HYDRAULICS AND SEDIMENTARY STABILITY OF COASTAL INLETS

by

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ABSTRACT

A method is presented for investigating the stability of coastal inlets against closure due to transport and deposition of sand in the inlet cross-section. The method utilizes earlier contributions by: (1) Keulegan representing the hydraulics of inlets, (2) O'Brien which describes an equilibrium relationship between the cross-sectional area of an inlet and the bay tidal prism, and (3) Escoffier which relates to the stability of an inlet under changes in conditions which tend to close or enlarge an inlet. A "stability index" is defined which incorporates the buffer storage area available in the inlet cross-section, prior to the onset of closure and also includes the capability of the inlet to transport excess sand from its cross-section. In order to apply the method, geometric and hydraulic data representing the inlet are necessary; the minimum data required include a survey of the inlet throat cross-section and the lag between high (or low) water in the ocean and the following slack water in the inlet. In addition, it is necessary to conduct measurements or make assumptions concerning the minor and gradual hydraulic loss coefficients. Based on assumed depositional patterns in the inlet, the method is applied to five real inlets and the stability indices are presented.

INTRODUCTION

Some inlet-bay systems are inherently more stable than others against closure due to sand deposition in the inlet cross-section. It is clear that the larger and jettied inlets are generally more resistant to closure than the smaller and unjettied inlets. A review of the histories of various inlets show that some inlet-bay systems appear to be marginally stable, with closure generally occurring within a period of several years to a decade or more often after opening by severe storm activity. Other water systems separated by a barrier island from a tidal sea are closed soon after breaching. A better understanding of the hydraulics and sedimentary responses of inlets is necessary in order to improve capabilities in the design and maintenance planning of these coastal features.

The subject of the hydraulic and sedimentary characteristics of inlets has been one of a great deal of previous investigation. The classic paper of E. I. Brown(1) provides a lucid description of the processes of importance in

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the vicinity of an inlet on a sandy coast and also presents an approach for calculating the hydraulics of tidal inlets. Brown's method considers the bay and ocean water level variations to be sinusoidal and the channel cross-sectional area to be constant with time. More recently, Keulegan(2) has extended the approach of Brown to include the effect of a non-sinusoidal bay tide; however, the other restrictions are the same. The methods of both Brown and Keulegan strictly apply for the case of a bay connected to the ocean by a single inlet and for a bay water level which rises and falls uniformly over the entire bay area, see Figure 1. Moreover, their considerations are limited to inlets forming a definite flow constriction, the discharge into and from the inlet being governed by a head loss which is quadratic in the velocity. For some large inlet features, such as the Chesapeake Bay Entrance, the flow is probably describable as due to a partial standing wave system. Keulegan’s results include the phase lag between bay and ocean tides, and dimensionless values of maximum inlet velocity and bay amplitude; these results are presented as functions of the so-called "repletion coefficient," \( K \), defined as

\[
K = \frac{T}{2\pi a_0} \left( \frac{A_C}{A_B} \right) \frac{\sqrt{2g a_0}}{\sqrt{K_{en} + K_{ex} + \frac{f_L}{4R}}}
\]

(1)

See Figures 2, 3 and 4 for the phase lag, \( \varepsilon \), dimensionless velocity and bay amplitude. Other hydraulic studies of inlets include those of Baines(3), Van de Kreeke(4), Moto Oliveira(5), and Shemdin and Forney(6).

O’Brien(7,8) has presented data summarizing the relationship between the inlet throat cross-sectional area, and the tidal prism passing through that inlet during spring tide conditions. Very briefly, these results indicate that equilibrium conditions of an inlet are represented by a balance of the tidal prism tending to enlarge the cross-section and the supply of sand transported to the inlet by waves and currents tending to reduce the cross-section.

The relationship, shown in Figure 5 can also be interpreted as a unique relationship of maximum spring tidal velocity, \( V_{\text{max}} \), versus inlet equilibrium cross-sectional area, \( A_{CE} \).

