CHAPTER 74

USE OF CRENULATE SHAPED BAYS TO STABILIZE COASTS

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ABSTRACT

Crenulate shaped bays are the rule rather than the exception on coastal margins of oceans, inland seas or lakes where sedimentary beaches exist between headlands. They have a particular orientation to the swell or resultant wave energy vector, such that the straight tangent section is downcoast and the curved portion upcoast. The latter is a logarithmic spiral at all stages of development of the bay. When fully stable, that is no littoral drift taking place, the constant of the log-spiral equation has a specific relationship to the approach angle of the waves to the headland alignment. In this condition it is shown that diffraction and refraction are involved when waves sculpture the curved beach in the lee of the upcoast headland. A further ratio to identify stable bays appears to be the ratio of indentation length to clearance between headlands. The application of crenulate shaped bays to stabilization of a reclaimed shoreline suffering strong littoral drift on Singapore Island is described.

INTRODUCTION

The crenulate shaped bay is a pronounced feature of any coastline, be it on an oceanic margin, the coast of an inland sea, the boundary of a lake, or the beach line of a hydraulic model. As long as there is a sedimentary shoreline interspersed with headlands and waves approaching the coast obliquely there will be such bays with half-heart or half spear-head shape.

Model work(1) has shown the sculpturing of this physiographic feature to be the work of waves, particularly those which are the most persistent in their incidence and direction. Such is the case of swell which arrives almost continuously from some storm zone of the ocean. The greatest source area for this energy is the 40° to 60° latitudes in both hemispheres.(2) Successive storms are generated there which travel from west to east. This provides almost continuous swell towards the margins of these oceans, but more particularly to the eastern margins, or the western coasts of continents. Even in enclosed seas (such as the Mediterranean, Baltic etc.) or large lakes (such as those in north America) the variable wind sequences will normally have a recognisable resultant at any specific zone of the coast as far as direction is concerned.

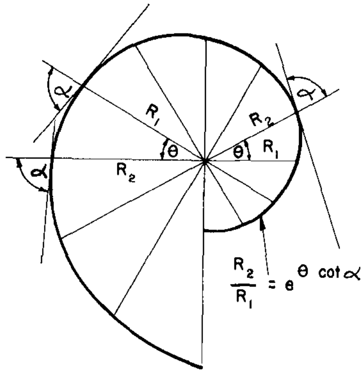
This oblique wave approach is directly related to the crenulate bay shape, since the orthogonal will be nearly normal to the straight section which passes through the downcoast headland. Downcoast here refers to the direction towards which sediment is moved by the wave action. The remainder of the bay comprises a curved zone, which is partly in the wave shadow of the upcoast headland. The indentation of the bay, from the headland alignment, depends upon the obliquity of the waves to this alignment and the amount of sediment available for transport through the embayment. In the final equilibrium, when little or no transport is taking place, the incoming waves are being diffracted and refracted into this shadow zone in such a way that simultaneous breaking occurs around the periphery.

BAY CURVATURE

It has been shown by Yasso(3) and subsequently proved by Silvester(4) that the curved water line can be defined by a logarithmic spiral. This is given by the relationship

$$R_2/R_1 = e^{\theta \cot \alpha}$$

where, as seen in Figure 1, R_1 and R_2 are radii from an origin that are at an angle of θ (radians) apart, and angle α is the constant angle of the radii to the tangents of the curve, with e as the exponential constant. It is the factor α which dictates the curvature and an infinite series could be drawn. When available on transparencies they can be used to define the shape of any sized bay, since scale is overcome by using various sections of a single curve.



1. Definition sketch of logarithmic spiral.

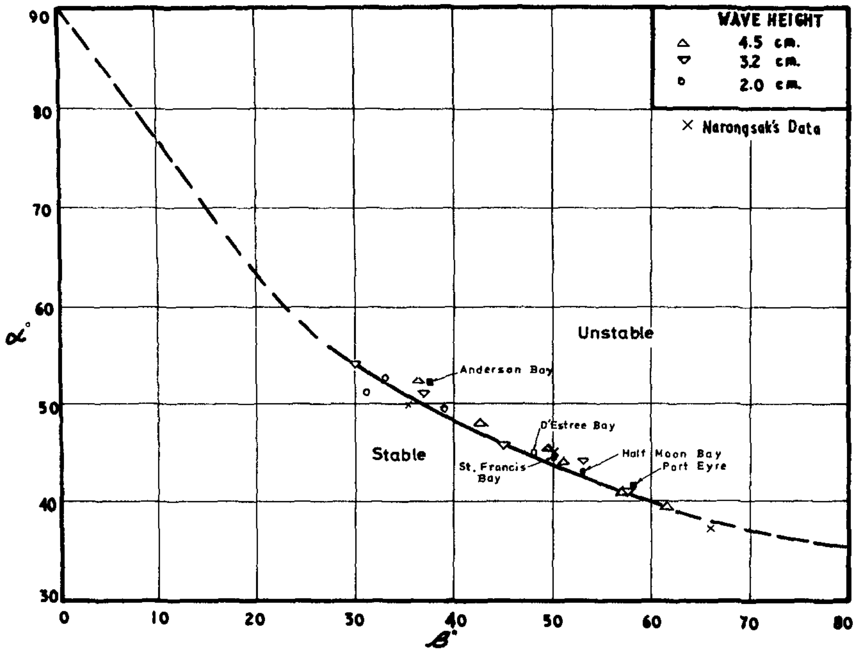
Model tests(4)(5) have shown that progressive sculpturing of a bay beachline from a straight coast between headlands causes a systematic variation in the curvature or the value of α. When equilibrium is reached for the case of no renourishment from upcoast, the final α value is related in a consistent manner with the angle between the incident wave crests and the headland alignment, designated β in Figure 2. This angle is also that between the tangent shoreline and the headland alignment, since in this equilibrium state the wave crests are parallel to the beach.

