CHAPTER 32

DISSIPATION OF WAVE ENERGY DUE TO OPPOSING CURRENT

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ABSTRACT

Energy dissipation and wave height attenuation were analysed theoretically for surface waves propagating against uniform flow. Energy dissipation was estimated from evaluation of work by internal and boundary shear stresses. Experiments were conducted in a test flume of 20m long, 0.8m wide and 0.5m high. Results showed that tested values of rate of wave height attenuation were comparable with theoretical values.

INTRODUCTION

When waves travel against current, wave energy is transfered from current by so called radiation stress at the one hand, but is dissipated by internal and boundary shear stress at the other hand. Thus the wave height variation in the course of travelling against the current is effected by these two controversing action and as the result, wave height may be increased in some time or may be decreased in othertime owing to their relative magnitude. In deep water, the shear stress can be neglected, for which several investigators such as Unna[1,2,3], Yu[4], Longuet-Higgins & Stewart[5], and Witham[6] have contributed. In shallow water and especially in estuary, the shear stress cannot be neglected. The friction factor for a current superimposed by waves was, due to the auther's knowledge, firstly treated by Ivar G. Jonsson[7] theoretically as,

$$f_{w+c} = \frac{U_b}{U + U_b} f_{wo} + \frac{U}{U + U_b} f_c \qquad (1)$$

where U, U_b mean current velocity and maximum amplitude of velocity of waves respectively, f_{wo} means a friction coefficient of waves in case of no current and f_c means that of current in case of no waves. In this paper, energy dissipation was estimated from evaluation of work by internal and boundary shear stresses.

Experiments were conducted in a test flume of 20m long, 0.8m wide and 0.5m high.

Analytical results showed that the attenuation rate due to boundary shear was nearly constant over full range of current velocity, but effect by internal friction become remarkable as the opposing current velocity increased. The latter dominated tendency of total dissipation.

THE ENERGY EQUATION FOR WAVES RIDING OVER CURRENT

The motion to be discussed will be assumed to occur in two dimensions on incompressible fluid.

Space co-ordinates x and y are chosen along and perpendicular to the bottom respectively (Fig. 1).

Basic equations for unsteady disturbance riding over uniform flow are,

$$\frac{\partial u}{\partial t} + (U+u)\frac{\partial u}{\partial x} + v\frac{\partial (U+u)}{\partial y} = g\sin\theta - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau}{\partial y} \qquad (2)$$

$$\frac{\partial v}{\partial t} + (U+u)\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -g\cos\theta - \frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (3)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4)$$

where u and v are the components of disturbance velocity in x and y directions respectively, U is the current velocity and is assumed as a function of y such as $U_1 f(y)$ in which U_1 means the velocity at $y=h$, ρ, g, θ, p and τ denote density, accerelation of gravity, angle of bottom to the horizontal, pressure intensity and shear stress.

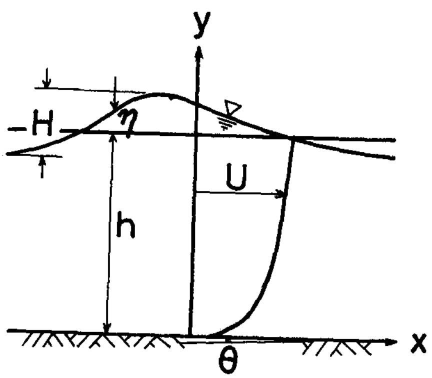

Fig.1

If τ is considered

to be the sum of shear stress due to current τ_c and that due to disturbance τ_w, then gradient of τ_c in y direction is compensated with $\rho g \sin\theta$ when flow is uniform without waves, because then remaining terms beside $g\sin\theta$ and $\frac{1}{\rho}\frac{\partial\tau_c}{\partial y}$ are all vanished in eq.(2).

Eq.(2), then, becomes

$$\frac{\partial u}{\partial t} + (U+u)\frac{\partial u}{\partial x} + v\frac{\partial(U+u)}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{1}{\rho}\frac{\partial \tau_w}{\partial y} \tag{2'}$$

Combination of eqs.(2)', (3) and (4) and integration from zero to $h+\eta$ with respect to y leads to the following equation for average values over a period,

$$\frac{\partial}{\partial t}\{\overline{\int_0^{h+\eta}\frac{\rho}{2}(u^2+v^2)dy+\frac{1}{2}\rho g\eta^2}\} + \frac{\partial}{\partial x}\overline{\int_0^{h+\eta}\{\frac{\rho}{2}(u^2+v^2)+p+\rho g(y-h)\}u dy}$$
$$+\frac{\partial}{\partial x}\overline{\int_0^{h+\eta}\frac{\rho}{2}(u^2+v^2)U dy} + \overline{U_{h+\eta}\frac{\partial}{\partial x}(\frac{1}{2}\rho g\eta^2)} + \overline{\int_0^{h+\eta}\rho u v\frac{\partial U}{\partial y}dy} = \overline{\int_0^{h+\eta}u\frac{\partial \tau_w}{\partial y}dy} \tag{5}$$

where bar means average over a period, h is the undisturbed depth and η is the surface elevation, and in the derivation of eq.(5), the surface condition (6) and the bottom condtion (7) are used with taking $\cos\theta$ as unity.

$$v = \frac{\partial\eta}{\partial t} + (U+u)\frac{\partial\eta}{\partial x} \qquad \text{at } y=h+\eta \tag{6}$$

$$v = 0 \qquad \text{at } y=0 \tag{7}$$

As $U_{h+\eta}$ is approximately equal to $U_h+\eta\frac{dU}{dy}\Big|_{y=h}$, the fourth term in the left side of eq.(5) is written as $\frac{\partial}{\partial x}(\overline{\frac{1}{2}\rho g\eta^2}U_h)$ to the second order in the amplitude.

