrm{sh}\,kH}\cos^2\sigma t -$$

$$-\frac{kh^2}{2}\,\frac{\mathrm{ch}\,2kH - \mathrm{ch}\,2k(H+y_o)}{\mathrm{sh}\,2kH}\cos 2\sigma t. \tag{12}$$

Since Eq.(12) gives pressure value in Lagrange variables, for finding the loading point calculated for a particle with $y_o = a$ we have to find the current ordinate of the particle by substituting $y_o = a$ into Eq. (7)*.

With unlimited depth Eqs. (12) and (7) become considerably simplified:

$$\frac{p}{\gamma} = -y_o - \frac{kh^2}{2}(1 - e^{2ky_o})\cos 2\sigma t \tag{12a}$$

$$y = y_o + he^{ky_o}\cos\sigma t + \frac{kh^2}{2}e^{2ky_o}\cos^2\sigma t \tag{7a}$$

Practically when $H \geqslant 0.4\lambda$ calculations can be begun using Eqs. (12a) and (7a).

For plotting an excess wave pressure diagram we have to set 5 to 7 y_o- values, then to calculate p/γ pressure values; then diagram of full pressure is plotted and the hydrostatic pressure subtracted.

The value of full pressue resultant can be derived

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*) In the publication of our relationship in Khaskhachik G.D. & O.M.Vanchagov's work (9) a misprint has slipped in: in the second term of Eq. (2) factor "cosσt" is erroneously omitted.

by integrating Eq. (12):

$$\frac{R}{\gamma} = \int_{0}^{-H} \frac{p}{\gamma} \frac{\partial y}{\partial y_o} dy_o \tag{13}$$

Excess wave pressure resultant will obviously take the form:

$$\frac{\Delta R}{\gamma} = \frac{R}{\gamma} - \frac{H^2}{2\gamma} \tag{14}$$

Omitting cumbersome calculations of this value we give here only the final equation:

$$\frac{\Delta R}{\gamma} = \frac{h^2}{2}\left(\frac{4}{kh} \text{ th } kH \cos 6t + 6kH \frac{\cos^2 6t}{\text{sh } 2kH} + \cos 26t + \right.$$

$$\left. + \cos^2 6t - 2kH \text{ cth } 2kH \cos 26t\right) \tag{15}$$

As seen from (15) the maximum of excessive wave pressure resultant is time-dependent. The value of phase which corresponds to maximum pressure is derived by taking a derivative of (15) and equalizing it to zero.

As a result we derive two radicals:

$$\cos 6t = 1 \tag{16}$$

$$\cos 6t = \frac{2 \text{ th } kH}{kh(4kH \text{ cth } 2kH - \frac{6kH}{\text{sh } 2kH} - 3)} \tag{17}$$

It was found that if cos $6t$ calculated from (17) yields a value $0 < \cos 6t < 1$, then this very moment will correspond to the maximum value of excessive pressure resultant. If however this condition is not fulfilled, then the maximum of resultant will occur when the first radical (16) is used, i.e. at the moment of maximum wave crest rise at the wall.

For a case of $H \geqslant 0.4\lambda$ Eq.(17) takes a simpler form:

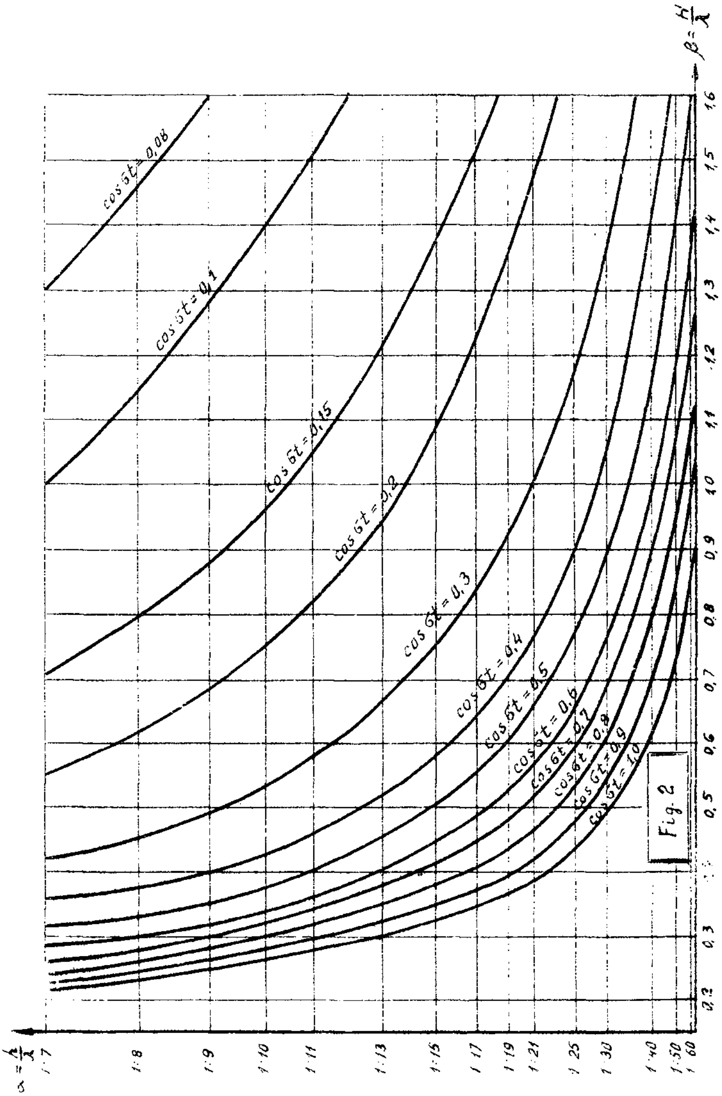
cos σt = 0,08
cos σt = 0,1
cos σt = 0,15
cos σt = 0,2
cos σt = 0,3
cos σt = 0,4
cos σt = 0,5
cos σt = 0,6
cos σt = 0,7
cos σt = 0,8
cos σt = 0,9
cos σt = 1,0
β = H'/λ
α = h/λ
0,2 0,3 0,5 0,6 0,7 0,8 0,9 1,0 1,1 1,2 1,3 1,4 1,5 1,6
1:7 1:8 1:9 1:10 1:11 1:13 1:15 1:17 1:19 1:21 1:25 1:30 1:40 1:50 1:60

Fig. 2

$$\cos \sigma t = \frac{\lambda}{H(8\frac{H}{\lambda} - 3)} \qquad (17a)$$

To simplify the calculations we plotted a diagram of relationships between cos t and relative wave heights $\alpha = h/\lambda$ and depths $\beta = H/\lambda$ (Fig.2). As seen from the diagram the increase in relative depth as a rule leads to $\cos \sigma t < 1$ and the maximum of excessive wave pressure resultant does not coincide with the moment of maximum wave crest rise at the wall.

If we derive by similar integration the resultant moment in relation to wall bottom and then determine the phase which corresponds to moment maximum, then it will be seen, that this phase does not coincide with that of the resultant maximum. The recommended method of calculations reflects all the peculiarities of pressure variations that were found by other investigators. Excess pressure at the bottom at $y = \infty$ will then be:

$$\frac{p_b}{\gamma} = -\frac{kh^2}{2} \cos 2\sigma t \qquad (18)$$

N.N.Zagriadskaya has collected all published data on experimental laboratory investigations of the action of standing waves upon a vertical wall, and made a comparison with theoretical data. She found (10) thereby that the method recommended in the present paper should be considered as preferable when compared to Miche - Biesel - Rundgren method and to that of Kuznetsov, since it gives the best agreement with the results of laboratory experiments.

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