STABILITY CRITERION

The connection between α and β is exhibited in Figure 2, which has been derived from two series of model tests.(5)(6) Also shown are some values from prototype bays, for which it can be rationalised that those nearest to the curve are closest to a stable condition. Points should fall above the line when there is adequate sediment still to be passed through the bay, or rivers are supplying it within the bay. Should such sand supply be decreased, or cut off altogether, the bay will become more indented as the shoreline and adjacent sea bed is eroded. Degradation will proceed until the curvature of the logarithmic spiral gives an α value which falls on the curve of Figure 2.

Evaluation of β by measuring the angle between the tangent shoreline and the headland alignment is only correct when waves are normal to this beachline. This cannot be so until all littoral drift has ceased. However, the extremity of this near-straight coastline reaches its equilibrium alignment long before the upcoast regions, so that even in early stages of bay development this angle β can be determined reasonably well.

It has been shown elsewhere(4) how the curve of Figure 2 can be utilised to test the long term stability of a bayed shoreline. The plot of α versus β falls above this line when the bay is not stable,



2. Relationship between log-spiral constant α and wave obliquity β.

implying that any reduction of sediment supply, from the upcoast direction or within the bay, will cause erosion of the beaches. This will be most pronounced in the curved section, particularly that part adjoining the tangent zone. This will not be prevented by groynes or sea walls in the long term, only by breaking the bay into two bays by the creation of an intermediate headland. The shape of the two subsequent stable bays are then predictable by using Figure 2 for the final equilibrium condition of no littoral drift.

WAVE PROPAGATION

It has been stated above that waves both diffract and refract into the shadow zone of the upcoast headland. The refraction is readily accepted due to the shoaling of the bed, but the existence of diffraction has not been proven in a prototype situation. Obviously, if the only waves present are short in period and the water between headlands is deep, the curving crests can only be created by diffraction. Under these conditions sculpturing the shoreline entails the formation of a shallow shelf close in shore, but this would have little effect on the overall shoreline shape in plan.

In order to test the occurrence of diffraction besides refraction, four bays were selected which appeared to be in equilibrium. Criteria used for this purpose were as follows:

(a) The downcoast tangential beach was sensibly parallel to the crests of the persistent swell.

(b) The crest alignment noted in (a) was checked against the tangential alignments of several adjacent bays when the coastline and Continental Shelf were sensibly straight.

(c) The proportionate length of the tangent section is greater when equilibrium is approached.

(d) No large rivers fed material to the bay.

(e) The wave energy level was high.

(f) Towards the downcoast end of the bay the bed contours spread, indicating a more normal approach of the waves.

(g) The approach angle β (measured between the downcoast shoreline and the headland alignment) and the logarithmic spiral factor α plotted close to the curve of Figure 2.

Details of the location and values of α and β for each bay are listed in Table I. The waterline and underwater contours are presented in Figure 3.

Table 1 - Summary of Equilibrium Bays

Name	Longitude	Latitude	β	α
Port Eyre	132°25'-133°00' E	31°57'-32°02' S	58°	41.5°
Anderson Bay	147°22'-147°38' E	40°50'-41°03' S	37.5°	52.5°
D'Estree Bay	137°35'-137°44' E	35°52'-35°59' S	48°	45°
St. Francis Bay	24°50'- 25°30' E	35°55'-34°15' S	50°	44.5°

Bed contours, or profiles normal to the beaches, of these several bays would influence refraction, but each would have also resulted from the wave climate and sediment characteristics of the area. Some average had therefore to be determined, for which purpose each bay was divided into equal distances along the coastline and the vertical profiles measured. The profiles are numbered 1 to 6 from the upcoast to the downcoast headland, and are presented on a logarithmic scale in Figure 4. The mean line through these points provided a mean profile at these locations, from which contours of an average bay in equilibrium could be derived. Such a typical bay for β = 50° is depicted in Figure 5, with bed contours shown in fathoms.