The last term is written as follow,

$$\overline{\int_0^{h+\eta}\rho u v\frac{\partial U}{\partial y}dy} = \int_0^{h}\rho\overline{uv}\frac{\partial U}{\partial y}dy + \overline{\eta(\rho u v\frac{\partial U}{\partial y})}_{y=h}$$

and is zero to the second order in the amplitude because $\overline{uv}$ is zero. Consequently eq.(5) is written as follows

$$\frac{\partial}{\partial t}[\overline{\int_0^{h+\eta}\frac{\rho}{2}(u^2+v^2)dy+\frac{1}{2}\rho g\eta^2}] + \frac{\partial}{\partial x}[\overline{\int_0^{h+\eta}\{\frac{\rho}{2}(u^2+v^2)+p+\rho g(y-h)\}u dy}$$
$$+\overline{\int_0^{h+\eta}\frac{\rho}{2}(u^2+v^2)U dy+\frac{1}{2}\rho g\eta^2 U_h}] = \overline{\int_0^{h+\eta}u\frac{\partial\tau_w}{\partial y}dy} \tag{8}$$

In eq.(8), the term in the first bracket is the energy density $\bar{E}$. That in the second bracket is the energy flux and is expressed by $\overline{E\cdot C_G'}$ physically, where C_G' is the energy transfer velocity. Then the physical expression of eq.(8) is simply as,

$$\frac{\partial \bar{E}}{\partial t} + \frac{\partial}{\partial x}(\overline{E \cdot C_G'}) = \overline{\int_0^{h+\eta} u \frac{\partial \tau_W}{\partial y} dy} \tag{9}$$

The term in the right side is the dissipated energy $\bar{E}_d$. Evaluation of $\bar{E}$ or $E \cdot C_G'$ must be given by disturbance velocity (u, v), pressure p, surface elevation η with current velocity assumption U. Strictly to say the energy transfer velocity is not necessarily equal to the group velocity in general,[8] which is defined by

$$\bar{C}_G = \frac{d(Cm)}{dm} \tag{10}$$

where m is the circular wave number $2\pi/L$ and L is the wave length. As this caluculation is very troublesome, C_G' is approximated by $\bar{C}_G$ in the following. Then,

$$\frac{\partial \bar{E}}{\partial t} + \frac{\partial}{\partial x}(\overline{E \cdot C_G}) = \overline{\int_0^{h+\eta} u \frac{\partial \tau_W}{\partial y} dy} \tag{9)'$$

Furthermore, $\bar{E}$ is approximated as that of regular sinusoidal waves,

$$\bar{E} = \frac{1}{8}\rho g H^2 \tag{11}$$

where H denotes wave height.

The term in the right side is dissipated energy $\bar{E}_d$ and contribution to this term is partly due to those in main part, $\bar{E}_{dI}$ and partly due to in the boundary layer, $\bar{E}_{dB}$.

For steady flow with regular waves, the first term in (9)' vanishes and putting $-2\bar{E}\cdot G$ for the right side of eq.(9)', eq.(12) is obtained.

$$\frac{d\bar{E}}{dx} = -2\bar{E}\cdot\frac{G}{C_G} \tag{12}$$

as C_G is constant.

Then,

$$\bar{E} = E_o e^{-\frac{2G}{C_G}x} \tag{13}$$

$$H = H_o e^{-\frac{G}{C_G}x} \tag{14}$$

Eq.(13) gives the rate of energy dissipation. Eq.(14) gives the wave height attenuation. G/C_G gives the physical meaning of Inman's a.[9]

Then,

$$G = -\frac{1}{2\bar{E}} \overline{\int_0^{h+\eta} u \frac{\partial \tau_W}{\partial y} dy} = -\frac{1}{\frac{1}{4}\rho g H^2}(\bar{E}_{dI} + \bar{E}_{dB}) \tag{15}$$

in which

$$\bar{E}_{dI} = -\overline{\int_\delta^{h+\eta} u \frac{\partial \tau_W}{\partial y} dy} \quad \text{and} \quad \bar{E}_{dB} = -\overline{\int_0^{\delta} u \frac{\partial \tau_W}{\partial y} dy} \tag{16}$$

where δ is the depth of the boundary layer. Evaluation is now to be divided into the main part of flow and the boundary layer.

DERIVATION OF WAVE DISTURBANCE ON CURRENT IN THE MAIN PART

As shown in Fig.2 local disturbance (u,v) is assumed to be the sum of disturbance velocities (u_1,v_1) neglecting effects of boundary layer due to the wave motion and that (u_2,v_2) to be corrected for effects of boundary layer. Then,

$$u = u_1+u_2\ , \qquad v = v_1+v_2 \tag{17}$$

To evaluate $\bar{E}_{dI}$ in the main part of flow, u can be replaced by u_1 because u_2 may be neglected. The shear stress in the main part is estimated to be,

$$\tau_w = \rho\varepsilon\frac{\partial u_1}{\partial y} \tag{18}$$

where ε is the eddy viscosity and is assumed as

$$\varepsilon=\kappa h U_*(1-\frac{y}{h})\frac{y}{h} \tag{19}$$

in which U_* is the shear velocity and κ is the Karman's universal constant.

