

CHAPTER 20

AIRY WAVE THEORY AND BREAKER HEIGHT PREDICTION

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ABSTRACT

Using a critical value for $\gamma_b = H_b / h_b$ as a wave breaking criterion, where H_b and h_b are respectively the wave breaker height and depth, applying Airy wave theory, and assuming conservation of the wave energy flux, one obtains

$$H_b = k g^{1/5} (T H_\infty^2)^{2/5}$$

relating H_b to the wave period T and to the deep-water wave height H_∞ . Three sets of laboratory data and one set of field data yield $k = 0.39$ for the dimensionless coefficient.

The relationship, based on Airy wave theory and empirically fitted to the data, is much more successful in predicting wave breaker heights than is the commonly used equation of Munk, based on solitary wave theory. In addition, the relationship is applicable over the entire practical range of wave steepness values.

INTRODUCTION

Engineers and scientists interested in the nearshore region often find it necessary to calculate the expected breaker heights of a wave train from its deep-water characteristics. Wave forecast procedures, for example, yield estimates of the deep-water wave height, H_∞ , and period T . From these values it is desirable to estimate the heights of these waves when they arrive and break on a particular beach.

The procedure that is commonly followed (CERC Tech. Report No. 4, 1966) is to utilize the theoretical equation

$$H_b = \frac{H_\infty}{3.3 (H_\infty / L_\infty)^{1/3}} \quad (1)$$

or its graphical equivalent, where H_b is the breaker height. This relationship, introduced by Munk (1949), is based on an evaluation of the breaker wave energy and celerity with the theoretical solitary wave equations of Boussinesq (1877). The application of the solitary wave was suggested by the obvious resemblance between the theoretically derived solitary wave profile and the observed profiles of oscillatory waves nearing the breaker zone.

Several studies, such as those of Ippen and Kulin (1954) and Kishi and Saeki (1966), have demonstrated that when a solitary wave travels up an inclined slope, as it would in approaching a beach, the observed changes in amplitude, celerity, wave profile, etc., deviate markedly from the theoretical values determined from solitary wave theory. Such results, plus the usual doubt in applying solitary wave theory to periodic oscillatory waves, casts doubts on the foundations and applicability of equation (1).

The purpose of this paper is to examine the applicability of linear Airy wave theory to evaluate wave breaker heights. Such an application is encouraged by the recent successes of Airy theory in examining longshore current generation (Bowen, 1969; Longuet-Higgins, 1970). In addition, based on the fit of the wave theories to the free surface boundary conditions, Dean (1970) determined that Airy wave theory may be applicable to a wide range of near breaking conditions.

Using Airy wave theory, a new relationship is deduced from which the breaker height H_b can be predicted from the deep-water wave parameters H_∞ and T . As will be seen, the resulting relationship can predict with excellent success breaker heights that agree with those observed.

THEORETICAL DEVELOPMENT

In his derivation of equation (1), Munk (1949) made use of the conservation of energy flux

$$(E Cn)_b = (E Cn)_\infty \quad (2)$$

where E is the wave energy and Cn is the wave group velocity, the rate at which the energy travels. This links the deep-water wave conditions (denoted by the subscript ∞) to the wave breaking parameters (denoted by the subscript b). As written, equation (2) does not include the effects of refraction. Munk evaluated the deep-water

parameters with Airy wave theory and applied solitary theory to the breaking wave. In this paper, Airy wave theory will be used for both the deep-water and breaking wave conditions. The energy of the breaking wave then becomes

$$E_b = \frac{1}{8} \rho g H_b^2 \quad (3)$$

where ρ is the density of water, and the celerity in shallow water is given by

$$C_b = \sqrt{g h_b} \quad (4)$$

where h_b is the water depth at breaking.
Munk (1949) made use of the substitution

