CHAPTER 73

ON THE FORMATION OF SPIRAL BEACHES

Paul H. LeBlond Institute of Oceanography University of British Columbia

ABSTRACT

The theory of wave-induced longshore currents is applied to problems of beach erosion. An erosion equation is derived, relating the local erosion (or deposition) rate to the form of the beach and to the characteristics of the incoming wave field. A numerical integration technique of the erosion equation is discussed and a specific example is examined: that of a linear coast line which is gradually eroded into a spiral-shaped beach in the lee of a headland.

Introduction

Hook-like beaches of the type shown in Figure 1 are quite common on exposed coasts. Such beaches have received various names: Silvester (1960) calls them "half-heart shaped bays" and Yasso (1965) "headland-bay beaches". Half-Moon Bay in California is one of the best known examples (Bascom, 1951). It was Yasso (1965) who discovered that the planimetric shape of many such beaches could be fitted very closely by a segment of a logarithmic spiral. The distance r from the beach to the center of the spiral increases with the angle ψ according to

$$r = r_{o} e^{\psi \cot \alpha}$$

(1)

in which α is called the "spiral angle" and determines the tightness of the spiral. Bremner (1970) has also shown the logarithmic spiral to give an excellent fit for each side of a recessed beach between two headlands.

It is extremely tempting to attribute the characteristic shape of these spiral beaches (as I call them here) to waveinduced erosional processes. To confirm this suspicion, I have attempted to show, using available theories of longshore currents and beach erosion, how a spiral beach could evolve from a linear wave-swept coastline.



Longshore currents

It will be assumed that any erosion or deposition at the shoreline will be due uniquely to transport of material by wave-induced longshore currents. The theory of such currents has recently risen out of a state of semi-empiricism following a re-examination of the role of waves in producing currents (Longuet-Higgins, 1970 a, b) and of the manner in which sand is transported by these currents (Komar and Inman, 1970; Komar, 1971). Longshore currents are produced by breaking waves in the surf zone; the amplitude and direction of the incoming wave are determined by offshore conditions which are assumed to be completely uncoupled with surf zone phenomena.

To be more specific, let's consider a straight section of beach, as shown in Figure 2. Approaching waves begin to break at the mean distance x b from the shore line, in a mean depth h_b ; their amplitude upon breaking is a_b and the angle made by their propagation vector with the x-axis is ϕ_b . The wave amplitude in the surf zone ($0 \leq x \leq x_b$) is taken as proportional to the mean depth: $a = \alpha$ h (Longuet-Higgins, 1970a). The local wave energy density is then $E = \frac{1}{2} \ \rho g \ \alpha^2 h^2$. In beach coordinates (x,y), the radiation stress of the waves has components

$$S_{ij} = \frac{E}{2} \quad \begin{cases} 3 \cos^2 \phi + \sin^2 \phi, -\sin 2 \phi \\ -\sin 2 \phi, 3 \sin^2 \phi + \cos^2 \phi \end{cases}$$
(2)

The divergence of the radiation stress S_{ij} provides the driving force for the longshore currents in the surf zone; there is no net forcing in the offshore area. Once a steady state has been reached, frictional forces will just balance the driving force and the mean longshore momentum equation will read

$$\frac{\partial}{\partial x} \left(\mu h \frac{\partial V}{\partial x} \right) - f V = \left(\frac{\partial}{\partial y} S_{yy} + \frac{\partial}{\partial x} S_{yx} \right) \qquad 0 \le x \le x_b$$

$$(3)$$

$$" = 0 \qquad x_b \le x < \infty$$

The assumptions and simplifications leading to these equations have been discussed by O'Rourke and LeBlond (1972). The lateral eddy friction coefficient μ is taken as increasing linearly with wave velocity and with distance from the beach:

$$\mu = N \rho x \sqrt{gh}$$
(4)

(N is a dimensionless constant). The bottom friction parameter f is proportional to the maximum orbital velocity

$$f = \frac{2}{\pi} \rho \alpha C \sqrt{gh}$$
 (5)

Where C is a friction coefficient for flow over rough plates. Assuming a linear depth profile which is uniform along the beach

$$h = Sx \tag{6}$$



and transforming to a scaled coordinate $\xi = x/x_h$, (3) may be rewritten

$$P \frac{\partial}{\partial \xi} \left(\xi^{5/2} \frac{\partial}{\partial \xi} \right) - \xi^{1/2} V = \frac{3}{\Sigma} V_{1} \xi^{1+1/2} \qquad 0 \le \xi \le 1$$

$$U = 0 \qquad 1 \le \xi \le \infty$$
(7)