Then $\bar{E}_{dI}$ is approximated as,

$$\begin{aligned}\bar{E}_{dI} &= -\int_\delta^h \overline{u_1\frac{\partial}{\partial y}(\rho\varepsilon\frac{\partial u_1}{\partial y})}dy \\ &= \int_\delta^h \rho\varepsilon\overline{(\frac{\partial u_1}{\partial y})^2}dy+[\frac{\rho\varepsilon}{2}\cdot\frac{\partial \overline{u_1^2}}{\partial y}]_{y=\delta} \\ &= \frac{1}{4}\rho g H^2\cdot G_1 \end{aligned} \tag{20}$$

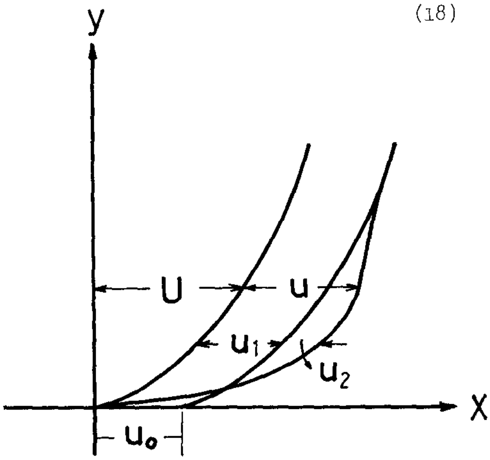

Fig.2

Derivation of (u_1,v_1) neglecting effects of boundary layer due to the wave motion is the next problem. For this purpose linearized equations of (2)' and (3) are used. Then,

$$\frac{\partial u_1}{\partial t} + U\frac{\partial u_1}{\partial x} + v_1\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \tag{21}$$

$$\frac{\partial v_1}{\partial t} + U\frac{\partial v_1}{\partial x} = -g - \frac{1}{\rho}\frac{\partial p}{\partial y} \tag{22}$$

and linearized surface conditions are

$$\left.\begin{aligned}\frac{\partial \eta}{\partial t} + U_1\frac{\partial \eta}{\partial x} &= v_1 \qquad (23)\\ \frac{\partial p}{\partial t} + U_1\frac{\partial p}{\partial x} &= v_1\rho g \qquad (24)\end{aligned}\right\} \quad \text{at } y=h$$

where U_1 is the current velocity at $y=h$ as already mentioned.

The stream function ψ is introduced as

$$u_1 = -\frac{\partial \psi}{\partial y} \qquad v_1 = \frac{\partial \psi}{\partial x} \tag{25}$$

Elimination of p between (21) and (22) leads to the equation

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\nabla^2\psi - U_1 f''(y)\frac{\partial \psi}{\partial x} = 0 \tag{26}$$

in which $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ and the primes denote differential with respect to y.

Putting

$$\psi = \phi(y)e^{im(x-ct)} \tag{27}$$

and substituting (27) in (26)

$$\frac{\partial^2\phi}{\partial y^2} - \left(m^2+ \frac{U_1 f''(y)}{U_1 f(y)-c}\right)\phi = 0 \tag{28}$$

As $\phi(0)=0$ from the bed condition (7), the solution of eq.(28) is

$$\phi(y) = A \sinh my + \frac{U_1}{m}\int_0^y \frac{f''(y)\sinh m(y-t)}{U_1 f(t)-C}\,\phi(t)\,dt \tag{29}$$

where A is constant to be determined.

Substituting $\phi(y)= \sum_0^\infty (U_1/\sqrt{gh})^n\cdot\phi_n(y)$ in eq.(29), and equating coefficients, the solution becomes as follows to the first order of $U_1/\sqrt{gh}$.

$$\phi(y) = A\left\{\sinh my + \frac{U_1}{m}\int_0^y \frac{f''(y)\sinh m(y-t)\sinh mt}{U_1 f(t)-C}dt\right\} \tag{30}$$

By putting $\eta=\frac{H}{2}\cdot e^{im(x-ct)}$, A is obtained from (23) as,

$$A = \frac{H}{2}\cdot\frac{U_1-C}{\sinh mh\left\{1+\frac{U_1}{m}\int_0^h \frac{f''(t)\sinh m(h-t)\sinh mt}{(U_1 f(t)-C)\sinh mh}\,dt\right\}} \tag{31}$$

Now the current field is assumed to be logarithmic in the main part and to be linear in the viscous sublayer. Thus,

$$U = \begin{cases}\left(5.5+\frac{1}{\kappa}\ln\frac{U_* y}{\nu}\right)U_*=U_1+\frac{U_*}{\kappa}\ln\frac{y}{h} & (y_s \leq y\leq h) \qquad (32)\\ \frac{U_*^2}{\nu}\,y & (0 \leq y\leq y_s) \qquad (33)\end{cases}$$

where κ is the Kármán's constant (=0.4), U_* is the shear velocity, ν is the kinematic viscosity coefficient and y_s is the depth of viscous sublayer.