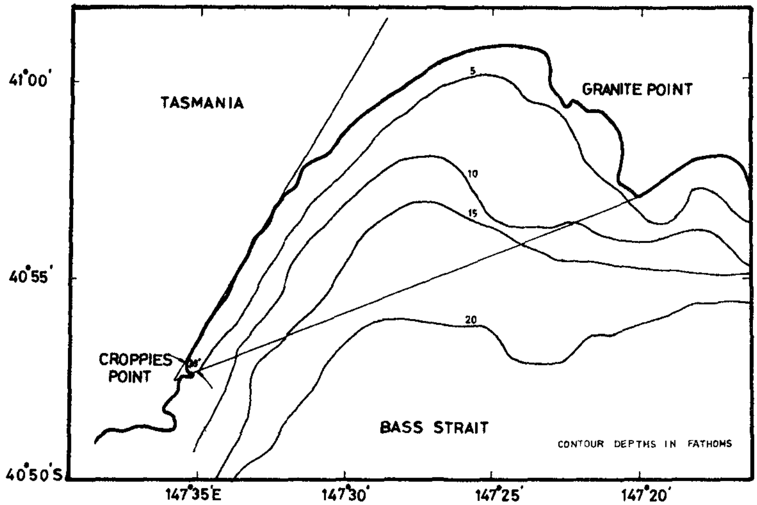


Fig 3A Plan of Anderson Bay Shoreline

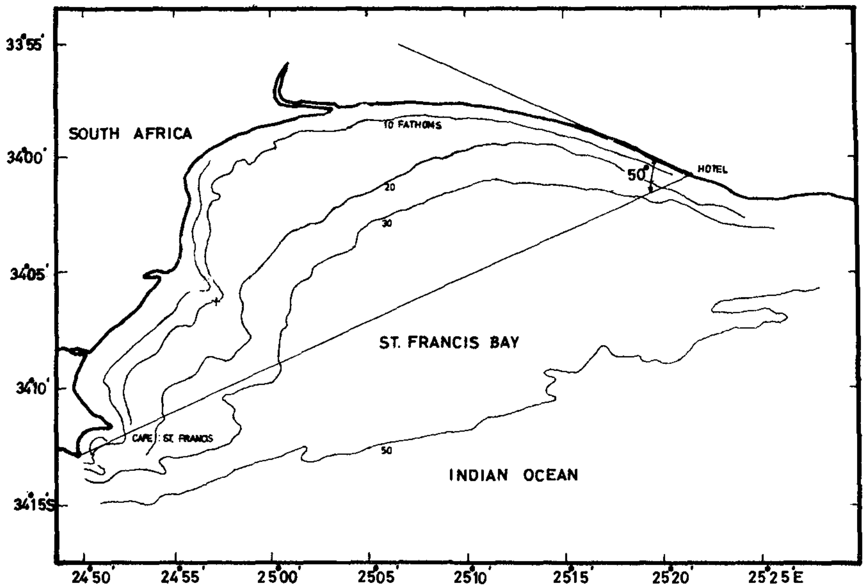


Fig.3B Plan of St.Francis Bay and fitted Logarithmic Spiral

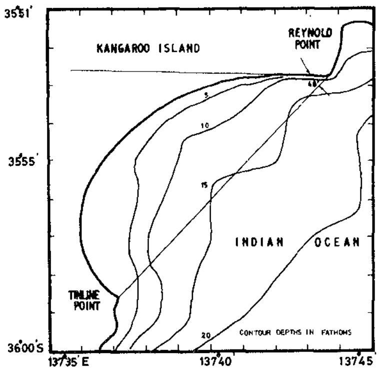


Fig.3C Plan of D'Estree Bay Shoreline

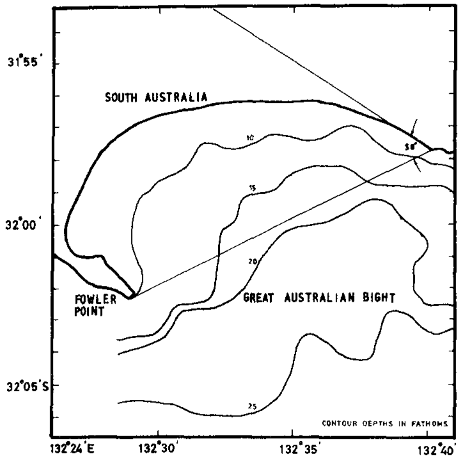
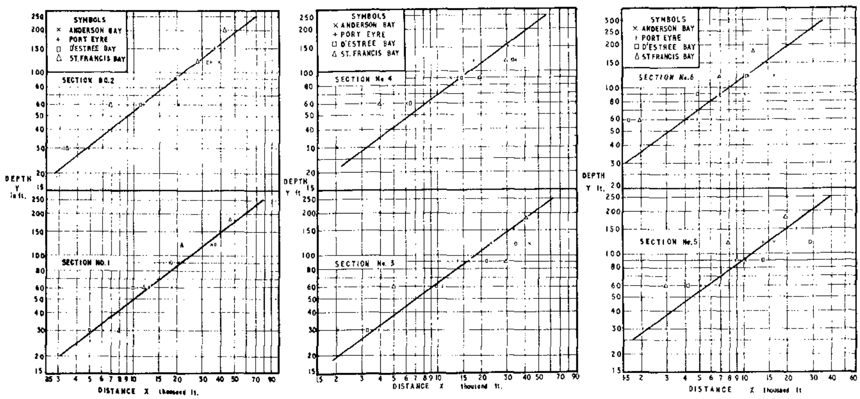


Fig.3D Plan of Port Eyre Shoreline



4. Bed profiles at points along the four bays detailed in Figure 3.

These bays on oceanic margins will suffer swell of various heights and periods throughout the year, but the predominant periods will be from 12 to 15 seconds, as discussed by Silvester(7)(8). These limiting bands were used in a refraction and diffraction analysis to test whether waves, whose crests in 30 fathoms are at 50° to the headland alignment, arrived at the breaker line normal to the beach. The angle $\beta = 50°$ at 30 fathoms was chosen because this depth contour essentially ran along the headland alignment and waves from there to the tangential shoreline had to traverse more or less parallel bed contours.