This is the same equation obtained by Longuet-Higgins (1970b), but with two more forcing terms. P is the ratio of lateral to bottom friction effects, $\left(P = \frac{\pi NS}{2\kappa C}\right)$

$$K = \frac{5\pi\alpha}{8C} g^{1/2} x_b^{1/2} S^{3/2}, \quad V_2 = \frac{2K}{5} \left(3 - 2\cos^2\phi_b\right) \frac{x_b}{a_b} \frac{\partial a_b}{\partial y}$$

$$V_1 = K\sin\phi_b \cos\phi_b \qquad V_3 = \frac{4K}{5} x_b \sin\phi_b \cos\phi_b \frac{\partial\phi_b}{\partial y}$$
(8)

The three forcing terms are due respectively to 1) the obliqueness of wave approach, 2) and the non-uniformity of wave amplitude and 3) of wave angle along the beach. The first term is usually the more important one.

The solution to (7) which keeps V finite and both V and $\partial V/\partial\xi$ continuous across the breaker line is

$$V = B_{1}\xi + 2\sum_{i=1}^{9} \frac{V_{i}\xi^{(1+i)/2}}{(1+4i)P-2} \qquad 0 \le \xi \le 1$$

$$P_{2}$$

$$B_{2}\xi \qquad 1 \le \xi \le \infty$$
(9)

in which

$$P_{1,2} = -\frac{3}{4} \pm \left(\frac{9}{16} + \frac{1}{P}\right)^{1/2}$$

$$B_{1} = \frac{2}{P(p_{1}-p_{2})} \quad \sum_{i=1}^{3} \frac{V_{i}}{2p_{1}-(1+i)}$$
$$B_{2} = \frac{2}{P(p_{1}-p_{2})} \quad \sum_{i=1}^{3} \frac{V_{i}}{2p_{2}-(1+i)}$$

Whenever P = 2/(1 + 4i) for any one of i = 1, 2, 3, a singularity appears in one of the coefficients of V in (9); the solution must then be modified. For i = 1, P = 2/5, the appropriate term is $5/7 V_1 \xi \ln \xi$; for i = 2, P = 2/9, $V_2 \xi^{3/2} \ln \xi$; and for i = 3, P = 1/7, -14/11 $V_3 \xi^2 \ln \xi$. We will avoid any worries by simply choosing a value of P different from 2/5, 2/9 and 1/7.

Sand transport

In relating sand transport to longshore currents I have simply assumed that the volume rate of sand transport T will be proportional to the total water transport in the surf zone, Q, times a "sand fraction" μ , so that T = Qµ. Q is defined by

$$Q = S x_b^2 \int_0^1 \xi V(\xi) d\xi$$
(10)

Sand transport is then an integral property of the surf zone, and is thus independent of ξ . It will thus not be possible with this model to examine variations in beach profile (h (ξ)) associated with differences in sand transport rates across the surf zone. Such refinements could well be brought into more advanced models, by making T an explicit function of V, as in Komar (1971).

It is useful to split Q into three parts, each one resulting from one and only one of the three forcing mechanisms mentioned earlier: thus,

$$Q = \sum_{i=1}^{3} Q_i$$
 (11)

in which

$$Q_{i} = Sx_{b}^{2} V_{i} \left(\frac{4}{(1+4i)P-2(5+i)} + \frac{2}{P(p_{1}+2)(p_{1}-p_{2})(2p_{1}-(1+i))} \right)$$
(12)

We recall the source of three components of the transport: Q_1 is caused by the obliqueness of the waves, Q_2 and Q_3 by the non-uniformities of the wave amplitude and angle of approach respectively.