$$y_s = 11.6\,\frac{\nu}{U_*} \tag{34}$$

Then, from (30), (31) and (32),

$$\phi(y) = \frac{H}{2}\cdot(U_1 - C)\cdot\frac{\sinh my}{\sinh mh}\cdot F_2(y) \tag{35}$$

$$F_2(y) = \frac{1+\dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{y}\frac{\sinh m(y-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2\sinh my}\,dt}{1+\dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2\sinh mh}\,dt} \tag{36}$$

From (25), (27) and (35), u_1 and v_1 are obtained as follows,

$$u_1 = \frac{H}{2}\cdot\frac{m(C-U_1)\cosh my}{\sinh mh}\cdot F_1(y)\cos m(x-Ct) \tag{37}$$

$$v_1 = \frac{H}{2}\cdot\frac{m(C-U_1)\sinh my}{\sinh mh}\cdot F_2(y)\sin m(x-Ct) \tag{38}$$

where

$$F_1(y) = \frac{1+\dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{y}\frac{\cosh m(y-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2\cosh my}\,dt}{1+\dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2\sinh mh}\,dt} \tag{39}$$

Elimination of p between (21) and (24) leads to the following equation.

$$(C-U_1)\frac{\partial\phi}{\partial y}\Big|_{y=h} + \{(C-U_1)\frac{\partial U}{\partial y}\Big|_{y=h} - g\}\phi_{y=h} = 0 \tag{40}$$

Substituting (35) in (40), C is obtained as

$$C = U_1 - \frac{U_*\tanh mh}{2\kappa mhF_1(h)} \pm \sqrt{\left(\frac{U_*\tanh mh}{2\kappa mhF_1(h)}\right)^2 + \frac{g\tanh mh}{m\,F_1(h)}} \tag{41}$$

Using eq.(10), the group velocity C_G is derived as

$$C_G = U_1 + \frac{(C-U_1)}{g+\{1+D_1(C-U_1)\}\{g-(C-U_1)(U_*/\kappa h)\}}\Big[\{2mh\,\mathrm{cosech}\,2mh + D_1(C-U_1) - D_2\}\{g-(C-U_1)\frac{U_*}{\kappa h}\} + g\Big] \tag{42}$$

where

$$D_1 = \frac{\dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}^2 t^2}\,dt}{\sinh mh + \dfrac{U_*}{\kappa m}\displaystyle\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}\,dt}$$

$$-\frac{\frac{U_*}{\kappa m}\int_{y_s}^{h}\frac{\cosh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}^2 t^2}dt}{\cosh mh+\frac{U_*}{\kappa m}\int_{y_s}^{h}\frac{\cosh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt} \tag{43}$$

$$D_2 = \frac{\cosh^2 mh+\frac{U_*}{2\kappa}\int_{y_s}^{h}\frac{t\cosh 2m(h-t)-(h-t)\cosh 2mt+h}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt}{\cosh^2 mh+\frac{U_*}{\kappa m}\cosh mh\int_{y_s}^{h}\frac{\cosh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt}$$

$$-\frac{\sinh^2 mh+\frac{U_*}{2\kappa}\int_{y_s}^{h}\frac{t\cosh 2m(h-t)+(h-t)\cosh 2mt-h}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt}{\sinh^2 mh+\frac{U_*}{\kappa m}\sinh mh\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt} \tag{44}$$

Evaluation of $\bar{E}_{dI}$ is now capable by putting (19) and (37) into (20) and approximating δ as y_s. Thus,

$$G_1 = \frac{U_*^3}{2\kappa gh}\left\{\frac{m(C-U_1)}{\sinh mh+\frac{U_*}{\kappa m}\int_{y_s}^{h}\frac{\sinh m(h-t)\,\sinh mt}{\{C-U_1-(U_*/\kappa)\ln(t/h)\}t^2}dt}\right\}^2$$

$$\times\Big[\int_{y_s}^{h}(h-y)y\{(\frac{\kappa m}{U_*}+\frac{1}{m\{C-U_1-(U_*/\kappa)\ln(y/h)\}y^2})\sinh my +$$

$$+\int_{y_s}^{y}\frac{\sinh m(y-t)\,\sinh m\,t}{\{C-U_1-(U_*/\kappa)\ln(y/h)\}t^2}dt\}^2 dy$$

$$+\frac{1}{2}\frac{\kappa}{U_*}(h-y_s)y_s(\frac{\kappa m}{U_*}+\frac{1}{m\{C-U_1-(U_*/\kappa)\ln(y_s/h)\}y_s^2})\sinh 2my_s\Big] \tag{45}$$

DERIVATION OF WAVE DISTURBANCE IN THE BOUNDARY LAYER

In the boundary layer, eq.(2)' can be linearized as,

$$\frac{\partial u}{\partial t}+U\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{1}{\rho}\frac{\partial \tau_w}{\partial y} \tag{46}$$

Putting eq.(17) into (46) and using eq.(21), the following equation is derived.

$$\frac{\partial u_2}{\partial t}+U\frac{\partial u_2}{\partial x} = \frac{1}{\rho}\frac{\partial}{\partial y}(\rho\varepsilon\frac{\partial u_2}{\partial y}) \tag{47}$$

in which v_2 and $\partial u_1/\partial y$ are neglected.