$$\gamma_b = H_b / h_b = 0.78 \quad (5)$$

for a breaking criterion. This value was determined theoretically by McCowan (1894) for solitary waves. Field measurements reported in Scripps Institution of Oceanography Wave Report 24 (1944) and again in Sverdrup and Munk (1946) confirm this value of γ_b for beaches with very low gradients. Several laboratory studies have demonstrated that γ_b actually varies with the beach slope, increasing as the slope increases. γ_b also varies somewhat with the deep-water wave steepness, H_∞ / L_∞ . In light of the poor showing of the solitary wave theory in studies such as those of Ippen and Kulin (1954) and Kishi and Saeki (1966) the success of γ_b as a breaking criterion must be fortuitous and cannot be taken as an indication of the success of the solitary wave theory as implied by Munk (1949).

Following Longuet-Higgins and Stewart (1964), Bowen et. al. (1968), Bowen (1969) and Longuet-Higgins (1970), we shall apply $\gamma = H/h$ as a similarity criterion without reference to the solitary wave theory. $\gamma_b = H_b / h_b$ will be accepted as a breaking criterion and used in conjunction with the Airy wave theory. This is commonly done in practice in computer programs for wave refraction.

Using γ_b as a breaking criterion, applying Airy theory, and assuming conservation of the energy flux (equation 2), one obtains the relationship

$$H_b = \left| \frac{\sqrt{g \gamma_b}}{4 \pi} T H_\infty^2 \right|^{2/5} \quad (6)$$

relating the breaker height H_b to the wave period T (assumed constant) and the deep-water wave height H_∞ . According to equation (6), if we plot H_b against $g^{1/5} (T H_\infty^2)^{2/5}$ we should obtain a straight line whose slope is dependent upon the value of γ_b . Since γ_b is known to vary with the beach slope, we might expect a separate straight line for each beach slope. However, the one-fifth power of γ_b is involved so that the expected variations in γ_b should not produce a very marked change in the line slope. A comparison with the data bears this out.

DATA TESTS

Three sets of extensive laboratory data and the one existent set of field data have been utilized to test the proposed relationship of equation (6).

The wave flume measurements of Komar and Simmons, collected in 1968, give a considerable range to the required wave parameters needed to test equation (6). This data, which has not been previously published, will be discussed fully in Gaughan (in prep). The technique of the study was very similar to that of the well-known study of Iverson (1951) and provided measurements of H_b and T and values of H_∞ computed from the wave height measurements in the constant depth portion of the wave channel. The corresponding measured values of H_b and the computed values of the parameter $g^{1/5} (T H_\infty^2)^{2/5}$ are plotted in Figure 1. It is seen that there is a good linear relationship as predicted by equation (6), which yields

$$H_b = 0.39 g^{1/5} (T H_\infty^2)^{2/5} \quad (7)$$

There is no apparent systematic dependence on the beach slope although the data extends over a range of slopes from 2 to 6 degrees.

The line slope value 0.39 corresponds to a $\gamma_b = 1.42$ in equation (6). This γ_b value is higher than those actually measured by Komar and Simmons (which ranged from 0.7 to 1.1). Apparently the line fitted to the data must empirically correct for the fact that Airy wave theory, when applied to the amplitude changes of a shoaling oscillatory wave near breaking, tends to give a predicted height lower than observed.

In Figure 2 are plotted the laboratory measurements of Iverson (1951), generally considered to be the best available laboratory data. The straight line shown is the same as that of Figure 1 and given by equation (7), established by the data of Komar and Simmons. It is apparent that there is good agreement between the two sets of data in establishing equation (7). The trend of the Iverson data does demonstrate a systematic dependence on the beach face slope, the higher gradient giving a somewhat higher line slope. Such a dependence is expected from the observed changes in γ_b with variations in the beach slope. Iverson found a much greater variation in γ_b with beach