Escoffier(9) has presented a concept of the stability of an inlet under the influences of depositional conditions which tend to enlarge or reduce the size of the inlet cross-section. The concept considers the maximum velocity in an inlet connected to a bay; this velocity varies with the inlet cross-sectional shape and area. A representative case is shown in Figure 6, in which it is seen that the \( V_{\text{max}} \) curve has a peak at some cross-sectional area, \( A_g \). This curve applies only for one tidal range, whereas in nature, the ratio of spring to neap tidal ranges can vary from a reasonably small factor (1.2 at Miami Beach, Florida) to a much larger factor (18 at Pensacola Bay Entrance, Florida). Referring to Figure 6, the following will be shown:
Note: $a_o = \text{Half Tidal Range in Ocean}$

FIGURE 1. DEFINITION SKETCH OF INLET/BAY SYSTEM
FIGURE 2. VARIATION OF PHASE LAG, $\epsilon$, WITH REPLETION COEFFICIENT, $K$. KEULEGAN'S METHOD

FIGURE 3. VARIATION OF DIMENSIONLESS MAXIMUM VELOCITY WITH REPLETION COEFFICIENT, $K$. KEULEGAN'S METHOD
FIGURE 4. VARIATION OF $\frac{a}{a_0}$ WITH REPLETION COEFFICIENT, K. KEULEGAN’S
METHOD

FIGURE 5. EQUILIBRIUM CROSS-SECTIONAL AREA AND TIDAL PRISM
RELATIONSHIP (FROM O’BRIEN)
FIGURE 6. ILLUSTRATION OF ESCOFFIER'S STABILITY CONCEPT
\[ A_c > A^* \rightarrow \text{This inlet is stable against changes in closure tendencies} \]

\[ A_c < A^* \rightarrow \text{This inlet is unstable against changes in closure tendencies} \]

Consider first the right hand side of the curve. If an unusual amount of littoral drift is carried into the inlet, the cross-sectional area will decrease, thereby resulting in an increase in inlet velocity and an increase in scouring capacity; therefore, the inlet tends to be stable by countering against any area change by a velocity change that will tend to reduce the area change. It is also noted that any enlargement in area will result in a decrease in velocity and an associated increase in deposition tendency, thereby causing the inlet to tend to stabilize about the equilibrium area. Consider next the left hand side of the curve. Any decrease in cross-sectional area will result in a decrease in velocity and a tendency for the area to decrease further. Also any increase in area will perpetuate this increase by an increase in inlet velocity. The cross-sectional area, \( A_c \), characterized by

\[
\frac{\partial V_{\text{max}}}{\partial A_c} = 0
\]

is denoted \( A^* \) and represents a division between stable and unstable conditions. The stable region is primarily governed by the changes in velocity resulting from a change in cross-sectional area, whereas the characteristics of the unstable region are due to the increasing friction with decreasing cross-sectional area, (and hydraulic radius, \( R \)). Finally, in concluding the discussion on stability, it is clear that in nature, the tidal ranges change with time and an equilibrium cross-sectional area as well as critical area would only be meaningful in terms of some average tidal range conditions. Furthermore, if the maximum velocity \( V_{\text{max}} \) associated with \( A^* \) is less than the "threshold velocity" required to move sand, it is clear that the inlet would tend to close under the depositional action of waves and currents. The presence of a net fresh water outflow would also be an important factor in favorably affecting the stability of an inlet.

**METHOD OF CALCULATING INLET STABILITY**

The method involved in calculating inlet stability requires some available information describing existing inlet conditions; these existing conditions are assumed to represent equilibrium conditions. Based on the equilibrium conditions and assumptions regarding minor and gradual head loss terms, it is possible to calculate the inlet stability characteristics for an assumed form of deposition in the inlet cross-section.

**Inlet Hydraulics**

It is assumed that the inlet hydraulic characteristics are adequately represented by the Keulegan method, even though the bay system may be interconnected to other bays or may be served by more than one inlet. Although
this is not strictly valid, it does allow calculation of the inlet stability and it can be shown that the effect is an overestimate of the stability.