Substituting $u_2=\hat{u}_2 e^{i(mx-\omega t)}$ in (47)

$$\frac{d}{dy}(\varepsilon\frac{\partial \hat{u}_2}{\partial y})+i\omega(1-\frac{U}{C})\hat{u}_2 = 0 \tag{48}$$

Expanding $\hat{u}_2$ as a power series of U_1/C and substituting in (48), the following equations are obtained.

$$\begin{cases} \dfrac{d}{dy}\left(\varepsilon\dfrac{d\hat{u}_{20}}{dy}\right) + i\omega\hat{u}_{20} = 0 & (49) \\ \dfrac{d}{dy}\left(\varepsilon\dfrac{d\hat{u}_{21}}{dy}\right) + i\omega\hat{u}_{21} = i\omega\left(\dfrac{U}{U_1}\right)\hat{u}_{20} & (50) \end{cases}$$

From eq.(18), ε is assumed as

$$\varepsilon = \begin{cases} \nu & \text{for } 0 \leq y \leq y_s \quad (51) \\ \kappa U_* y & \text{for } y_s \leq y \quad (52) \end{cases}$$

because $y/h \ll 1$.

Substituting in (49) and (50), the equations to be solved are

$$\begin{cases} \dfrac{d^2\hat{u}_{20}}{dy^2} + \dfrac{i\omega}{\nu}\hat{u}_{20} = 0 & \text{for } 0 \leq y \leq y_s \quad (53) \\ \dfrac{d^2\hat{u}_{20}}{dy^2} + \dfrac{1}{y}\dfrac{d\hat{u}_{20}}{dy} + \dfrac{i\omega}{\kappa U_*}\dfrac{1}{y}\hat{u}_{20} = 0 & \text{for } y_s \leq y \quad (54) \end{cases}$$

$$\begin{cases} \dfrac{d^2\hat{u}_{21}}{dy^2} + \dfrac{i\omega}{\nu}\hat{u}_{21} = \dfrac{i\omega}{\nu}\dfrac{U}{U_1}\hat{u}_{20} & \text{for } 0 \leq y \leq y_s \quad (55) \\ \dfrac{d^2\hat{u}_{21}}{dy^2} + \dfrac{1}{y}\dfrac{d\hat{u}_{21}}{dy} + \dfrac{i\omega}{\kappa U_*}\dfrac{1}{y}\hat{u}_{21} = \dfrac{i\omega}{\kappa U_*}\dfrac{1}{y}\dfrac{U}{U_1}\hat{u}_{20} & \text{for } y_s \leq y \quad (56) \end{cases}$$

The boundary conditions are

$$\hat{u}_{20} = -\hat{u}_1(0) = -\hat{u}_o \qquad \text{at } y=0 \qquad (57)$$

$$\hat{u}_{21} = 0 \qquad \text{at } y=0 \qquad (58)$$

where

$\hat{u}_{20}$, $d\hat{u}_{20}/dy$, $\hat{u}_{21}$, $d\hat{u}_{21}/dy$ are to be continuous at $y=y_s$ respectively (59)

In addition to them, $\hat{u}_{20}$ and $\hat{u}_{21}$ must be bounded at large y.

The solutions become finally as,

$$u = u_1 + u_2 = u_1 + R_e\{(\hat{u}_{20}+(U_1/C)\hat{u}_{21})e^{i(mx-\omega t)}\}$$

$$= \begin{cases} u_1 - \hat{u}_o R_e\Bigg\{\Bigg[\dfrac{H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-y)+\dfrac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-y)}{H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s+\dfrac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s} \\ -\dfrac{\beta_1}{\{H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s+\dfrac{\beta_1}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s\}^2}\{(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-y) \\ +\dfrac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-y))\displaystyle\int_0^{y_s}\left(\dfrac{U}{C}\right)(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-\xi) \\ +\dfrac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-\xi))\sin\beta_1\xi\, d\xi \end{cases}$$

$$+(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s)\int_y^{y_s}(\frac{U}{C})(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-\xi) + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-\xi))\sin\beta_1(y-\xi)d\xi\}\Big]e^{i(mx-\omega t)}\Big\}$$

for $0 \leqq y \leqq y_s$ (60)

$$u_1 - \hat{u}_0 R_e\Bigg\{\Bigg[\frac{H_0^{(1)}(2\beta_2\sqrt{y})}{H_0^{(1)}(2\beta_2\sqrt{y})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s} - \beta_1\frac{H_0^{(1)}(2\beta_2\sqrt{y})\int_0^{y_s}(\frac{U}{C})(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-\xi) + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-\xi))\sin\beta_1\xi d\xi}{\{H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s\}^2}$$

$$-\frac{i\pi\beta_2^2}{2}\frac{\int_{y_s}^{y}(\frac{U}{C})H_0^{(1)}(2\beta_2\sqrt{\xi})(H_0^{(1)}(2\beta_2\sqrt{y})H_0^{(2)}(2\beta_2\sqrt{\xi}) - H_0^{(2)}(2\beta_2\sqrt{y})H_0^{(1)}(2\beta_2\sqrt{\xi}))d\xi}{H_0^{(1)}(2\beta_2\sqrt{y})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s}\Bigg]e^{i(mx-\omega t)}\Bigg\}$$

for $y_s \leqq y$ (61)

where

$$\beta_1 = \sqrt{\frac{\omega}{\nu}}e^{i\frac{\pi}{4}} \qquad \beta_2 = \sqrt{\frac{\omega}{\kappa U_*}}e^{i\frac{\pi}{4}} \tag{62}$$

in which $H_\nu^{(1)}(z)$ and $H_\nu^{(2)}(z)$ are the first and the second kind of Hankel function respectively.