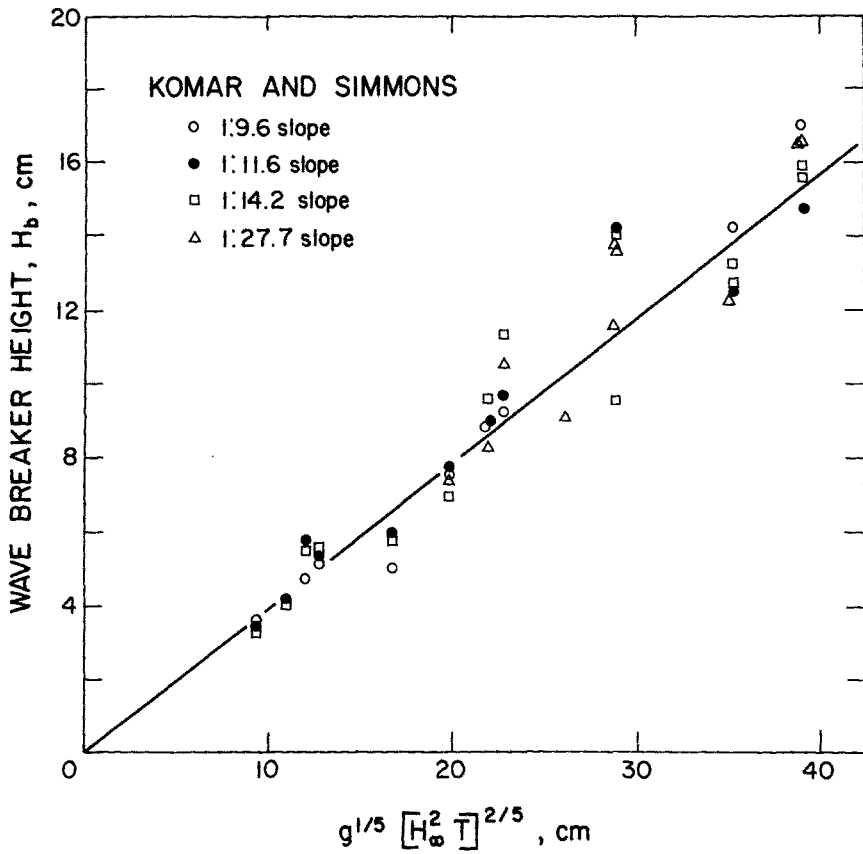


Figure 1: Laboratory breaking wave data of Komar and Simmons. The straight line, fitted by eye, has a slope of 0.39 and yields equation (7).