The wave orthogonal arriving at the tip of the upcoast headland from the 30 fathom depth is refracted across the parallel contours in front of it. The depth in the headland zone is aroung 10 fathoms so that the orthogonal direction can be obtained readily for the 12 and 15 second trains(9). These were 32.5° and 30° respectively to the normal of the breakwater or the bed contours. The waves were then diffracted and refracted over the contours within five wave lengths of the headland tip, as indicated for the 15 second wave in Figure 6(10). From there to the shoreline, refraction was determined by a computer programme, for which purpose the typical bay was divided into squares with 9840 ft sides (see Figure 5). Waves were so traced to a depth where breaking occurred, as assessed from H_o = 4.75 and 4.55 ft respectively for the 15 and 12 second waves respectively, with d_b = 1.28 H_b, where H_b was calculated from the diffraction and refraction process. The final breaking angle of the waves was compared with the orientation of the beach at that point on the bay (see Figure 5) and an error value derived. There are two methods of finding beach alignment, (a) by direct measurement (E_m) and (b) by the orientation of the radius from the logarithmic spiral centre less the angle α of the specific bay curvature (E_c). For the 20 orthogonals treated (as detailed in Figure 5) the angular errors are listed in Table II.

Observation of Table II indicates that the errors are least for the 15 second waves, indicating that the most predominant swell is closer to this period than 12 seconds. The differences in angle are all of the same sign, such that waves are breaking at a slight angle which would cause transport of sediment downcoast. The orthogonals with the greatest error, namely 1 and 17 to 20, are those traversing bed contours at very acute angles, making for reduced accuracy in the refraction procedure. Also the strong curvature of the bed contours in this zone could be susceptible to error in position or location. Also any misjudgement of bay shape will vary the length of these orthogonals the greatest. However, the modest nature of the errors indicates that both diffraction and refraction are involved in the sculpturing process of crenulate shaped bays. Although the typical bay examined is close to equilibrium it could be rationalised that similar processes occur prior to this condition.