The Erosion Equation

The rate of erosion will be directly proportional to the divergence of sand transport QU. As we have lost all information about the details of sand transport across the surf zone by relating the total sand transport directly to the integral of the longshore current, it will be reasonable

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to simply assume that the beach profile is not modified by erosion. The slope S retains the same value, the beach being shifted laterally by sand removal or accretion, as shown in Figure 3. The parallelogram of sand (of density $\rho_{\rm S}$) removed by erosion in a time Δt has a mass $\rho_{\rm Shb}\Delta x_{\rm o}$. With $x_{\rm o}(t)$ the position of the mean shore line, the rate of sand removal is then $\rho_{\rm S} {}^{\rm b}_{\rm S} {}^{\rm b}_{\rm O} {}^{\rm c}_{\rm t}$. This is balanced by the divergence of sand transport in the surf zone

$$\frac{\partial}{\partial y} (Q\mu) + \rho_{s} h_{b} \frac{\partial x_{o}}{\partial t} = 0$$
(13)



This derivation is strictly correct only for linear beaches. We will extend its applicability to curved beaches by appealing to the results of O'Rourke and LeBlond (1972) who found that, for semi-circular beaches, the supplementary forcing terms occuring in (3) because of beach curvature were negligible provided the radius of curvature of the beach remained much greater than the width of the surf zone. When that condition is satisfied, (13) still describes the erosional processes in a short enough section of the beach. We may now redefine local variables (x, y) as shown in Figure 4; the orientation of the beach segment to fixed axes (X, Y) will be specified by the angle θ . The rate of displacement of a point P(X, Y) which always remains on the beach may then be found from (13) as

$$\frac{\partial X}{\partial t} = -\frac{\cos\theta}{\rho_{s} h_{b}} \frac{\partial}{\partial y} (Q\mu)$$
(14a)

$$\frac{\partial Y}{\partial t} = \frac{\sin\theta}{\rho_{\rm s}h_{\rm b}} \frac{\partial}{\partial x} \quad (Q\mu) \tag{14b}$$



Figure 4. Definition of local coordinates for extension to curved beaches.

Computational Scheme

The initial planimetric shape of a beach may be specified by giving the coordinates (X, Y) of a sufficient number of points P_j (j = 1, . . . N). Given the characteristics of the incident wave field (ϕ_{i} , ϕ_{i}), the initial beach geometry (S, x_b , ρ_s) and the empirical coefficients α , N, C, μ it will be possible to calculate the sand transport Q μ and its divergence along the beach. Integrating the erosion equations (14) over a finite time interval yields values Δx_j , Δy_j by which the position of the j^{th} point is to be modified because of erosion or deposition during that time span. Repetition of this process gives a series of positions for the forms Pj and hence a series of planimetric shapes for the beach. This apparently simple-minded integration scheme is alas full of pitfalls! Having fallen victim to many of them I would like to discuss the origin and the means of avoiding the worst of them before presenting actual computational results.

First of all, there arises the problem of stability. The natural processes described by this model occur over time-scales ranging from months to centuries. The integration time-step should then be chosen large enough to make it possible to witness the evolution of the beachwithin a reasonable lapse of computing time. A long time step is thus desirable; it is also dangerous. Imagine for a moment a linear section of beach with a hump on it. With a uniform incident wave field $(\phi_b \neq 0, \partial_{a_b}/\partial_y = 0, \partial\phi_b/\partial_y = 0; Q_1 \neq 0, Q_2 = 0 = Q_3)$ there will be a tendency for this hump to be eroded away. A strong divergence of sand transport will occur on the hump, leading from (13) to a large rate of erosion $(\partial_{x_0}/\partial t < 0)$. If the time step is small enough, the hump will gradually be reduced to insignificance. Should the time step be chosen too large however the correction Δ_{x_0} to the beach shape during the time Δt may be large enough to transform the hump into a hole! This is already nonsense, but not yet instability. If the hole is less deep than the hump was high, an oscillatory approach to equilibrium will result.

Only when the hole is deeper than the original height of the hump will instability occur. It is clearly not sufficient to choose a time step small enough to avoid instability of this type; one must avoid over-correction: violent transformation of humpsinto holes and vice-versa within a single time step. The computer model should behave very much in the same way as a real beach does, and be endowed with a similar tendency to gradually minimize its curvature, except possibly at a few well understood and identified points, such as sand spits.

The problem of stability has to do with the mechanics of numerical computation, and is readily taken care of. A more fundamental difficulty is that of correcting the incident wave field to account for the change in planimetric shape of the beach. As indicated earlier, one of the fundamental premises of the theory of generation of longshore currents is that there is no coupling between the longshore currents in the surf zone and the wave field in the offshore zone. The longshore currents are caused by the incident wave field but do not in turn influence it. This may well be so on a time scale short compared to that during which significant modifications of the sea shore occur, but as the planimetric shape of the beach departs more and more from its initial form, the incoming wave field will suffer from refraction or diffraction to a significant extent and the forcing function for the longshore currents will be altered. There is thus a larger-time-scale coupling between longshore currents and the incident wave field, and it must be taken into account in problems of beach evolution.