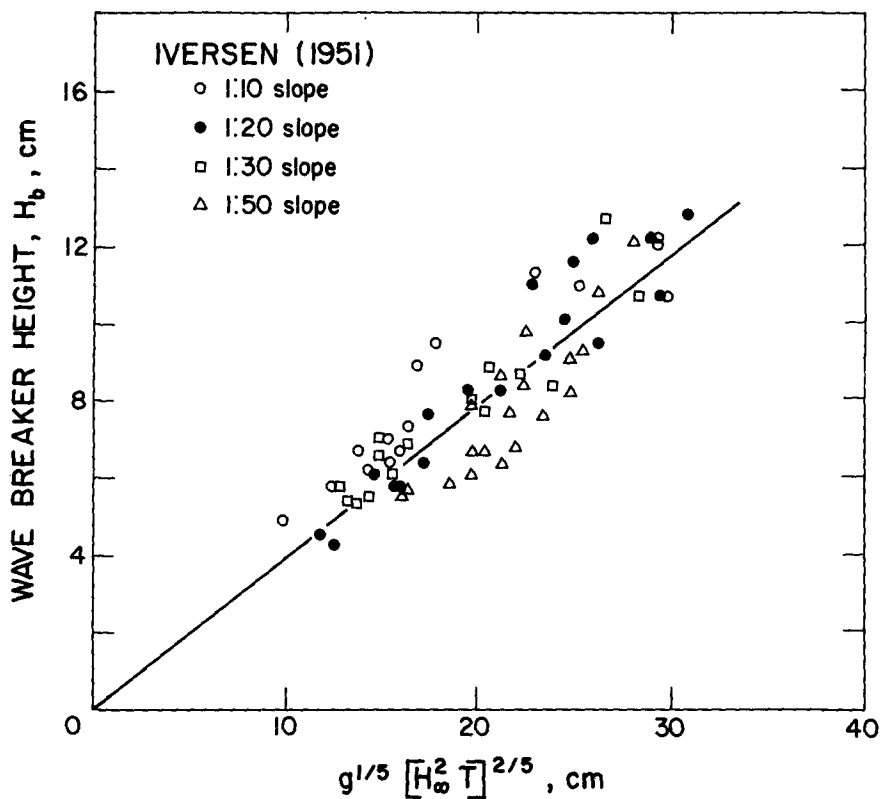


Figure 2: The straight line of equation (7) in comparison to the laboratory breaking wave data of Iversen (1951).

slope than did Komar and Simmons (see Gaughan, in prep) and this greater dependence is reflected in the plots of Figure 1 and 2.

The laboratory data of Galvin (1969) are also utilized to test equation (6); the results are shown in Figure 3. The data does not follow quite as well the straight line based on the data of Komar and Simmons. The main difficulty here is that Galvin defined his breaker heights differently than did Iverson or myself, and so should systematically plot above the line. In addition, he calculated the deep-water wave heights directly from the paddle stroke rather than from wave measurements in the constant depth portion of the channel; this may account for the increased scatter in the data plot. No systematic dependence on the beach slope appears.

The real test for the relationship of equation (6) comes in examining the available field data. The only field data of breaking waves suitable for such a test is that reported by Munk (1949). The data is referred to as Leica, Types 1 and 2. The Type 2 data represents waves which break behind a bar in water of increasing depth while the more normal breaking conditions are included in the Type 1 data. Both sets of field data are plotted in Figure 4 along with the highly extrapolated straight line of equation (7) obtained from Figure 1. The degree of agreement between the trend of the data and the straight line is remarkable in view of the degree of extrapolation involved. To illustrate the extent of the extrapolation, the data of Komar and Simmons and the field data of Munk are plotted together on a log-log graph in Figure 5.

Linear regression analysis of the above laboratory and field data yields the equation

$$H_b = 0.383 g^{1/5} (T H_\infty^2)^{2/5} + 0.73 \text{ cm} \quad (8)$$

with a sampling correlation coefficient of $r = 0.98$. This remarkably high value of the correlation coefficient confirms our visual approval of the correlation.

COMPARISON WITH SOLITARY WAVE THEORY

By using the relationship $L_\infty = g T^2 / 2\pi$ between the deep-water wave length L_∞ and the period T , equation (7) can be modified to the dimensionless form