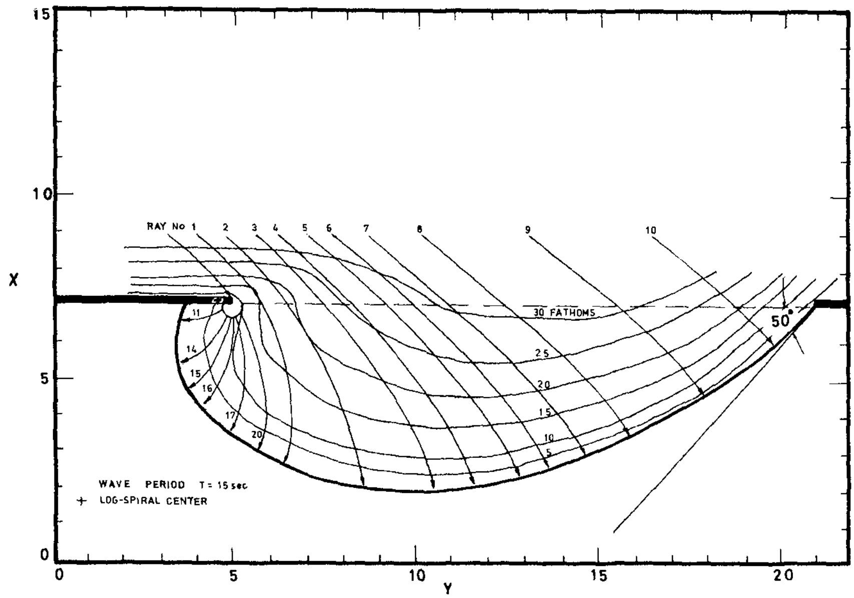


Fig.18 Typical Prototype Bay showing Refracted Orthogonals

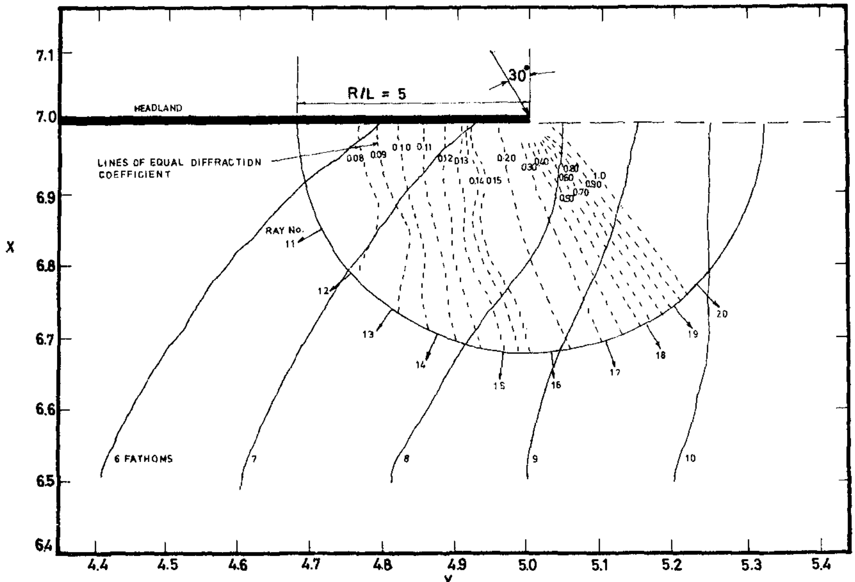


Fig. 6 Diffraction Diagram at Headland Tip for 15 sec. waves

Table II - Angular Difference in Degrees Between Orthogonal of Breaking Wave and Normal to the Beach.

Wave Period →	12 Second		15 Second	
Ray H_o	E_m	E_c	E_m	E_c
1	5.64	6.64	3.41	3.99
2	2.72	5.22	2.76	3.76
3	4.39	6.89	2.08	1.58
4	0.40	2.40	3.32	4.82
5	3.50	4.50	0.62	0.12
6	2.58	4.08	3.11	2.11
7	0.96	1.96	1.19	0.69
8	0.65	2.15	0.64	0.86
9	0.22	0.72	0.55	0.95
10	2.91	5.41	1.79	5.29
11	4.49	4.99	2.50	8.50
12	-	-	-	-
13	-	-	-	-
14	3.19	7.19	2.51	6.51
15	-	-	0.06	3.46
16	0.03	2.03	0.96	1.46
17	6.97	8.47	4.91	9.91
18	6.60	5.10	-	-
19	-	-	-	-
20	10.92	10.42	4.55	6.05

INDENTATION RATIO

Although the tangential angle β and the logarithmic spiral constant α should be sufficient to define any crenulate shaped bay, it is sometimes difficult to position the curved zone of the bay. This is because the upcoast end of the tangent section cannot be identified readily. It seems reasonable, therefore, to seek a relationship between indentation and width of bay. Such a ratio of a/b is illustrated in Figure 7

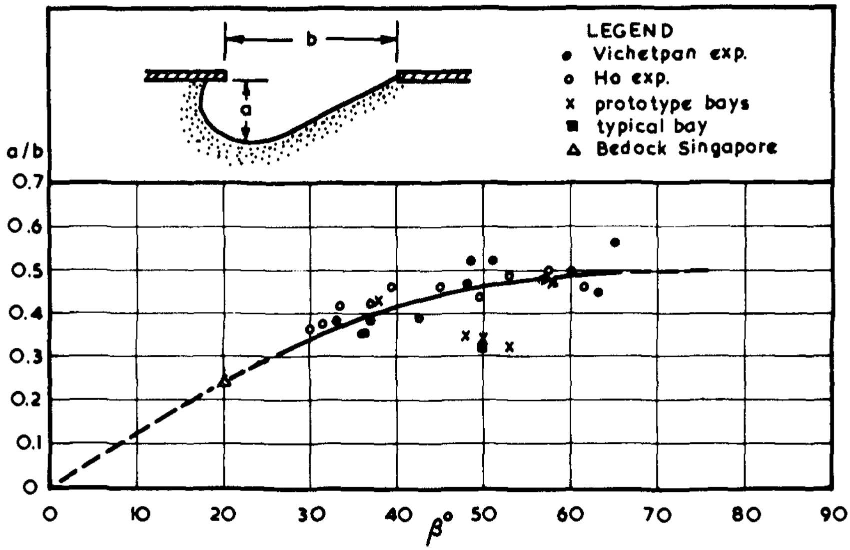
(a) for experiments conducted by Ho[6] and Vichetpan[5]

(b) the four prototype bays referred to previously, besides Half Moon Bay in California (also considered in equilibrium),

(c) the typical bay as derived before, and

(d) a bay at the upcoast end of the reclamation project in Singapore designated as Bedok (this should also be in equilibrium).

The dotted extension of the mean line through the points in Figure 7 should naturally pass through the origin (since for $\beta = 0$ $a = 0$).