From eq.(16), (17) and (18),

$$\bar{E}_{dB} = -\overline{\int_0^\delta u_1\frac{\partial}{\partial y}(\rho\varepsilon\frac{\partial u_2}{\partial y})dy} - \overline{\int_0^\delta u_2(\frac{\partial}{\partial t}+U\frac{\partial}{\partial x})u_2 dy} \tag{63}$$

in which the second term is derived from eq.(47) and vanishes due to the wave periodicity. Assuming that $\partial u_1/\partial y$ is approximately equal to zero in the layer and $\partial u_2/\partial y$ is approximately zero at the outer edge of the layer, $\bar{E}_{dB}$ becomes from eq.(63) as,

$$\bar{E}_{dB} = \mu u_1\frac{\partial u_2}{\partial y}\Big|_{y=0} = \frac{1}{4}\rho g H_1^2\, G_2 \tag{64}$$

$$G_2 = \frac{\nu}{2g}\{\frac{m(C-U_1)F_1(0)}{\sinh mh}\}^2 R_e\{\beta_1 \frac{H_0^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s - \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s}{H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s}$$

$$- \beta_1^2 \frac{\int_0^{y_s}(\frac{U}{C})(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1(y_s-\xi) + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1(y_s-\xi))^2 d\xi}{(H_0^{(1)}(2\beta_2\sqrt{y_s})\cos\beta_1 y_s + \frac{\beta_2}{\beta_1\sqrt{y_s}}H_1^{(1)}(2\beta_2\sqrt{y_s})\sin\beta_1 y_s)^2}\} \tag{65}$$

Thus, the rate of the wave height variation can be calculated from eq.(14) by using eq.(42) for C_G, eq.(45) for G_1 and (65) for G_2 , in which

$$G = G_1 + G_2 \tag{66}$$

is used.

EXPERIMENT

A glass walled flume, 20m long, 0.8m wide and 0.5m high shown in Fig.3 was used for experiments. Flow was circulated and regular waves were generated by a plunger type generator from the most downstream end. Velocities were measured by a propeller type current-meter in which electric pulses were transduced by a photo-electric cell for each rotation of the propeller. An electric resistance type wave gauge and an electro-magnetic oscillograph were used to record water surface. To avoid errors due to difference of characteristics between different gauges, only one wave gauge was used to measure waves at different points, for which a gauge carrier on wheels was used. Wave heights at each points were determined by averaging those of eight waves which become steady just after the beginning of movement of the generator.

As shown in Table-1, surface velocities were between 6 cm/sec and 33 cm/sec and undisturbed depth was in the range between 10 and 15 cm. Wave periods were 0.85~1.25 sec and wave heights were 1~4 cm. Water temperature was adjusted between 13°C and 16°C.

Fig.4 shows the wave height variation with travel distance, which is exponential as shown eq.(14).

Fig.5 shows the relationship between the rate of wave height attenuation $\alpha L_0 = \frac{G}{C_G}L_0$ and U_1/C_0 in which C_0 and L_0 are the wave celerity and the wave length without current. Full lines represent theoretical results for the water temperature of 15°C. White circles are experimental values.

It is shown that the rate of wave height attenuation becomes larger as the opposing current velocity increases. The attenuation rate due to boundary shear α_2 is nearly constant over full range of current velocity. Although as shown by $\alpha_1 L_0$ effect by internal friction can be neglected in compare with that by boundary shear when the current velocity is small, the former becomes remarkable as the opposing current velocity increases, which dominates tendency of total dissipation.

However evaluation of energy dissipation in the boundary layer is too low which suggests turbulence due to wave motion must be analysed.

Fig.5 shows velocity distributions calculated in the boundary layer.

As given in eq.(31) and (32), discontinuity is shown at $y=y_s$. However y_s is too small to give serious effects by this continuity.

CONCLUSION

Mechanics of energy dissipation of waves traveling against currents are analysed theoretically in this paper. The rate of energy dissipation or wave height attenuation is given physical meaning by eq.(14). This is the ratio of the dissipated energy and the energy transfer velocity.

Energy dissipation in the boundary layer is nearly constant over full range of current velocity. Effect of internal eddy viscosity is negligible when current velocity is small, but become remarkable as the opposing current velocity increases. In this case, this internal eddy viscosity dominates thendency of total dissipation.

The effect of the wave motion on turbulence in the boundary layer is not treated in this paper. However experimental results showed they should be taken into consideration.