$$H_b / H_\infty = \frac{0.56}{(H_\infty / L_\infty)^{1/5}} \quad (9)$$

which indicates that H_b is a function of the deep-water wave steepness

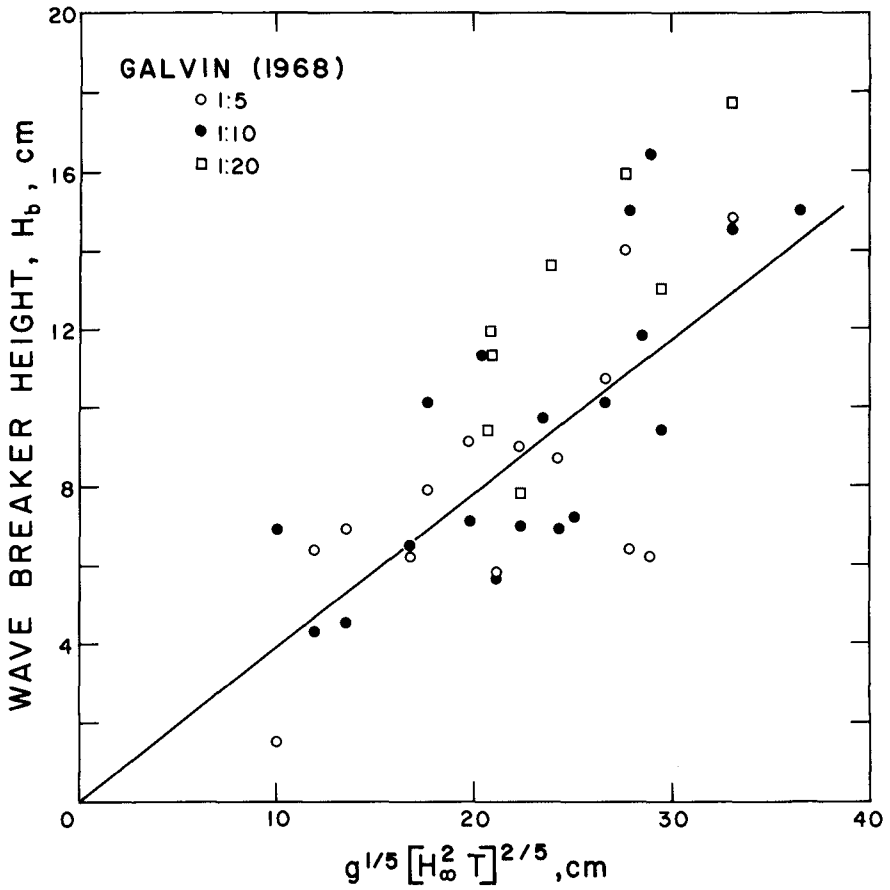


Figure 3: The straight line of equation (7) in comparison to the laboratory breaking wave data of Galvin (1968).

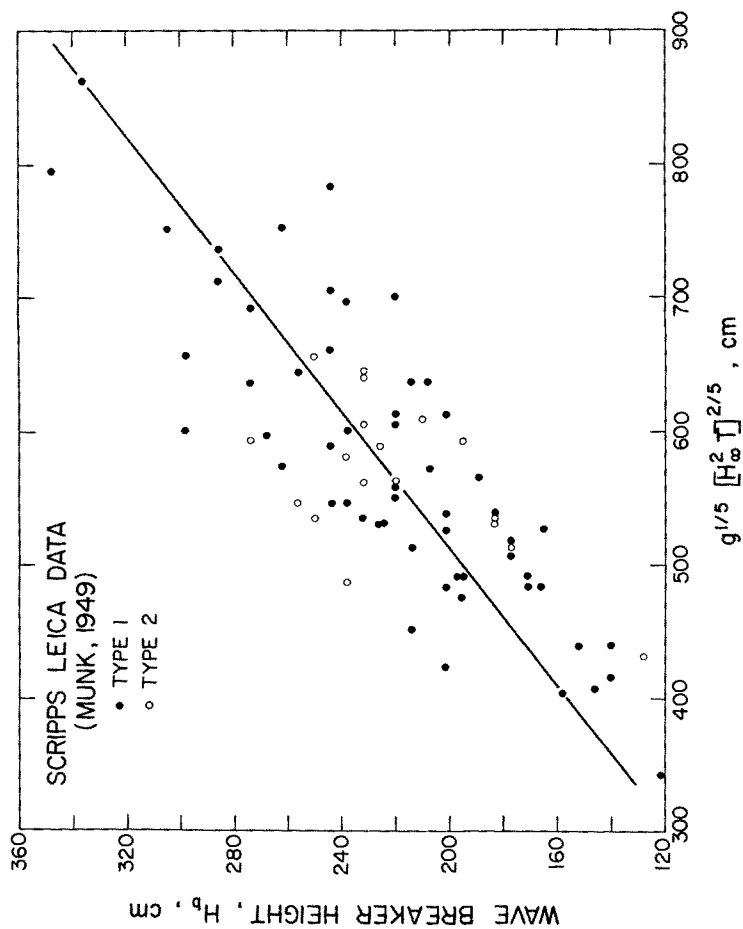


Figure 4: The straight line of equation (7) in comparison to the field breaking wave data reported in Munk (1949). The sampling correlation coefficient is 0.81.