7. Indentation ratio for a range of wave obliquity.

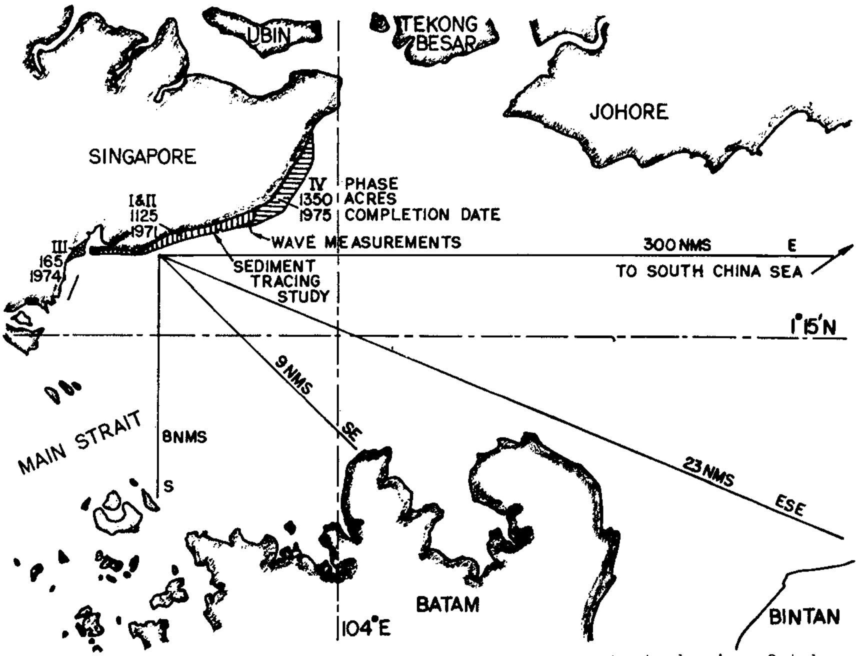
It is seen in Figure 7 that the prototype bays of D'Estree, St. Francis, Half Moon and the typical one derived, fall below those given by experiments. This might be through slight lack of equilibrium, or the impedance to this shape by local outcrops of beach rock which cannot be identified in the scale of reproduction of the bay. The apparent lack of equilibrium in the typical prototype bay may have been the reason for the greater errors in the orthogonals numbered 1 and 17 to 20 previously noted.

SINGAPORE RECLAMATION

In order to provide suitable flat ground for large accommodation complexes the Singapore Housing Board is undertaking a large scale reclamation of shoreline to the east of the main harbour. As seen in Figure 8, this is being accomplished in four phases, the first two of which are already complete, the third due for completion in 1974 and the fourth in 1975.

Wave Incidence

Phases I and II were along a coastline facing south which is subject to wave action from the south to the east. This results in a net littoral drift from east to west, whether the waves are generated locally or are swell from the South China Sea entering Main Strait from the east (see Figure 8). The wind rose for the area indicates a predominance of winds from the NE quadrant. The largest proportion of



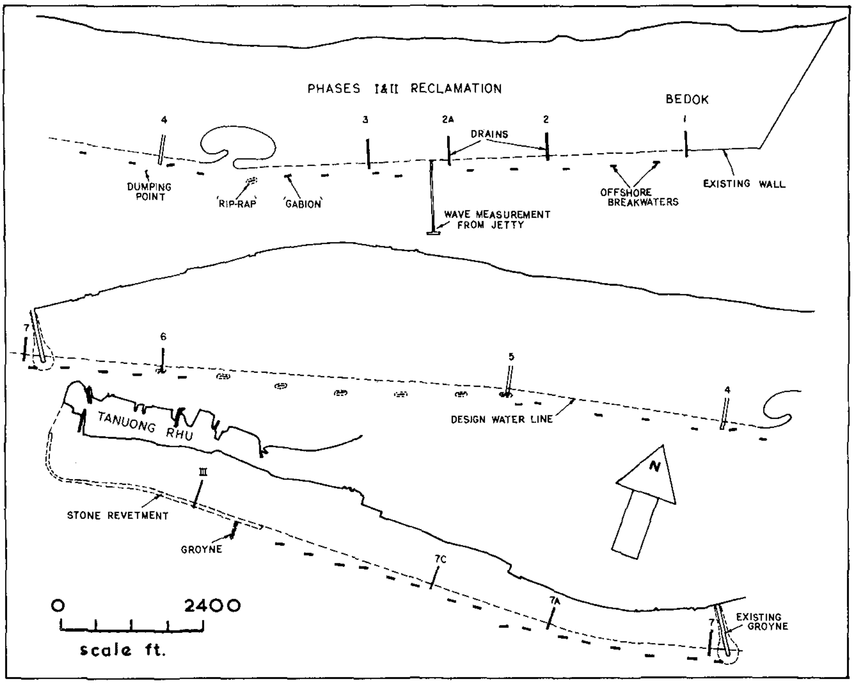
8. Location map of Singapore reclamation project showing fetch lengths for locally generated waves.

these are winds of less than 15 knots. Winds in excess of this velocity from this NE quadrant occur for less than 2% of the time, whilst from the SE quadrant none occur at all.

The major wave incidencc would thercfore appear to be swell arriving from the South China Sea. The predominance of this wave action is indicated by the crenulate shaped bays on the southern shoreline of Johore State (see Figure 8). Thesc clearly indicate an east to west drift of sediment, even though locally generated waves have a reasonable fetch in the SW quadrant at this point. During March 1972 waves were recorded at the end of the jetty marked in Figure 8(11). They never exceeded 3 ft in hcight and were mainly around 1 ft. Their direction was essentially SE in 13 ft of water and their period around 6 seconds. Considering the general orientation of the bcd contours at this point, a refraction computation indicates a deepwater approach from an easterly direction. The wave period is close to that reported in the South China Sea of 5 to 7 seconds.(11) Such swell is predominant in the months from Decomber to March when the NE monsoons exist in the area.

Coastal Defense

The initial proposal for protecting the reclaimed land was to construct a rock revetment with seawall capstone along the straight shoreline. Such a structure was commenced at the eastern or Bedok end of the Phase I reclamation (see Figure 9). Construction was effected in the dry by overfilling the area and then trenching and dewatering the site of the seawall. However, erosion of the protecting dike was rapid and undermining of the finished wall severe, particularly at the exposed eastern corner.



