

## CHAPTER 72

### PREDICTING CHANGES IN THE PLAN SHAPE OF BEACHES

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#### ABSTRACT

A mathematical model is described that is capable of predicting changes in the plan shape of a beach following the construction of sea defences or an alteration in the wave climate. The rate of change is calculated. The technique is illustrated by comparing model predictions with results from experiments carried out in a wave basin. The importance of the method in estimating coastal changes due to offshore dredging is mentioned, and the future development of the mathematical approach is outlined.

#### INTRODUCTION

A frequently recurring problem in coastal engineering is the prediction of changes in the plan shape of a beach following alterations in the wave climate. Such changes can be induced by various means. The construction of groynes, breakwaters and harbour moles has a direct influence on the beach plan shape. Offshore dredging for sand and gravel may also affect the waves by changing the refraction pattern.

Changes in the plan shape of a beach and the rate of this change are of particular interest in Great Britain. Large deposits of sand and gravel exist close to the shore. The use of these offshore natural resources is becoming necessary as stocks on land are now less economically obtained.

Hence methods to forecast changes affecting general beach stability are of immediate concern to the civil engineer.

This paper describes the early development of a mathematical model to study such situations. It illustrates the application of the method to the problem of predicting beach changes following the construction of a long groyne. Results from experiments carried out in a wave basin to check predictions made by this mathematical approach are described.

#### MATHEMATICAL MODEL

Essentially, the mathematical model is a finite difference solution of the continuity equation in the alongshore direction:-

$$\frac{\delta Q}{\delta x} + \frac{\delta A}{\delta t} = 0$$

where Q = volume rate of alongshore sediment transport  
 x = distance in the alongshore direction  
 A = beach cross-sectional area  
 t = time

When the cross-sectional area is represented as the product of a beach ordinate, y, perpendicular to x, and a depth, D, as shown in Figure 1, this becomes:-

$$\frac{\delta Q}{\delta x} + D \frac{\delta y}{\delta t} = 0 \quad (1)$$

The solution of this equation requires a means of calculating the rate of alongshore sediment transport, Q, given wave and beach properties at an instant in time. Analysis of the results of previous tests in the wave basin indicated that the Scripps Equation, as modified by Komar in Reference 1, described the submerged weight sediment transport rate reasonably well for uniform waves in the absence of tidal currents:-

$$I_L = 0.35 E (nC) \sin 2 \alpha$$

where  $I_L$  = submerged weight rate of alongshore sediment transport

E = energy density of breaking waves,  $\frac{1}{8} \rho g H^2$

$\rho$  = mass density of water

g = acceleration due to gravity

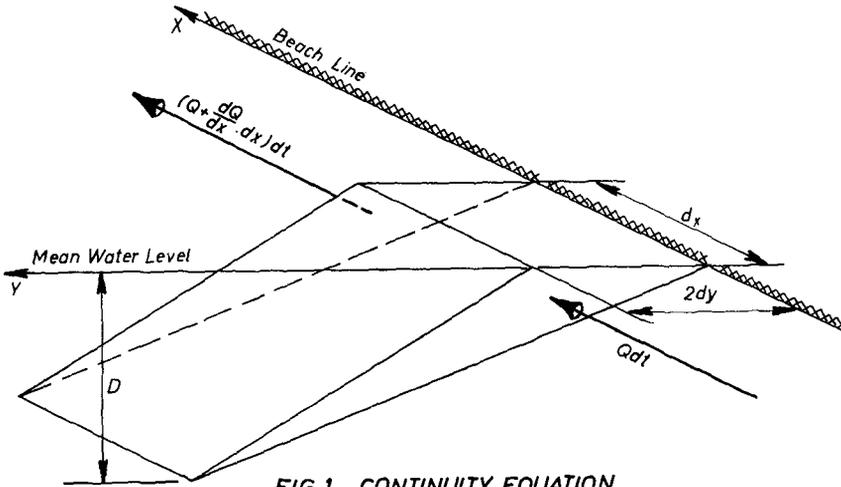


FIG.1 CONTINUITY EQUATION

- H = wave height, trough to crest, at breaking
- (nC) = group velocity of waves at breaking
- $\alpha$  = angle between breaking wave front and the beach

Converted to a volume rate of transport, this becomes:-

$$Q = \frac{0.35}{\gamma_s} E (nC) \sin 2 \alpha \quad (2)$$

in which  $\gamma_s$  = submerged density of beach material in place

It is assumed that, for the given wave conditions, (nC) is a constant, E is a function of x only, and  $\alpha$  is a function of x and t.