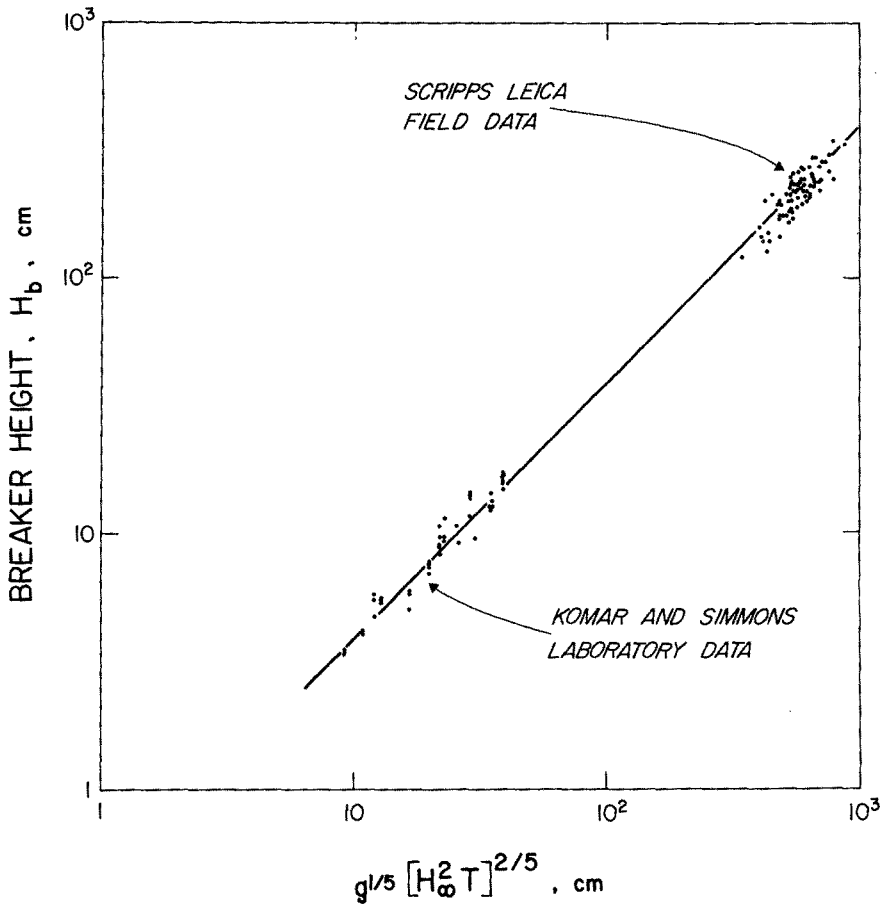


Figure 5: A log-log plot of the laboratory data of Komar and Simmons and the field data of Munk (1949). The straight line is that of equation (7).

H_{∞} / L_{∞} . This relationship is similar to equation (1) obtained by Munk (1949) using solitary wave theory, the principal difference being that H_{∞} / L_{∞} is to the $-1/5$ power rather than to the $-1/3$ power. It is also very close to the empirical equation of Le Mehaute and Koh (1967) which gives H_{∞} / L_{∞} to the $-1/4$ power.

Figure 6 is the well-known graph of H_b / H_{∞} versus H_{∞} / L_{∞} from Munk (1949) showing the line from solitary wave theory, equation (1), fitting the data best for low H_{∞} / L_{∞} values and a line at high wave slopes from regular Airy wave theory. Connecting the two, at intermediate values is an empirical line through the data. Superimposed on this graph is the line (solid) corresponding to equations (7) and (9). It is seen that this curve fits the data very well over the entire range of H_{∞} / L_{∞} values, nearly lying atop the empirical curve of Munk. Because of this success over the entire range of wave slope values, equation (9) should be very useful for engineering design and field application.