Estimate of the Repletion Coefficient, $K$

There are several types of field observations which can be used to yield an estimate of the repletion coefficient, $K$. Referring to Figures 2, and 4, it is seen that estimates could be obtained as shown in Table I.

<table>
<thead>
<tr>
<th>Field Measurements</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lag between ocean high (or low) tide and following slack water in the inlet, $e$</td>
<td>2</td>
</tr>
<tr>
<td>Ratio of bay to ocean amplitudes, $a_B/a_o$</td>
<td>4</td>
</tr>
</tbody>
</table>

After comparing various methods, it appears that the most convenient and reliable field measurement is the lag between the ocean tide extreme and the following slack water in the inlet. Tide and current tables contain adequate information for some inlets. For example, Figure 7 represents the lag, $e$, from tide (10) and current (11) tables for Government Cut, Florida for the month of January, 1972. The different lags following high and low ocean tides represent a departure from the Keulegan predictions, and it is recommended that an average value be taken. The values of $K$ as determined from Figure 2 are also presented in Figure 7 for two values of $e$. The representative value of $e$ and $K$ for Government Cut were taken as $56^\circ$ and 0.53 respectively.

Estimate of Inlet and Bay Characteristics

Considering the inlet throat cross-sectional area to be known from surveys, and to be in equilibrium with the tidal prism, $P$, in accordance with O'Brien's relationship, it is possible to determine the head loss coefficient $K_{en} + K_{ex} + f_{x} \frac{1}{4R}$. Expressing the repletion coefficient, $K$

$$K = \frac{T}{2\pi a_o} \frac{\sqrt{2ga_o}}{\left[K_{en} + K_{ex} + f_{x} \frac{1}{4R}\right]^{1/2}}$$

in which the tidal prism $P$ has been equated to $2a_B a_o$ and $\cos e = a_B/a_o$. It should be recognized that the bay area as defined is not the actual bay area but an area consistent with the tidal prism and the bay tidal amplitude, $a_B$. 
FIGURE 7. LAG BETWEEN HIGH AND LOW WATERS AND FOLLOWING SLACKS FOR GOVERNMENT CUT, FLORIDA DETERMINED FROM TIDE AND CURRENT TABLES

FIGURE 8. DEPOSITIONAL PATTERN IN INLET
just inside of the inlet. Because the tidal amplitude within the bay is generally less than this value, it follows that the effective bay area will be less than the actual bay area.

Considering the inlet cross-section to be in equilibrium with the tidal prism, Equation (2) may be solved for the head loss terms, i.e.

\[ K_{en} + K_{ex} + \frac{f_L}{4R} = \left( \frac{T}{\pi} \sqrt{2gA_{pm} \cos \epsilon \frac{A_c}{P_m}} \right)^2 \]  

One slight refinement is introduced in the calculation indicated by Equation (3). The equilibrium area-tidal prism relationship applies for spring or diurnal tides, whereas the head loss terms are solved for the mean tidal range. The values of \( a_0 \) and the tidal prism, \( P \), in Equation (3) should therefore refer to mean tide conditions. Denoting \( P_s \) and \( P_m \) as the tidal prisms associated with spring and mean tides, respectively, it can be shown that

\[ P_m = P_s \left( \frac{a_0}{a_0} \right)_m \frac{n}{n} \]

in which \( 0.5 < n < 1.0 \) depending on the repletion coefficient, \( K \), and the subscripts \( m \) and \( s \) refer to mean and spring tidal ranges respectively. The tidal prism value employed in Equation (3) is \( P_m \) with \( n \) taken to be 0.5.

The hydraulic radius is known from the inlet cross-sectional surveys and if \( K_{en} + K_{ex} \) and \( f \) can be estimated, than an equivalent inlet length, \( \lambda \), based on an inlet cross-section of uniform area with length, is estimated as

\[ \lambda = \left( \frac{\sqrt{2gA_{pm} \cos \epsilon \frac{A_c}{P_m}}}{K_{en} + K_{ex}} \right) \frac{4R}{T} \]  

where the terms in the square brackets are estimated from Equation (3) and \( K_{en} + K_{ex} \) and \( f \) were estimated to be 1.3 and 0.03, respectively.