Let us see how the above theory of longshore currents and the scheme of integration of the erosion equation may be applied to account for the presence of spiral beaches. We shall assume that at some initial time (t = 0) a completely uniform wave field is incident at an angle ϕ_0 upon a linear beach (Figure 5). To simulate the presence of a rocky headland it will be assumed that half the beach (Y \leq 0) is a rocky strip, from which no sand can be eroded, and hence on which $\mu = 0$. The longshore water transport Q is thus initially uniform, but not the sand transport Q μ , which has a discontinuity at the origin.



It is simple to see, at least qualitatively, how the beach will evolve. Because of the sand fraction discontinuity at the origin, the sand transport divergence will be large and positive there and, from equation (13), erosion will occur rapidly. The beach will be gradually deformed as shown in Figure 5. As the beach changes its orientation in the (X, Y) plane, so that the angle θ in (14) is no longer zero, the local angle of attack ϕ_b will change from its original value of ϕ_o to a new value $\phi_b = \phi_o - \theta$. Those parts of the transport which depend on the angle ϕ_b (Q_1) or on the longshore variation of ϕ_b (Q_3) will be changed accordingly. Further, as the nick dug in the beach gets deeper and the beach segment near the origin approaches the "shadow line", the influence of diffraction by the tip of the rocky strip (which has now become a headland) will become more noticeable, and the amplitude and direction of the waves arriving at the breaker line will have to be modified to account for the changing geometry of the shore line. The variation of Q along the beach will gradually become as important as the initial discontinuity in μ in determining the erosion rate.

The actual corputations have been made following the procedure outlined earlier. A number of points, labelled j = 1, 2 . . . N, are initially strung along the half line Y \geq 0, and are gradually displaced according to equations (14). The beach thus consists of N - 1 linear segments, the jth segment being between the jth and (j + 1)th points, and making an angle θ , with the Y-axis. Transports are calculated at intermediate points and are characteristic of a segment, not of a point (see Figure 6).

The rate of displacement of a labelled point will be proportional in magnitude to the difference in the transport in the segments on either side of it. The direction in which the point moves must be defined with more care; since the labelled points are at the intersection of beach segments, where there is usually a discontinuity in slope, as characterized by the angle θ , one cannot use equations (14) in exactly the form in which they appear. We define an angle α which is the average slope at a point:



and allow the erosion to proceed at the $(j+1)^{th}$ point as if the beach had an orientation given by the angle α_{j+1} there. In finite difference form, (14) now reads:

$$\lambda x_{j+1} = \frac{-\mu \cos \alpha_{j+1}}{\rho_{sb}} \quad \begin{pmatrix} Q_{j+1} - Q_j \\ \Delta_{j+1} \end{pmatrix}$$
(15a)

$$\Delta Y_{j+1} = \frac{\mu \sin \alpha_{j+1}}{\rho_s h_b} \quad \begin{pmatrix} q_{j+1} - q_j \\ \Delta_{j+1} \end{pmatrix}$$
(15b)

where
$$2(\Delta_{j+1})^2 = \sqrt{(X_{j+2} - X_{j+1})^2 + (Y_{j+2} - Y_{j+1})^2} + \sqrt{(X_{j+1} - X_j)^2 + (Y_{j+1} - Y_j)^2}.$$

Since there can be no erosion of the rocky spit, the first point (j = 1) does not move: $(X_1, Y_1) = (0, 0)$ at all times.

Qualitative considerations

Even before proceeding with the integration of (15) it is possible and advisable to consider what kind of qualitative results are expected. First, as already indicated, the beach erodes at the corner, as shown in Figure 5. From the definition of α , it should be clear that erosion at point j = 2 should take place in a direction which will take it towards decreasing values of Y, so that erosion behind the rocky strip will ultimately result. This back-cutting is indeed seen to occur in the computed configurations (Fig. 7, 8) and is a necessary step towards attaining spiral shape (or anything which resembles a spiral).