CONCLUSIONS

Both the available laboratory and field data support the relationship in equation (7), derived from Airy wave theory and the use of $\gamma_b = H_b / h_b$ as a breaking wave criterion. The relationship is successful over the entire practical range of wave steepness values and therefore is much more useful than the standard relationship derived from solitary wave theory by Munk (1949) which is limited only to small H_{∞} / L_{∞} values. The proposed relationship can also replace the empirical curve given by Munk for the intermediate range of wave slope values.

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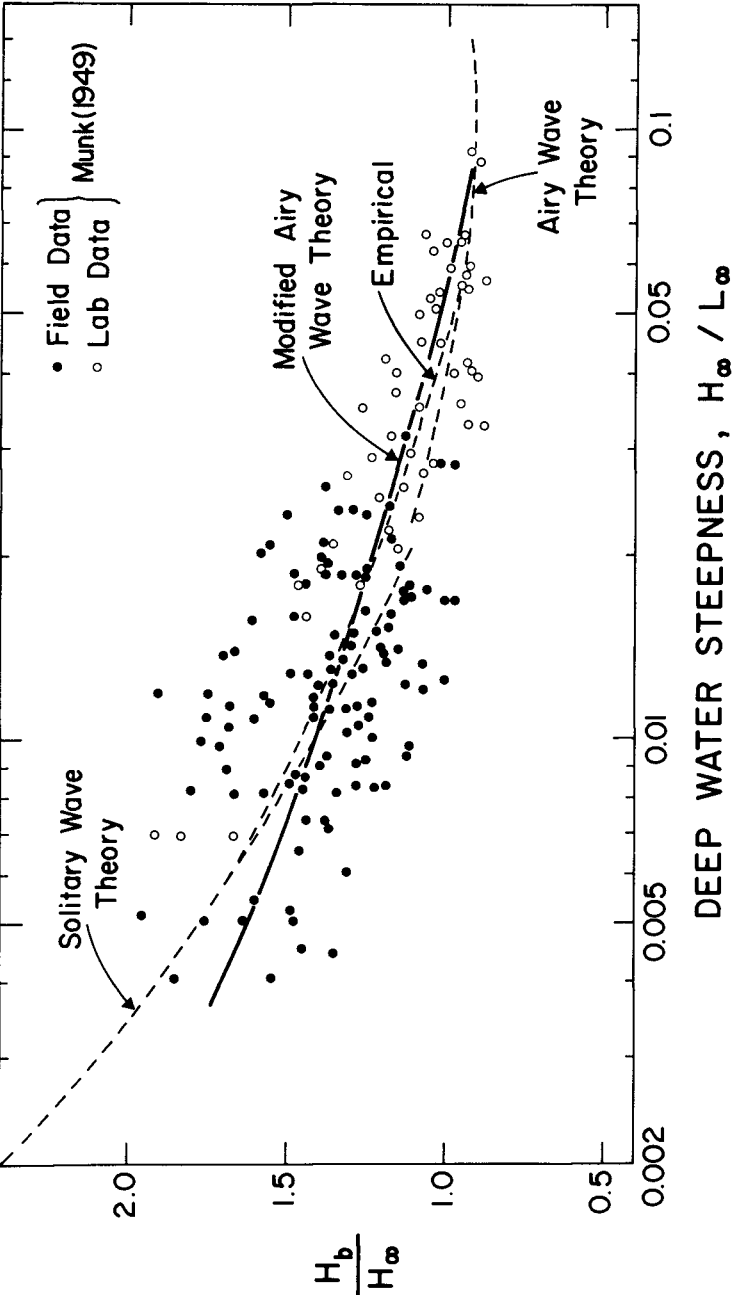


Figure 6: Wave breaker height related to the deep-water wave steepness (after Munk, 1949). The dashed curves are the theoretical and empirical curves given by Munk (1949). The solid curve corresponds to equation (9), the dimensionless form of equation (7).

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