In summary of the calculated inlet-bay properties, it is possible to estimate: \( K, A_B, \lambda \) in which \( A_B \) and \( \lambda \) are "effective" or "equivalent" properties.

**Stability Calculations**

With the calculated values of the existing inlet characteristics and the assumption that the inlet is in equilibrium, the response of an inlet to deposition of sand will be investigated.

Consider a quantity of sand driven into an inlet by waves and currents and deposited, thereby reducing the inlet cross-section. The constriction of the cross-section will tend to increase the inlet currents whereas the
increase in friction due to the lesser depths will tend to decrease the currents. It therefore is apparent that the manner in which the inlet is loaded with sand is of importance. Obviously, in carrying out stability calculations, it would be preferable to assume deposition in each inlet in a manner that is consistent with the particular hydrography and wave climate of the vicinity. This type of information, however, is generally not available and, for purposes of consistency, it was decided to "load" all inlets in the same manner which is reasonably realistic; the results would therefore represent the response of a group of inlets to the same depositional pattern.

As noted previously, the pattern of deposition affects the response of the inlet to a decrease in area. For example, if the sand is deposited primarily along the sides of a channel, the hydraulic radius will be affected only slightly and the resulting maximum velocity will be larger relative to the case where the deposition resulted in a depth decrease but no decrease in width. Similarly, deposition along the entire inlet length will influence the response differently than the same reduction in cross-sectional area, but the decrease limited to a portion of the inlet length. In the calculations, it was assumed that the reduced cross-sectional area was geometrically similar to the equilibrium (measured) cross-sectional area. Denoting the equilibrium values by the subscript, E,

\[ A_c = \kappa A_{cE} \]  

where \( \kappa \) is to be specified as the parameter of area reduction. It can be shown that the hydraulic radius is related to the equilibrium hydraulic radius \( R_E \) by

\[ R = \sqrt{\kappa} R_E \]  

**Governing Equations**

Referring to Figure 8, it can be shown that for a given area reduction and depositional length, denoted by \( \kappa \) and \( \Delta L \), respectively, the repletion coefficient is given by

\[ K = T \frac{A_{cE}}{A_B} \frac{\sqrt{2ga}}{\kappa^2} \left[ \left( \frac{K_E + K_{ex}}{\kappa^2} \right) + f \left( \frac{\Delta L}{\kappa} \right) \right]^{\frac{1}{2}} \]  

\[ \left( \frac{2\left( \frac{\Delta L}{\kappa} \right)}{4R_E^2 + \frac{f \left( \frac{\Delta L}{\kappa} \right)}{4R_E^2}} \right) \]  

in which it is assumed that no change in the effective bay area, \( A_B \), occurs due to the deposition and the same hydraulic minor loss coefficients apply at the transition from \( A_{cE} \) to \( A_c \), except the coefficients are multiplied by the difference in velocity heads. With the repletion coefficient known for the considered deposition, \( V_{\text{max}}' \) is determined from Figure 3 and \( V_{\text{max}} \) is determined
This is the maximum velocity that would pertain in the deposition region and would be the velocity of importance in tending to restore the area to equilibrium conditions. In employing Equations (7) and (8), the spring tidal range was used as the condition considered most effective in governing stability.

**Stability Index, $\beta$**

A measure of stability, $\beta$, was defined to represent the capacity of an inlet to remain stable under conditions of deposition. The best definition of the stability index is not apparent, however, it is clear that the definition should recognize that inlets with equilibrium areas much larger than the critical area $A_c^*$ have more storage area and therefore, will be more resistant to closure. Also the definition should reflect the capacity of large velocities to transport sand out of an inlet.

The relationship between sediment transport, $q_s$, and water velocity is not precisely known, however it is generally agreed that the sediment discharge is proportional to some power, $j$, of the velocity

$$q_s = C(V - V_T)^j$$

where $C$ is some constant (or function) and $V_T$ represents a "threshold velocity" for sand transport.