From the nature of the erosion and deposition processes, it is also clear that humps and holes (regions of high curvature) will be rapidly smoothed away in a real beach, and should suffer the same fate in our model. Such regions of sharp curvature would appear wherever the sand transport $Q\mu$, has maxima or minima along the beach. We thus expect that once an equilibrium profile has been reached, there will be no such extrema in $Q\mu$, which will increase <u>monotonically</u> from zero at the headland to a maximum value at the far end of the beach. From the very beginning, the eroded beach is concave seawards near the headland, convex seawards further on. If $Q\mu$ increases monotonically from the situation will prevail at all times and there will be only one point of inflexion.

If there exists a planimetric shape which the headland beach asymptotically approaches, it must have the following properties: 1) it is first concave outwards, near the headland, and then convex outwards; 2) the sand transport increases monotonically along it; 3) erosion, by causing the beach to be displaced (inwards) normally to itself, does not change the qualitative shape of the beach. This last statement requires some explanation. If a planimetric shape is defined as a curve $f(X, Y, a_1 \dots a_n) = 0$ where $a_1 \dots a_n$ are parameters which define the centre, the size, the orientation, etc., of the curve, then what is stated is that a displacement of the curve normal to itself produces another curve of the same n-parameter family. A trivial example of such a curve is a circle $(x-x_0)^2 + (y-y_0)^2 = r^2$, which is a three-parameter curve; displacing every point of the circle normally outwards by an equal amount gives another circle of greater radius. A more appropriate example would be that of a logarithmic spiral displaced normally to itself by a distance L proportional to the radius vector from the origin: $L = \beta r$. Such a shift, as could be caused by erosion, transforms a spiral into another one of the same angle but different intercept r_0 . The spiral does not fulfill the first condition however. There must exist more complicated curves satisfying all of the above three conditions, and one cannot decide a priori which one will be the equilibrium one.

Because of the diffractive influence of the headland on the incident wave field, the region in the shadow of the headland, i.e. the head of the hook, is the most difficult to describe. All three terms (Q_1, Q_2, Q_3) may be important in the longshore volume flow there and it is not clear which one will dominate. The tail of the beach on the other hand should behave in a much simpler fashion since the wave field there should be nearly uniform in the longshore direction. The only contribution to longshore transport will come from Q_1 , which, from (12), (8) and $\phi_b = \phi_0 - \theta$, may be written

$$Q_1 = Q_sin(2(\phi_0 - \theta))$$

In the tail region, θ is a small angle, and $\tan\theta \simeq dX/dY$. For the same reason, $\frac{\partial}{\partial w} \simeq \frac{\partial}{\partial W}$, so that

$$\frac{\partial Q_1}{\partial \mathbf{v}} = -2Q_0 \frac{\partial \theta}{\partial \mathbf{v}} \approx -2Q_0 \frac{\partial^2 \mathbf{x}}{\partial \mathbf{y}^2}$$

The shape of the beach may then be found from

$$\frac{\partial^2 X}{\partial Y^2} = -\frac{1}{2Q_0} \frac{\partial Q_1}{\partial Y}$$

Analytic determination of the solution curve which will satisfy the three fundamental criteria in this region and connect to an equally satisfactory curve in the head region is beyond the present effort. We may choose for the moment any Q_1 which will tend to Q_0 so as to make X and all of its Y derivatives vanish as Y tends to infinity. For example, the tail region at t = 80 hours in Figure 7 is very closely fitted by $X = -X_0 e^{-\beta (Y-Y_0)^2}$, with $X_0 = -143 \text{ m}$, $Y_0 = 130 \text{ m}$, and $\beta = 3.15 \times 10^{-6} \text{m}^{-2}$, so that $Q_1 = Q_0 (1 - 4\beta X_0 Y e^{-\beta (Y-Y_0)^2})$ in that region.

Computed Beaches

The computed beach profiles fall into two categories according to the type of approximations made in describing the wave field in the vicinity of the headland.