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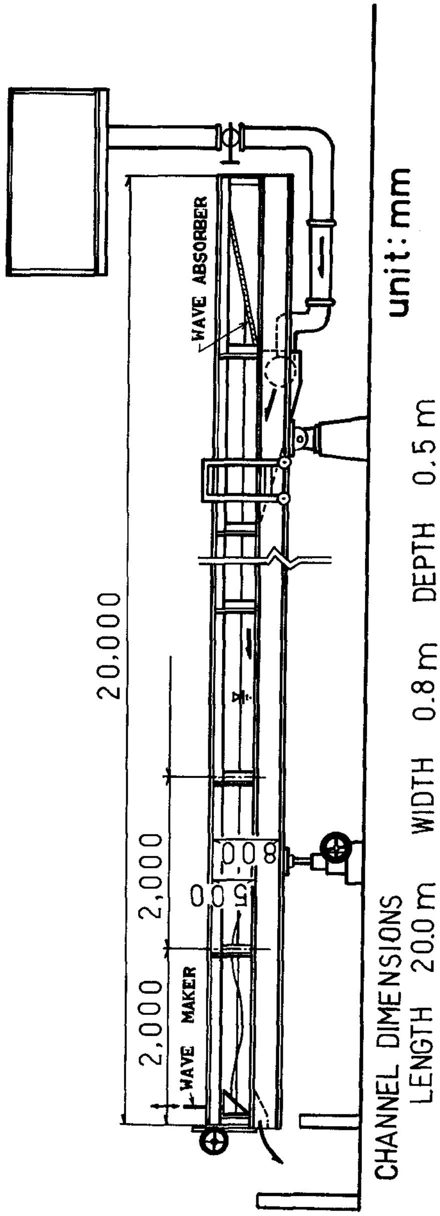


Fig.3 Experimental apparatus

Table-1

h (cm)	T (sec)	U_1 (cm/sec)	H_0 (cm)	α (cm^{-1})	Temp. (°C)
10.0	0.85	6.3	2.02	0.79×10^{-4}	14.2
10.0	0.94	6.4	1.45	2.13	15.3
10.0	0.94	6.4	2.16	1.85	15.3
10.0	0.94	12.0	1.53	1.91	14.4
10.0	0.96	33.2	1.63	4.51	14.3
11.4	1.02	16.1	1.91	1.37	16.0
10.0	1.05	12.0	1.63	3.29	14.4
11.4	1.05	23.1	3.15	4.26	16.0
10.0	1.07	6.3	2.42	2.27	14.2
10.0	1.08	33.2	1.66	4.53	14.3
10.0	1.10	10.4	1.48	2.11	13.2
10.0	1.10	10.4	2.04	1.68	13.2
12.0	1.15	23.1	2.71	3.15	16.0
12.0	1.15	23.1	3.33	3.77	16.0
11.5	1.15	27.1	2.92	4.89	10.8
10.0	1.25	10.4	1.70	2.54	13.2
10.0	1.25	10.4	2.13	1.81	13.2
10.0	1.24	33.2	1.38	6.42	14.3
15.0	0.85	6.3	2.34	0.52	14.1
15.0	0.85	13.1	1.37	2.09	14.9
15.0	0.86	14.6	2.36	2.14	14.3
15.0	0.84	19.5	1.33	2.68	15.1
15.0	0.84	19.5	2.12	3.48	15.1
15.0	0.83	20.3	1.49	2.97	15.0
15.0	0.85	29.4	1.36	2.76	15.0
15.0	0.95	6.3	1.91	1.74	14.1
14.8	0.94	13.1	1.21	1.54	13.1
15.0	0.95	14.6	1.95	2.67	14.3
15.0	0.94	16.9	2.07	3.88	12.4
15.0	0.94	19.5	1.37	2.42	15.1
15.0	0.93	20.3	1.72	2.11	15.0
15.0	0.94	29.4	1.50	2.64	15.0
15.0	1.05	6.3	2.16	1.91	14.1
15.0	1.06	13.1	1.20	1.61	15.0
15.0	1.05	19.5	1.44	2.75	15.1
15.0	1.06	20.3	1.86	2.71	15.0
15.0	1.08	14.6	1.47	3.00	14.3
15.0	1.08	16.9	3.46	2.72	12.4
15.0	1.23	6.3	2.00	0.91	14.1
15.0	1.24	13.1	1.11	1.33	15.0
15.0	1.26	16.9	2.15	2.93	12.4
15.0	1.24	19.5	1.24	1.08	15.1
15.0	1.23	20.3	2.51	2.11	15.0