In consideration of the sediment discharge relationship, and the factors noted, the stability index, $\beta$ was defined as

$$\beta = \int_{A_c^*}^{A_{CE}} (V_{max} - V_T)^3 dA_c$$

The index $\beta$ has units of $(\text{length})^5/(\text{time})^3$.

**Example Calculation**

To illustrate the effects of various deposition lengths, $\Delta A$, consider a hypothetical inlet and the following parameters:

- $R_E = 8 \text{ ft.}$
- $A_{CE} = 10,000 \text{ ft.}^2$
- $a_0 = 2 \text{ ft.}$
- $c_E = 60^0$
The equilibrium repletion coefficient, $K_e$, determined from Figure 2 is 0.465 and the equilibrium tidal prism from Figure 5 is $4.86 \times 10^8$ ft.$^3$. The effective bay area $A_B$ is determined as

$$A_B = \frac{P_E}{2a_B^2} = \frac{P}{2a_o \cos \epsilon} = \frac{4.86 \times 10^8}{2(2)(0.5)} = 2.43 \times 10^8 \text{ ft}^2.$$

The head loss coefficients are determined from Equation (3) as

$$[K_{en} + K_{ex} + \frac{fP}{4R}] = 12.74$$

from which the effective length is (Equation 4)

$$\lambda = 12,200 \text{ ft}.$$

This completes the determination of existing conditions. The stability calculations for $V_{max}$ as a function of $A_c$ for various $\Delta z$ values are carried out in accordance with Equations (7) and (8) and are plotted in Figure 9.

From Figure 9 it is seen that the stability results depend markedly on the deposition length, $\Delta z$. For small deposition lengths, the inlet is more stable because the total friction is less; hence the velocities are higher than for the larger deposition lengths. Based, in part, on inspection of photographs of deposition along inlets, a standard deposition length was taken as 1000 ft. in all calculations related to natural bay-inlet systems.

The stability indices for the five values of $\Delta z$ presented in Figure 9 and determined from Equation (10) are summarized in Table II.

**TABLE II**

<table>
<thead>
<tr>
<th>Relative Deposition Length ($\Delta z$%)</th>
<th>$A_c/A^*$</th>
<th>Stability Index, $\beta$ (ft.$^3$/sec$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>$\infty$</td>
<td>$16.4 \times 10^5$</td>
</tr>
<tr>
<td>0.2</td>
<td>3.7</td>
<td>$1.38 \times 10^5$</td>
</tr>
<tr>
<td>0.4</td>
<td>2.1</td>
<td>$0.58 \times 10^5$</td>
</tr>
<tr>
<td>0.6</td>
<td>1.8</td>
<td>$0.34 \times 10^5$</td>
</tr>
<tr>
<td>0.8</td>
<td>1.4</td>
<td>$0.16 \times 10^5$</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
FIGURE 9. STABILITY CONSIDERATIONS FOR EXAMPLE INLET, EFFECT OF DEPOSITION LENGTH
RESULTS OF CALCULATIONS FOR NATURAL INLET-BAY SYSTEMS

Calculations were carried out for four Florida inlets and one inlet along the New York shoreline.

1. Government Cut, Florida

Government Cut, Florida, is a jettied and dredged inlet at the south end of the barrier island on which Miami Beach is located. The existing (1970) cross-sectional area, $A_{CE}$, was determined by survey to be 19,600 ft.\(^2\). Other characteristics are summarized in Table III. The results of the stability calculations are presented in Figure 10 where it is seen that the critical cross-sectional area is 3000 ft.\(^2\). The calculated stability index, $\beta$ is $11.5 \times 10^5$ ft.\(^5\)/sec.\(^3\).