It was first simply assumed that there was no diffraction whatsoever and hence no wave energy behind the "shadow line" Y = -X tan ϕ_0 . The transport along the beach segment which intersects the shadow line (at the point P_S with coordinates (X_S, Y_S)) was reduced in proportion to the fraction of it that lies in the shadow. The reduction factor is

$$\left[\left(\mathbf{X}_{j+1} - \mathbf{X}_{s} \right)^{2} + \left(\mathbf{Y}_{j+1} - \mathbf{Y}_{s} \right)^{2} \right]^{1/2} / \left[\left(\mathbf{X}_{j+1} - \mathbf{X}_{j} \right)^{2} + \left(\mathbf{Y}_{j+1} - \mathbf{Y}_{j} \right)^{2} \right]^{1/2}$$
(16)

Only the oblicity component Q_1 was retained in this case, to lighten the computational burden. Even under such gross approximations the results are encouraging. The successive beach shapes shown in Figure 7 were computed for $P=0.2,\ S=0.02,\ x_b=62.5\ m.,\ C=10^{-2},\ \alpha=0.4,\ \phi_0=\pi/10,\ a_b=1\ m.,\ \mu=1\ kg\ m^{-3},\ \rho_s=2\ x\ 10^3\ kg\ m^{-3}$ and a time step Δt of 1 minute. The small time step was necessary to avoid instabilities. Except for the incredibly rapid rate of erosion of the beach, which may be attributed to the presence of an unrealistic discontinuity at the origin as well as to the absence of the Q_2 and Q_3 terms, the erosion proceeds in a reasonable fashion. Note in particular that the first two of the qualitative criteria established earlier for an equilibrium shape are satisfied: 1) the beach is concave outwards at first, convex afterwards; 2) the transport increases monotonically along the beach.

Encouraged by this moderate success I have started computations in which diffraction effects are included. As it is not in general possible to obtain closed-form solutions to the diffracted wave problem for arbitrary coast geometries, any diffraction correction to the incident wave field will necessarily be an approximation. The most obvious correction, Sommerfeld's solution to wave diffraction by a wedge (Stoker, 1957, p. 109), is too complicated for practical computations. Once the beach has been eroded back sufficiently behind the rocky coastal strip, one would expect the solution for a wedge of zero angle, i.e. a thin barrier, to be a useful approximation. The theoretical results for that case have been verified experimentally (Putnam and Arthur, 1948). In order to take into account the fact that the barrier is not infinitely thin, nor the corner angle identically zero, I have used the following correction for the wave field: if $F_{0}(X, Y)$ is the wave amplitude function for the thin barrier case (as given by Putnam and Arthur) then the correction factor for the amplitude used in the computations was

$$F(X, Y) = 1 - (1 - F_0(X, Y)) (\theta_1 / \pi)^n$$
(17)

where θ_1 is the angle of the first segment (j=1), as defined in Figure 4 and n is an adjustable exponent. Clearly, for $\theta_1=\pi, F=F_0$ and the wave field is that behind a thin barrier; for $\theta_1=0,$ F=1, and the waves have the same amplitude everywhere. Inside the geometrical shadow (behind the shadow line: $Y < \sim X \tan \varphi_0$) the waves are now assumed to radiate from the origin (i.e. the headland), and $\varphi_b = \tan^{-1} (-Y/X) - \theta$; outside the shadow area, $\varphi_b = \varphi_0 - \theta$, as before. An upper bound φ_{max} was also imposed on the angle of incidence.

Computations including all components of transport (Q $_1,$ Q $_2$ and Q $_3) as well as diffraction corrections have not so far been very successful.$





As a matter of fact, they have been plagued with instabilities to such an extent that it will be necessary to review the whole of the computational scheme. One of the more successful efforts is shown in Figure 8. In this computation, P = 0.3, S = 0.083, $x_b = 32$ m, $C = 10^{-2}$, $\alpha = 0.42$, $\phi_0 = 0.17$, $a_b = 1$ m., $\mu = 1$ kg m⁻¹, $\rho_S = 2 \times 10^3$ kg m⁻³, $\Delta t = 10$ sec and n = 3, $\phi_{max} = 0.35$. The computation was first run for 1 hour without diffraction corrections and with $Q_2 = Q_3 = 0$; the shape arrived at then (t = 0) provided the base line for the more sophisticated calculations. It is clear from Figure 8 that, after some adjustment (t = 5, 10), the beach digs in behind the headland, as it did in Figure 7. The transports do not become monotonic, as they did in the previous example however and a catastrophic instability occurs soon after t = 80 min.

Conclusions

I have shown, using the theory of wave-induced longshore currents and a simple model for beach erosion, how waves obliquely incident upon a nonuniform coast could initiate beach erosion which would eventually lead to the formation of hook-like beaches. I have however not succeeded in explaining why these beaches are such good fits to segments of logarithmic spirals, although I have presented qualitative arguments which indicate that such a shape would satisfy the requirements of an equilibrium planimetric shape.

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