9. Location of coastal defense structures for Phase I and II reclamation.

At this time the concept of the crenulate shaped bay to stabilize coastlines was conveyed to the Board, who immediately instigated the design of offshore breakwaters or headlands to prevent erosion. These have been spaced in front of the new shoreline, as illustrated in Figure 9, and are progressively forming bays between them, as material is transported in the nearshore zone towards the west.

The tidal range in Main Strait is around 10 feet, which exposes a width of shoreline around 50 ft, for sufficient time to permit the construction of headlands near LWL by land-based equipment. Two types of rock-fill structure have been utilised, namely, the "gabion" (with stones in steel mesh baskets) and "rip-rap" (consisting of a compacted soil mound faced with layers of rock).

The gabions consist of steel cages of 2 cubic metre capacity filled with stones of about 1 ft dimension. These rise in 4 or more layers from 2 ft below LWL to 1 ft above HWL. They extend about 100 ft along the shoreline and are located about 200 ft. from the proposed straight alignment of the reclaimed area and spaced about 800 ft apart.

The rip-rap structure is constructed in the dry by overfilling the area and compacting the soil. The mound is then shaped to give a seaward slope of 1 : 2 and landward slope of 1 on $1\frac{1}{2}$. A vinylon sheet is then spread over this, on which are placed a 6 inch layer of 2 to 6 inch stones, a second 6 inch layer of 6 to 9 inch stones, and lastly a single layer of 2 ft stones properly pitched. The structure extends from 2 ft below LWL to 3 feet above HWL, which is the same as the finished ground level. The crown of these structures is 120 ft in length and some 200ft from the design shoreline. Their spacing is similar to the gabion headlands.

As seen in Figure 9, the alignments are such as to be parallel to the incoming swell waves at high tide. Also shown in the figure are the locations of drains, which should enter the predicted bays in the lee of the upcoast headlands,(12) so as not to be silted up by the action of high waves.

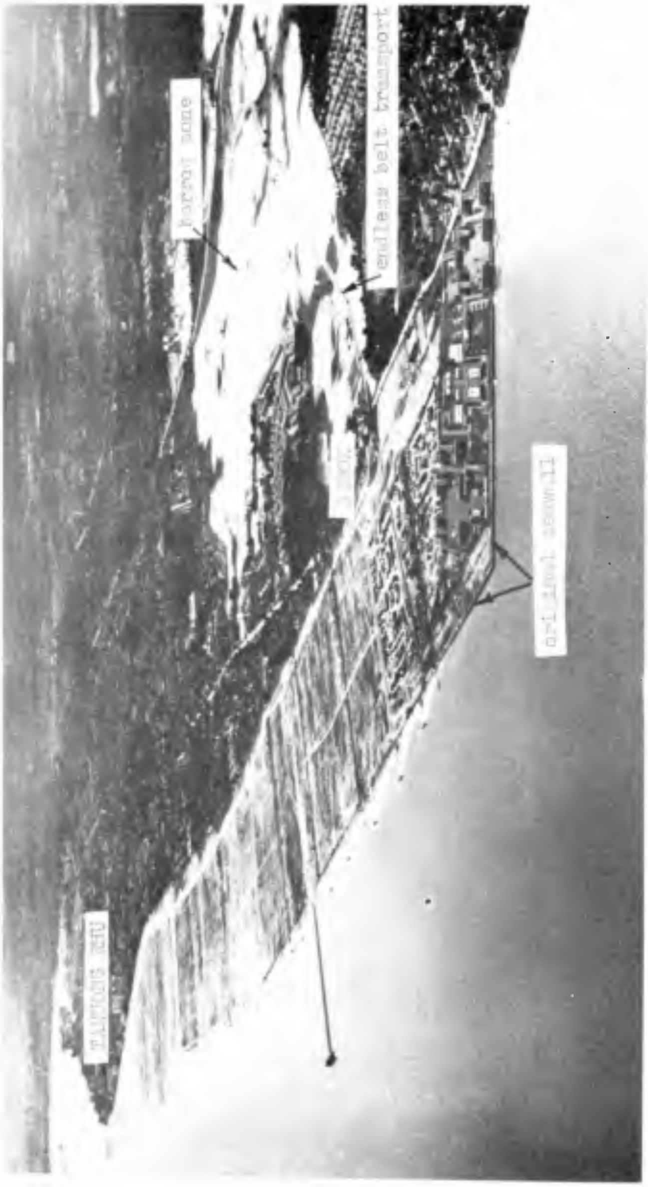
Tracer Test

To check that the predominate direction of sediment transport was to the west, a tracer test was carried out at the location indicated in Figure 8 in March 1970. Fluorescent sand was dumped just westward of drain no. 4 (see Figure 9) at the low water mark, and sampled at four points across the exposed tidal beach face on normals both upcoast and downcoast. The results indicated that the predominant motion was westerly, or parallel to the shoreline. Not until the second headland was reached did the higher concentration of tracer appear at the high water mark. Downcoast of this (i.e. westwards) the tracer again moved out to the low water mark, and onwards to the next headland. Although tombolo's then existed at high tide level, the bays in question had not reached their predicted crenulate shape. They have since done this.