Two methods of solution have been used. In the first, Equation (2) is differentiated with respect to x and substituted in Equation (1) to obtain:-

$$\frac{0.35}{\gamma_s} (nC) \left[ \sin 2 \alpha \frac{dE}{dx} + 2 E \cos 2 \alpha \frac{\delta \alpha}{\delta x} \right] + D \frac{\delta y}{\delta t} = 0 \quad (3)$$

In difference form, this is

$$\frac{0.35}{\gamma_s} (nC) \left\{ \sin 2 \alpha [n, t] \frac{E[n+1] - E[n-1]}{2\Delta x} \right. \\ \left. + 2 E [n] \cos 2 \alpha [n, t] \frac{\alpha [n+1, t] - \alpha [n-1, t]}{2\Delta x} \right\} \\ + D \frac{y[n, t + \frac{1}{2}] - y[n, t]}{\Delta t} = 0$$

in which  $[n, t]$  refer to the number of  $\Delta x$  and  $\Delta t$  steps respectively from the origin.

Thus rewriting:-

$$y[n, t + \frac{1}{2}] = y[n, t] - \frac{0.35(nC)\Delta t}{2\gamma_s D \Delta x} \left\{ \sin 2 \alpha [n, t] (E[n+1] \right. \\ \left. - E[n-1]) + 2 E [n] \cos 2 \alpha [n, t] (\alpha [n+1, t] \right. \\ \left. - \alpha [n-1, t]) \right\} \quad (4)$$

which can be solved, given an expression for  $\alpha$ , see Figure 2:-

$$\alpha = \alpha_x - \tan^{-1} \frac{\delta y}{\delta x} \quad (5)$$

where  $\alpha_x$  = the angle between the breaking wave front and the x-axis, a function of x only.

$$\text{Thus, } \alpha [n, t + \frac{1}{2}] = \alpha_x [n] - \tan^{-1} \frac{y[n+1, t] - y[n-1, t]}{2\Delta x} \quad (6)$$

The second method of solution is much simpler. In this technique, Equation (1) is converted immediately to differences:-

$$\frac{Q[n+1, t] - Q[n, t]}{\Delta x} + D \frac{y[n + \frac{1}{2}, t + \frac{1}{2}] - y[n + \frac{1}{2}, t - \frac{1}{2}]}{\Delta t} = 0$$

or:-

$$y[n + \frac{1}{2}, t + \frac{1}{2}] = y[n + \frac{1}{2}, t - \frac{1}{2}] - \frac{\Delta t}{D\Delta x} \left\{ Q[n+1, t] - Q[n, t] \right\} \quad (7)$$

in which:-

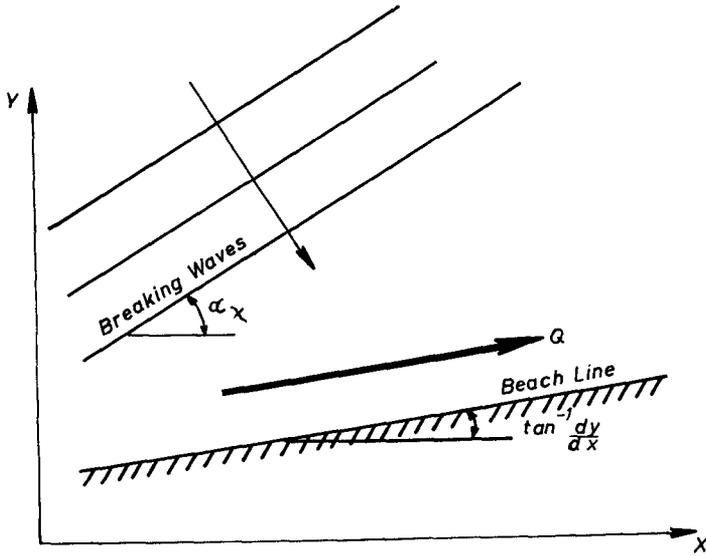


FIG.2 BREAKER ANGLE

$$Q[n, t + 1] = \frac{0.35E[n](nC)}{\gamma_s} \sin 2 \alpha [n, t + 1] \quad (8)$$

$$\alpha [n, t + 1] = \alpha_x [n] - \frac{\tan^{-1} \frac{y[n + \frac{1}{2}, t + \frac{1}{2}] - y[n - \frac{1}{2}, t + \frac{1}{2}]}{\Delta x}}{\Delta x} \quad (9)$$

This method has three principal advantages over the first:

1. Within the limitations of the alongshore transport calculation, it is 'exact'. That is, the budget of beach material is maintained because

$$\frac{Q[n + 1, t] - Q[n, t]}{\Delta x}$$

completely describes  $\frac{\delta Q}{\delta x}$  over the length  $\Delta x$ .

2. It can be used with alongshore transport equations that are difficult or impossible to differentiate.
3. It is more economical in computer time.

The second advantage described is probably the most important, and the second technique has already been used with a rather more complex transport equation, incorporating transport from

areas of high to areas of low wave energy when the angle of incidence is zero. This has not yet been tested but will form the subject of a future paper.

The only drawback to the second method is that, because  $Q$  and  $y$  are not known at the same points along the beach, boundary conditions are difficult to define. However, the advantages outweigh the disadvantage, and no further development of the first technique is contemplated.