2. Broad Creek, Florida

Broad Creek comprises a natural system of inlet channels through a limestone reef, although sedimentary material is present along much of the bottom of these channels. The inlet characteristics and results of the stability calculations are summarized in Table III, and Figure 11. The equilibrium and critical areas are 9200 and 1800 ft.\(^2\) respectively. The calculated stability index is $1.7 \times 10^3$ ft.\(^5\)/sec.\(^3\).

3. Boca Raton Inlet, Florida

Boca Raton Inlet is a natural inlet with very short jetties. The inlet has a history of closure in an approximate period of 1-3 years if maintenance dredging is not carried out. The information of interest and results of the stability calculations are presented in Table III and Figure 12. The stability index for Boca Raton Inlet is $0.12 \times 10^5$ ft.\(^5\)/sec.\(^3\).

4. Stump Pass, Florida

Stump Pass is a small inlet located on the west coast of Florida. The inlet cross-sectional area determined by a 1972 survey\(^{[12]}\) was 4940 ft\(^2\) and the stability index, $\beta$, determined as described is $0.75 \times 10^5$ ft.\(^5\)/sec.\(^3\), see Figure 13.

5. Shinnecock Inlet, New York

Considerable data are available describing the hydraulic characteristics and history of Shinnecock Inlet. The 1955 inlet cross-sectional area\(^{[13]}\) is assumed to represent equilibrium conditions; see Table III and Figure 14 for the summarized results. The stability index for Shinnecock Inlet is $0.96 \times 10^5$ ft.\(^5\)/sec.\(^3\).
TABLE III
SUMMARY OF NATURAL INLET COMPUTATIONS

<table>
<thead>
<tr>
<th>Inlet</th>
<th>$A_{CE}$ (ft.$^2$)</th>
<th>$A^*$ (ft.$^2$)</th>
<th>Stability Index, $\delta$ (ft.$^3$/sec$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Cut, Florida</td>
<td>19,600</td>
<td>3,000</td>
<td>$11.5 \times 10^5$</td>
</tr>
<tr>
<td>Broad Creek, Florida</td>
<td>9,200</td>
<td>1,800</td>
<td>$1.7 \times 10^5$</td>
</tr>
<tr>
<td>Boca Raton Inlet, Florida</td>
<td>1,410</td>
<td>230</td>
<td>$0.12 \times 10^5$</td>
</tr>
<tr>
<td>Stump Pass, Florida</td>
<td>4,940</td>
<td>900</td>
<td>$0.75 \times 10^5$</td>
</tr>
<tr>
<td>Shinnecock Inlet, New York</td>
<td>5,500</td>
<td>1,400</td>
<td>$0.96 \times 10^5$</td>
</tr>
</tbody>
</table>

SUMMARY

Based on earlier concepts and techniques relating to inlet hydraulics, equilibrium inlet conditions, and inlet stability, as developed by Keulegan, O'Brien and Escoffier respectively, and assuming idealized depositional patterns within the inlet, a method has been developed to calculate the stability of an inlet as affected by deposition. A stability index, $\delta$, has been defined, based on considerations of idealized depositional patterns, allowing the comparison of stabilities of various inlet systems. The stability indices are calculated and compared for five natural inlet systems.

ACKNOWLEDGEMENT

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Critical Area = 3000 ft$^2$

Note: Assumed Deposition Length = 1000 ft.

Assumed Equilibrium Area = 19,600 ft$^2$

Figure 10. Stability considerations for Government Cut, Florida

Critical Area = 1800 ft$^2$

Assumed Equilibrium Area = 9200 ft$^2$

Note: Assumed Deposition Length = 1000 ft.

Figure 11. Stability considerations for Broad Creek, Florida
FIGURE 12. STABILITY CONSIDERATIONS FOR BOCA RATON INLET, FLORIDA

FIGURE 13. STABILITY CONSIDERATIONS FOR STUMP PASS, FLORIDA
FIGURE 14. STABILITY CONSIDERATIONS FOR SHINNECOCK INLET, NEW YORK

- Critical Area = 1400 ft$^2$
- Assumed Equilibrium Area = 5500 ft$^2$
- Assumed Deposition Length = 1000 ft
REFERENCES