Overall Result

The success of the headland measure can be gauged by the aerial photos of Figures 10 to 13, which show the development of the stabilising bays at the two extremities of the Phase I and II reclamation. The advantages of this approach over the seawall originally proposed may be listed as follows:

(a) The headlands are much more economical initially. The seawall had precast capping stones on a stone revetment and required overfill for its construction. The savings have been



10. View of phase I and II reclamation looking west in May 1971.

11. View of bay development looking west from drain 7A (Fig. 9) in February 1972 at medium tide.

12. View of gabion headlands near Tanuong Rhu looking east in Febru 1972 at high tide.

13. Close-up view of gabion headland showing bay formation between drains 7A and 7C looking west at low tide.

estimated as around 50% for gabion headlands and 25% for the rip-rap headlands.

(b) The maintenance cost on the seawall would have been high, due to the action of the reflected waves in expediting the scouring in front of it(13). This rapid transit of material westward would have created a silting problem west of Tanuong Rhu.

(c) The residents of the new housing estate will have a sandy shoreline for their recreation. This is divided into many small embayments which are safe for children to use. The alternative revetment would have had water of some depth adjacent to it, with submerged rocks to add to the danger for infants.

(d) The reclaimed shoreline is protected by the buffer zone of the bay, with offshore bed slopes mild in character. These help break waves prior to reaching the shoreline. Less spray will be generated than if waves were smashing up against a stone revetment and coping stone. This should minimise corrosion of fittings in dwellings and the concentration of salt in the lawns of the compounds. Overtopping of the wall by standing storm waves is obviated, so reducing the cost of drainage structures to cope with such eventualities.

CONCLUSIONS

1. Crenulate shaped bays are ubiquitous features of the coastline and give a lead to man in his attempts to minimise longshore drift.

2. Such bays have a definite orientation with respect to the predominant swell on oceanic margins, and to the resultant energy vector in enclosed seas, where locally generated waves assume importance.

3. At all stages of development the sedimentary coast between two successive headlands consists of a tangential zone downcoast and a logarithmic spiral section upcoast.

4. Where full equilibrium is reached and zero longshore drift is achieved, the constant in the log-spiral equation has a specific relationship to the angle of the wave crests to the headland alignment, this latter angle equating to that between the tangent beach line and this same headland alignment.

5. The relationship noted in (4) above can serve as a measure for equilibrium of any bay on a coast, so that the limit of erosion can be determined in the event of sand supply to the bay being cut off by natural or man-made means.

6. Waves arriving at the shorelines of crenulate shaped bays both diffract and refract into the shadow zone of the upcoast headland, so producing the log-spiral shape of this zone of the bay.

7. Wave propagation as in (6) results in a normal approach to the beach at the breaker line when the bay is in complete equilibrium, so preventing a littoral current.

8. To completely predict the outline of a stable bay, when criteria (4) and (7) above are fulfilled, the ratio of indentation length to distance between headlands can be ascertained from a knowledge of the wave obliquity as designated in (4) above.

9. Application of the stable bay principle to a reclamation project in Singapore appears to have provided an economic solution to a problem involving strong littoral drift.

REFERENCES

1. Silvester, R. "Stabilisation of Sedimentary Coastlines". Nature 188, 1960, 467-469.

2. Davies, J.L. "A Morphogenic Approach to World Shorelines" Ann. of Geomorph. 8, 1964, 127-142.

3. Yasso, W.E. "Plan Geometry of Headland-bay Beaches" J. of Geology 73, 1965, 702-714.

4. Silvester, R. "Growth of Crenulate Shaped Bays to Equilibrium" Proc.ASCE. 96 (WW2), 1970, 275-287.

5. Vichetpan, N. "Development of Crenulate Shaped Bays". Thesis No. 280 submitted to the Asian Institute of Technology, Bangkok, in partial fulfilment of Master of Eng. degree, 1969.

6. Ho, S.K. "Crenulate Shaped Bays" Thesis No. 346 submitted to the Asian Institute of Technology, Bangkok, in partial fulfilment of Master of Eng. degree 1971.

7. Silvester, R. "Design Waves for Littoral Drift Models" Proc.ASCE 89, (WW3), 1963, 37-47.

8. Silvester, R. and S. Vongvisessomjai "Computation of Storm Waves and Swell" Proc. I.C.E. 48, 1971, 259-283.

9. Silvester, R. "An Aid to Constructing Wave Refraction Diagrams" Trans.I.E. Aust, CE8, 1966, 123-127.

10. Blue, F.L. Jr. and J.W. Johnson "Diffraction of Water Waves Passing Through a Breakwater Gap" Trans.Am.Geoph.Un. 30, 1949, 705-718.

11. Private correspondence from S.Y. Chew of Singapore Housing Board.

12. Bascom, W.N. "The Control of Stream Outlets by Wave Refraction" J. of Geology 62, 1954, 600-610.

13. Silvester, R. "Wave Reflection at Seawalls and Breakwaters" Proc. Inst.Civil Engrs. 51, 1972, 123-131.

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