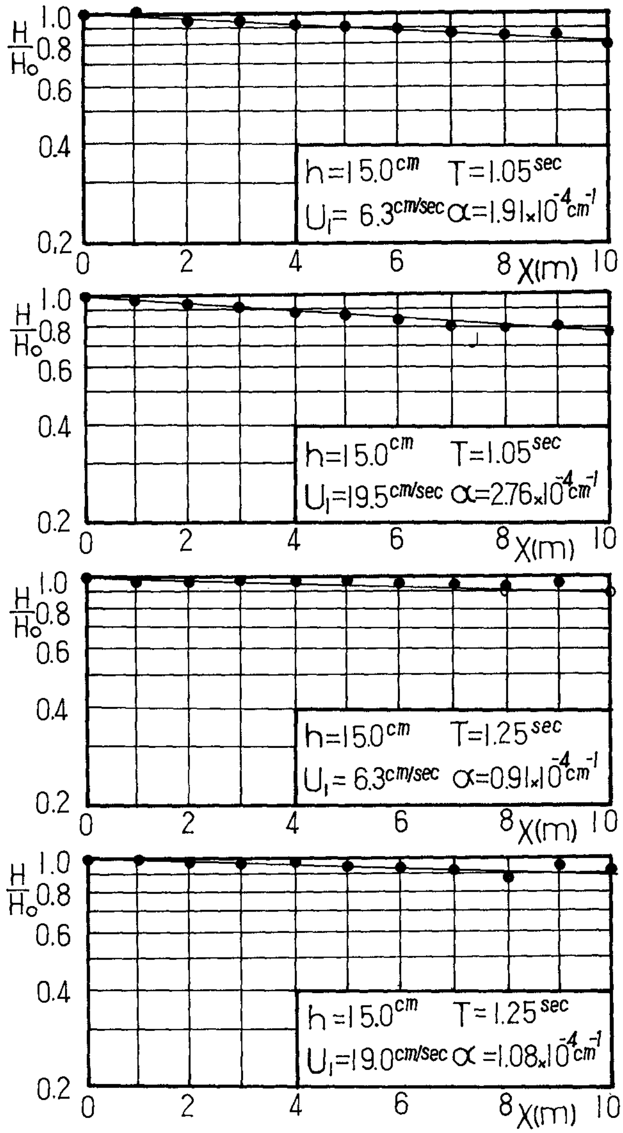
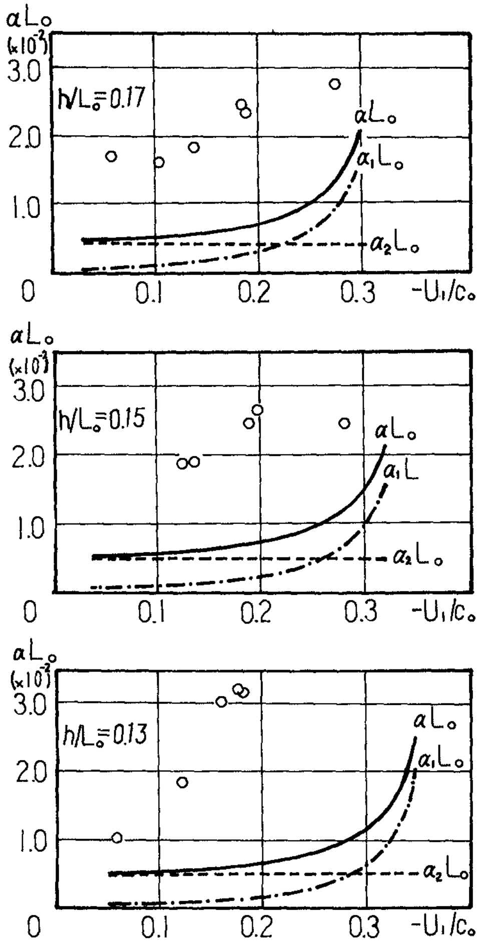


Fig.4

Wave height variation with distance


αLo
(x10⁻²)
3.0
2.0
1.0
0
h/Lo=0.17
αLo
α1Lo
α2Lo
0.1
0.2
0.3
-U1/co
αLo
(x10⁻²)
3.0
2.0
1.0
0
h/Lo=0.15
αLo
α1L
α2Lo
0.1
0.2
0.3
-U1/co
αLo
(x10⁻²)
3.0
2.0
1.0
0
h/Lo=0.13
αLo
α1Lo
α2Lo
0.1
0.2
0.3
-U1/co

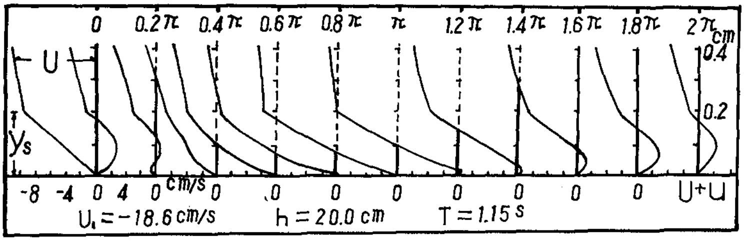


Fig.6

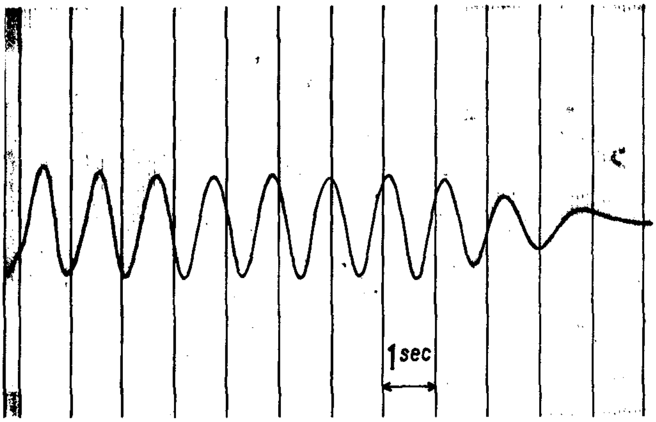


Fig.7

Example of wave record

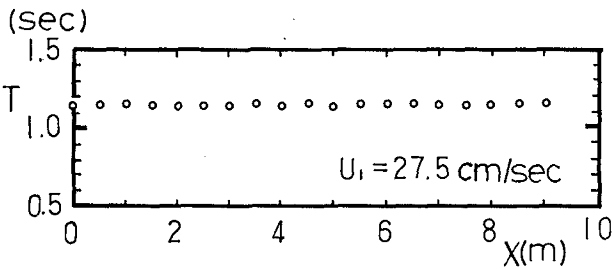


Fig.8

Wave period variation with travel distance