Input to the mathematical model is the wave height,  $H$ , angle of incidence,  $\alpha_x$ , and the initial beach shape,  $y$ , all as functions of  $x$ ; an estimated celerity at breaking,  $nC$ , based on the observed water depth and the wave period; and an estimated depth,  $D$ , beyond which alongshore transport no longer takes place. In the present case all these were measured in the physical model, but in practice it is intended that they would be provided by an analysis of observed or forecasted wave conditions.

#### WAVE BASIN TESTS

A beach of crushed coal - specific gravity 1.35 and  $d_{50} = 0.8$  mm - was subjected to waves having a period of 1.15 s and a constant height of 40 mm - the initial angle between the breakers and the beach being  $4^\circ$ . Material was fed to the updrift beach at the calculated alongshore transport rate. The beach was moulded to an arbitrary profile, and waves were generated for an hour to allow the profile to adjust to the waves. A long groyne was then constructed across the foreshore as shown in Figure 3. For comparison with the mathematical predictions, beach ordinates were obtained by averaging the 0, 25, 50, 75 and 100 mm depth contours at the following times:

- before the groyne was constructed. This was the initial plan shape used in the mathematical model.
- 1 hour after construction of the groyne.
- 3 hours after construction of the groyne.
- 6 hours after construction of the groyne.

#### DISCUSSION

A comparison of beach plan shapes predicted by the mathematical model and those measured in the wave basin is shown in Figure 4. There is close agreement for a period of 3 hours after the introduction of the groyne. After 6 hours

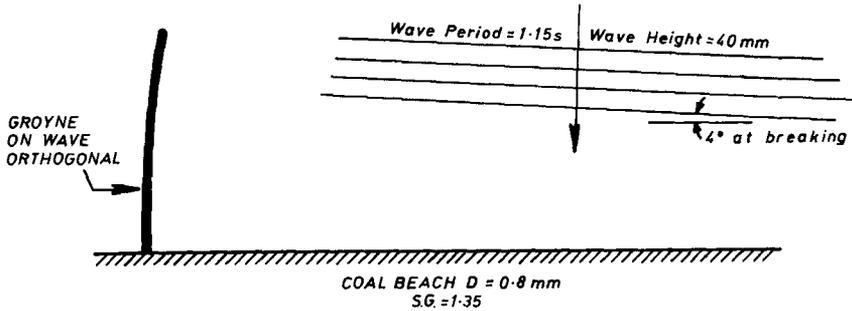


FIG. 3 TESTS IN THE WAVE BASIN

the build-up of coal near the structure sufficiently altered the wave refraction pattern to invalidate the input wave data. A complex boundary condition had also developed at the updrift end of the wave basin. No further useful comparisons could therefore be made.

The experiment to date has shown that the mathematical model, operated with the Komar equation for alongshore sediment transport, is capable of predicting the plan shape and rate of change of the plan shape of a beach with reasonable accuracy. Greater accuracy can be obtained if the relationship between alongshore sediment transport and wave height and direction can be verified for the beach in question.

The next development of the mathematical model is to link it with an existing refraction program to take account of the effect of beach changes on the waves. Further research will then be necessary to move away from the bulk flow sediment approach, as used by Komar, to a detailed description of sediment flux in terms of waves and currents; this will enable hydraulic and sediment conditions to be described for a point in space and time.

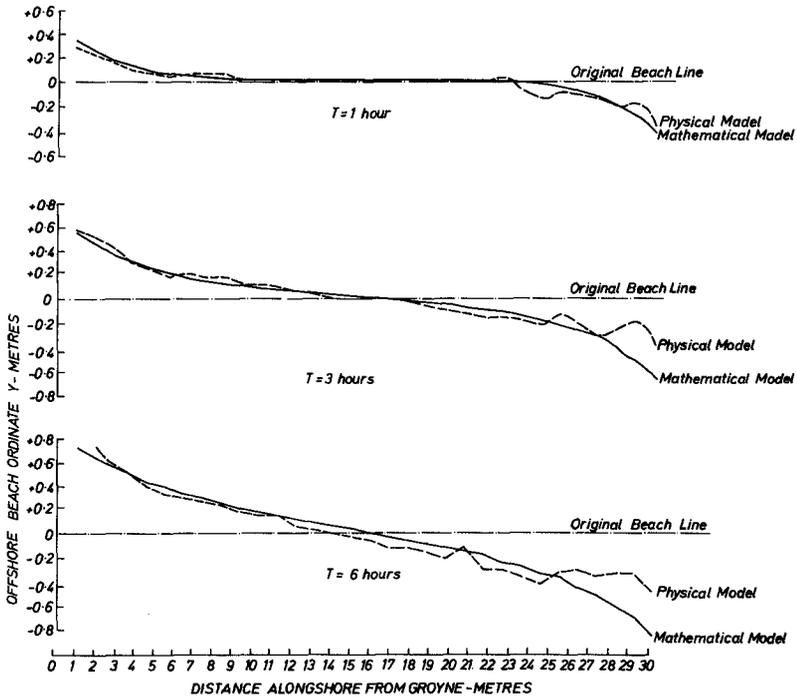


FIG. 4 CHANGES IN BEACH PLAN SHAPE

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